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# Modeling the effects of climate change on the population dynamics of mosquitoes that are vectors of infectious diseases

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#### Abstract

We incorporate almost periodic functions in a mosquito model to take into account a loss of synchronicity in the population dynamics of mosquitoes due to climate change. The model takes into account the skip oviposition strategy that is associated with the mosquitoes that are vectors of infectious diseases as dengue, malaria and leishmaniasis. We prove existence and uniqueness of a stable almost periodic solution for some conditions over the parameters of the model. Numerical simulations are performed using values estimated for the life cycle of Aedes albopictus gathered in literature. The results show that the vector population can be underestimated or overestimated if an almost periodic dynamics is approximated by a periodic dynamics. Therefore, using an almost seasonal model can be more adequate to design breeding habitat-targeted mosquito control strategies when seasonal drivers are modeled since climate-mediated shifts can induce a loss of periodicity in environmental drivers.

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**Keywords:** Almost periodic function, cooperative systems, skip oviposition, vector-borne diseases.

## 1. Introduction

The population dynamics of many species that transmit infectious diseases is strongly related to the phenomenon of seasonality. Since the Hippocratic era, the relationship between the increase in the incidence of infectious diseases and seasonality has been emphasized. In this sense, the Corpus Hippocraticum mentions that the increase in infectious cases is related to environmental and social factors. For example, Hippocrates relates the appearance of influenza virus infection to the arrival of winter. It is widely accepted that seasonality can derive from changes in the environment, changes in the host behavior, changes in the host immune response, and the appearance and disappearance of particular pathogens. However, even if it is widely accepted that the appearance of new infectious cases has a seasonal component, we are still far from understanding the epidemiological and ecological mechanisms that generate seasonal diseases.

Knowing the population dynamics of species that transmit infectious diseases is of great interest to decision makers. Particularly, for decision makers, it is desirable to know the population dynamics of mosquitoes that are vectors of infectious diseases as dengue, zyka, chikungungya, yellow fever, leishmaniasis and malaria, among others [20, 3, 18]. In this sense, it is known that the species spreading these infectious diseases have a seasonal population dynamics. Commonly, seasonal phenomena are related in a natural way with periodic behavior; however, climate-mediated shifts have induced a loss of periodicity in environmental drivers such as weather. Seasonality allows the development and implementation of breeding habitat-targeted mosquito control strategies; however, if there is a loss of synchronicity due to climate-mediated shifts, then forecasting a vector population blowup may fail if the loss of periodicity is not considered.

Many models of mathematical ecology incorporate seasonality into population growth rates through forced oscillators [19, 6]. Seasonal models show that the resulting dynamics may be more varied and interesting. Seasonality is usually modeled through a  $\sin(t)$  function, even though it is clear that the ecological effects of seasonality on the phenomenon are far from having a pattern as simple as  $\sin(t)$ . As a possibility, it could be assumed that each factor involved is described by a function  $\sin(t)$  and that the periods of these functions are not synchronized. Then, the total effect produced by several factors can be incorporated into the model through a linear combination of those factors. In this sense, it is of paramount importance to understand how the duration and form of seasonal forcing have an effect over the dynamics of the species. The aim is to have models that incorporate more realistic characteristics of the phenomenon and are useful for decision makers when designing population control strategies.

In this work, we will analyze an almost periodically forced seasonal model using the theory of almost periodic functions [1]. We study a variant of a model for mosquito population given in [9]. However, such a model can also be used to describe a wide variety of species that show the skip oviposition strategy; such as, *Aedes aegypti* and *Aedes albopictus* that are transmitting vector of dengue, zyka, chikunguya and yellow fever; *Anopheles coluzzi*, *Anopheles gambie*, and *Anopheles arabiensis* that are transmitting vectors of malaria and *Phelebotomus papatasi* and *Phlebotomus orientalis* that are transmitting vectors of leishmaniasis [15, 24, 25, 20, 2]. To show the population dynamics given by the proposed model, we will use some values of the parameters estimated for the *Aedes albopictus* [5].

### 2. The Model

In this section, we explain the terms and the parameters used in the construction of the model proposed. At any given time, t, the mosquito population is divided into two compartments: the adult mosquito population that is denoted by A and the aquatic population (this class includes eggs, larvae and pupae) that is denoted by L. We will study the following model:

(2.1) 
$$\begin{aligned} \frac{dL}{dt} &= rb(t)\left(1-\frac{L}{K}\right)A - (\nu+\mu_L)L,\\ \frac{dA}{dt} &= \nu L - \mu A, \end{aligned}$$

where the sex ratio is denoted by r, the death rate of the aquatic class is denoted by  $\mu_L$ , the death rate of the female mosquito is denoted by  $\mu$ , the transition rate from the aquatic class to the adult class is denoted by  $\nu$ , b(t)is the fertility of a female mosquito and K is the larval carrying capacity.

is the fertility of a female mosquito and K is the larval carrying capacity. The term  $rb(t)\left(1-\frac{L}{K}\right)A$  in (2.1) describes the intraspecific competition that exists between the female mosquitoes. That is, this term models the competition for places where the female mosquitoes oviposit. As mentioned in the introduction, the mosquitoes of the species Aedes, Anopheles and Phlebotomus show a skip oviposition behavior. That is, the female mosquitoes seek oviposition places with low intraspecific competition and high food content. The population dynamics of these species of mosquitoes has been associated with a seasonal behavior because the time of breading and oviposition is strongly related to abiotic variables such as climate, temperature, humidity among other environment drivers [12, 24, 25, 14, 11, 18, 15, 13, 17, 3, 16]. Recently, the daily and seasonal temperature fluctuations have been object of studies to forecasting the evolution of the malaria [10]. Since some biotic variables are changing due to climate change and there is a loss of synchronicity in the ecological behavior of mosquitoes, we propose an almost periodic model to describe the population dynamics of some mosquitoes that are vectors of infectious diseases. To do this, we use an almost periodic function to describe the number of egg laid by female mosquitoes.

### 3. Almost periodic functions in cooperative systems

We refer the reader to [1, 4] for more details about almost periodic functions as well as to [21, 22] for a detailed description of competitive and cooperative systems.

**Definition 1.** We say that a continuous function  $\phi \in C^0(\mathbf{R})$  is almost periodic if given  $\varepsilon > 0$  there exists a set of translation numbers  $T(\varepsilon) \subseteq \mathbf{R}$ and a length  $l(\varepsilon) > 0$  such that  $I \cap T(\varepsilon) \neq \emptyset$  for any interval I whose length is  $l(\varepsilon)$ . Moreover,

$$|\phi(x+\tau) - \phi(x)| < \varepsilon, \qquad \forall x \in \mathbf{R}, \, \forall \tau \in I \cap T(\varepsilon).$$

For every  $\phi \in C_{ap}(\mathbf{R}) \subset C^0(\mathbf{R})$  in the subspace of almost periodic functions we have a Fourier series:

$$\phi \sim \sum_{n \in \mathbf{N}} c(\phi, \lambda_n) e^{i\lambda_n t}$$

as well as a mean

$$\mathcal{M}(\phi) := \lim_{T \to \infty} \frac{1}{T} \int_0^T \phi(t) \, dt,$$

which is a linear bounded functional,  $\mathcal{M} : \operatorname{AP}(\mathbf{R}) \to \mathbf{R}$ , which is positive in the following sense,  $\phi \geq 0$  implies  $\mathcal{M}[\phi] \geq 0$ . The corresponding Parseval's Theorem for almost periodic functions states that

$$\mathcal{M}\left[|\phi|^2\right] = \sum_{n \in \mathbf{N}} |c(\phi, \lambda_n)|^2.$$

We consider a planar system

(3.1) 
$$\begin{aligned} x'(t) &= u(t, x(t), y(t)) \\ y'(t) &= v(t, x(t), y(t)), \end{aligned}$$

in a domain  $(x, y) \in D \subseteq \mathbb{R}^2$ . We say that the C<sup>1</sup> functions, u(t, x, y) and v(t, x, y) are uniformly almost periodic with respect to  $(x, y) \in C$  for every compact  $C \subseteq D$ , if f (resp. g) has generalized Fourier series,

$$u(t, x, y) \sim \overline{u}(x, y) + \sum_{n \in \mathbf{N}} a(u, \lambda_n; (x, y)) \cos(\lambda_n t) + b(u, \lambda_n, (x, y)) \sin(\lambda_n t),$$

where the uniform mean  $\overline{u}(x, y) := \mathcal{M}[u(t, x, y)]$  as well as the frequencies  $\lambda_n$  do not depend on (x, y).

**Definition 2.** We will say that (3.1) is of cooperative type [21], if for every  $t \in \mathbf{R}$ ,

$$\frac{\partial u}{\partial y}(t,x,y) \ge 0, \ \frac{\partial v}{\partial x}(t,x,y) \ge 0$$

Moreover,  $(\alpha(t), \beta(t))$  is sub-solution if

(3.2)  $\xi'(t) \leq u(t,\xi(t),\eta(t))$ 

(3.3) 
$$\eta'(t) \leq v(t,\xi(t),\eta(t))$$

Analogously, we define a super-solution  $(\Xi(t), H(t))$  by reversing inequalities. A pair  $(\xi(t), \eta(t))$  and  $(\Xi(t), H(t))$  is ordered if

$$\xi(t) \le \Xi(t), \quad \eta(t) \le H(t), \quad \forall t \in \mathbf{R}.$$

The following result will be used and was proved in [7] as Theorem 2.

**Theorem 1.** Suppose that  $(\xi(t), \eta(t))$  and  $(\Xi(t), H(t))$  is a sub-super-solution ordered pair of the competitive ODE (3.1). Then there exists an almost periodic solution satisfying  $\xi(t) < x(t) < \Xi(t)$  and  $\eta(t) < x(t) < H(t)$ . The set of almost periodic solutions, having initial conditions in the rectangle  $\xi(0) < x(0) < \Xi(0)$  and  $\eta(0) < y(0) < H(0)$  is totally ordered, provided there is no equilibrium. If  $(\check{x}(t), \check{y}(t)), (\hat{x}(t), \hat{y}(t))$ , denote the minimal and maximal almost periodic solutions. Then

$$\check{x}(t) \le x(t) \le \hat{x}(t), \quad \check{x}(t) \le y(t) \le \hat{y}(t).$$

We just remark that in the case that there is an equilibrium point we could have an equilibrium, instead of a genuine almost periodic orbit.

### 4. Results

Given an almost periodic function  $u: \mathbf{R} \to \mathbf{R}$ , we denote

$$u_* := \inf_{t \in \mathbf{R}} u(t)$$
 and  $u^* := \sup_{t \in \mathbf{R}} u(t)$ 

**Lemma 1.** Any solutions of (2.1) starting in the positive quadrant  $\mathbf{R}^2_{>0}$  either approach, enter, or remain in the subset defined by

(4.1) 
$$\Gamma := \{ (L, A) \in \mathbf{R}^2 : 0 \le L \le K, \ 0 \le A \le \frac{\nu K}{\mu} \}.$$

**Proof.** Indeed, for any initial condition (L(0), A(0)) with  $L \ge K$ , in particular  $\frac{L}{K} \ge 1$ , then on the right side of L' we get

$$rb(t)\left(1-\frac{L}{K}\right)A - (\nu+\mu_L)L \le 0$$

thus L is decreasing. For the above, we can assume  $L \leq K$ . Now, we consider  $A \geq \frac{\nu K}{\mu}$ , then we get that A is decreasing. Therefore, any solution with initial conditions in  $\mathbf{R}^2_{>0}$  remains or eventually enters into region  $\Gamma$ .  $\Box$ 

Thus it suffices to consider solutions in the feasible region  $\Gamma$ .

Define the basic offspring number  $\mathcal{N}$  of system (2.1) when b is constant as

$$\mathcal{N} := rac{
u r b}{(
u + \mu_L) \mu}.$$

The basic offspring number represents the average number of larvae produced over the lifetime of an individual. Motivated by this, we consider  $\mathcal{M}(\mathcal{N})$  for system (2.1) as follows

$$\mathcal{M}(\mathcal{N}) := rac{
u r \mathcal{M}(b)}{(
u + \mu_L) \mu}.$$

We also consider

$$\mathcal{N}_* := rac{
u r b_*}{(
u + \mu_L)\mu}$$

Our main result gives conditions which warranty globally asymptotic convergence of the population towards an almost periodic solution whenever there exists an almost periodic forcing in the birth rate. Now we state our main result. **Theorem 1.** Assume  $r, \nu, \mu, \mu_L > 0, K > 1$  and b(t) a non constant continuous almost periodic function with  $b_* > 0$ . Suppose that

$$(4.2) \mathcal{N}_* > 1.$$

Then we get

- i) Under (4.2) there is, at least, one almost periodic solution (L, A) of (2.1) whose components are positive.
- ii) Under (4.2), the almost periodic solution is unique in  $\mathbf{R}^2_{>0}$ .
- iii) Any positive solution of (2.1), tends asymptotically to (L, A) when  $t \to \infty$ .

**Proof.** For i). We first observe that (2.1) is cooperative in  $\Gamma$ . We find a super-solution,

$$(\Xi(t), H(t)) = \left(K, \frac{\nu K}{\mu}\right).$$

These functions satisfy the inequalities.

(4.3) 
$$\Xi'(t) = 0 \geq rb(t) \left(1 - \frac{K}{K}\right) \left(\frac{\nu K}{\mu}\right) - (\nu + \mu_L)K$$

(4.4) 
$$H'(t) = 0 \ge \nu K - \mu \left(\frac{\nu K}{\mu}\right) = 0.$$

Therefore this is a super-solution.

To define a sub-solution, first we write  $K = 1 + \delta$  with  $\delta > 0$  and choose  $0 < \epsilon < \delta$  such that  $\mathcal{N}_* > 1 + \epsilon$ , doing

$$(\xi(t), \eta(t)) = \left(\epsilon, \frac{\nu(1+\epsilon)\epsilon}{\mu \mathcal{N}_*}\right).$$

Using  $\frac{1}{1+\delta} \ge \frac{1}{K}$ , these functions satisfy the inequalities in (3.2)

$$\begin{aligned} \xi'(t) &= 0 &\leq (\nu + \mu_L)\epsilon \left[ \frac{b(t)}{b_*} \left( 1 - \frac{\epsilon}{1+\delta} \right) (1+\epsilon) - 1 \right] \\ &\leq rb(t) \left( 1 - \frac{\epsilon}{K} \right) \left( \frac{\nu(1+\epsilon)\epsilon}{\mu \mathcal{N}_*} \right) - (\nu + \mu_L)\epsilon. \\ \eta'(t) &= 0 &\leq \nu\epsilon \left( 1 - \frac{1+\epsilon}{\mathcal{N}_*} \right). \end{aligned}$$

The right side is positive because  $\epsilon < \delta$ . Thus, we have a sub-solution pair. Therefore, by Theorem 1 the set of almost periodic solutions for system (2.1) with initial conditions in  $\Gamma$  is non-empty.

ii) For uniqueness, we consider a maximum  $(\hat{L}, \hat{A})$  and a minimum  $(\check{L}, \check{A})$  in the set of almost periodic functions of (2.1), ordered according to the partial ordered defined by the cooperative condition with initial condition within the region  $\Gamma$ . We just need to prove that  $\hat{L}(t) = \check{L}(t)$  and  $\hat{A}(t) = \check{A}(t)$ .

We get

(4.5) 
$$\hat{L} = rb(t)\hat{A} - r\frac{b(t)\hat{L}\hat{A}}{K} - (\nu + \mu_L)\hat{L}, \dot{\check{L}} = rb(t)\check{A} - r\frac{b(t)\check{L}\check{A}}{K} - (\nu + \mu_L)\check{L},$$

and

(4.6) 
$$\begin{aligned} A &= \nu L - \mu A, \\ \dot{\check{A}} &= \nu \check{L} - \mu \check{A}. \end{aligned}$$

We consider the Lyapunov type function  $V: \mathbf{R}^4_{>0} \to \mathbf{R}$ ,

$$V(L_1, A_1, L_2, A_2) = A_2 L_1 - A_1 L_2,$$

so that

$$V(t) = V\left(\check{L}, \check{A}; \hat{L}, \hat{A}\right) = \hat{A}\check{L} - \check{A}\hat{L},$$

is almost periodic and

$$V\left(\check{L},\check{A};\hat{L},\hat{A}\right) = \dot{A}\check{L} + A\dot{\check{L}} - \dot{A}\hat{L} - \dot{A}\dot{\hat{L}}$$
$$= -\mu A\check{L} + \mu \check{A}\hat{L} - (\nu + \mu_L)\check{L}\hat{A} + (\nu + \mu_L)\hat{L}\dot{A}$$
$$+ (\nu + \mu_L)\hat{L}\check{A} + \frac{rb(t)}{K}A\check{A}\hat{L} - \frac{rb(t)}{K}\check{A}A\check{L}.$$

Hence, by

$$\check{L} \le \hat{L} \le K, \quad \frac{\nu}{\mu \mathcal{N}_*} \epsilon (1+\epsilon) \le \check{A} \le \hat{A} \le \frac{\nu}{\mu} K$$

and by (4.2) we obtain

$$V = -(\mu + \nu + \mu_L)V + \left(\frac{rb(t)}{K}\hat{A}\right)\check{A}\hat{L} - \left(\frac{rb(t)}{K}\check{A}\right)\hat{A}\check{L} \leq -(\mu + \nu + \mu_L)V + \left(\frac{r\nu b(t)}{\mu}\right)\check{A}\hat{L} - \left(\frac{rb(t)}{K}\check{A}\right)\hat{A}\check{L} = -(\mu + \nu + \mu_L)V - rb(t)\frac{\check{A}}{K}\left[\left(\frac{\nu K}{\mu\check{A}}\right)\hat{A}\check{L} - \check{A}\hat{L}\right] \leq -(\mu + \nu + \mu_L)V - rb(t)\frac{\check{A}}{K}\left[\left(\frac{\nu K}{\mu(\nu K^*/\mu)}\right)\hat{A}\check{L} - \check{A}\hat{L}\right].$$

Therefore,

$$\dot{V} \le -(\mu + \nu + \mu_L + rb^* \check{A}^*/K)V \le -c \cdot V,$$

or

$$V(t) \le V(0)e^{-ct}, \qquad \forall t \ge 0,$$

where  $\mu + \nu + \mu_L + \frac{rb^*\nu}{\mu} = c > 0$ . We remark that V(t) is almost periodic. If V(0) > 0, then the exponential decay  $V(0)e^{-ct}$  implies that  $V(t) \le 0$ . We claim that  $V \equiv 0$ . Indeed, if on the contrary  $V \le 0$ , then

$$\frac{1}{2}\frac{d}{dt}\left[\left(\check{A}\right)^2 - \left(\hat{A}\right)^2\right] = \nu V \le 0.$$

Hence,  $(\check{A})^2 - (\hat{A})^2$  would be monotone decreasing. Moreover,  $\check{A} - \hat{A}$  would also be monotone decreasing. But the difference of two almost periodic functions is almost periodic and the only chance of being monotone is  $\check{A} - \hat{A}$  being a constant. By further considerations about solutions of an homogeneous linear system obtained from (4.5) and (4.6), we conclude that  $\hat{L} - \check{L}$  would necessarily remain constant. Furthermore,  $\check{A} = \hat{A}$  and  $\hat{L} = \check{L}$ 

Finally, by Theorem 1 and doing  $\epsilon$  arbitrarily small in the construction of the sub-solution in ii) we have a single attractor almost-periodic orbit in the set  $\Gamma^0$ . But, since  $\Gamma$  is an attractor of  $\mathbf{R}^2_{>0}$  by Lemma 1, then the almost-periodic orbit is attractor at  $\mathbf{R}^2$ . This concludes iii) and therefore ends the proof of the Theorem.

A situation of special interest arises when considering in particular that b(t) is a T periodic function, in this case we get

**Corollary 2.** Assume  $r, \nu, \mu, \mu_L > 0, K > 1$  and b(t) > 0 a non constant continuous *T*-periodic function. If  $\mathcal{N}_* > 1$ , then there is at least one almost *T*-periodic solution (L, A) of (2.1) whose components are positive. In the case above, condition holds, there exists a globally stable almost periodic solution of (2.1).

### 5. Numerical examples

In this section we show different scenarios for the solutions of model (2.1). To do this, we consider values of some parameters of the model gathered in literature. Particularly, the rates associated to mortality, the rates of transition from larvae to mosquitoes and the rate of fertility. We use arbitrary values for the parameters associated to the seasonal drivers.

As in [9, 8], we use the values of the rates  $\mu_L$ ,  $\nu$  and  $\mu$  estimated in [5]. In that work, it is analyzed a cohort of *Aedes albopictus* from the immature development to the adult stage. The authors in [5] measured fecundity, survival, longevity and gonotrophic cycles. Also, they show the sex ratio observed at five constant temperatures.

Table 5.1: Values of the parameters  $\mu_L$ ,  $\nu$  and  $\mu$  for *Aedes albopictus* at different temperatures.

Parameters	10 °C	$15 \ ^{\circ}\mathrm{C}$	20 °C	$25 \ ^{\circ}\mathrm{C}$	$30 \ ^{\circ}\mathrm{C}$	$35 \ ^{\circ}\mathrm{C}$	$40 \ ^{\circ}\mathrm{C}$	Source
$\mu_L$	1	0.96	0.48	0.62	0.65	0.99	1	[9]
$\nu$	0	0.0236	0.0578	0.0667	0.0645	0.0515	0	[8]
$\mu$	—	0.0286	0.0400	0.0400	0.0385	0.0556	—	[8]

Table 5.2: Sex ratio of *Aedes albopictus* at eight constant temperatures.

Parameters	$15 \ ^{\circ}\mathrm{C}$	$20 \ ^{\circ}\mathrm{C}$	$25 \ ^{\circ}\mathrm{C}$	$30 \ ^{\circ}\mathrm{C}$	$35 \ ^{\circ}\mathrm{C}$	Source
Sex ratio $(r)$	47.5%	43.5%	41.0%	46.3%	66.6%	[5]

In Table 5.1, there are shown different values for the death rates and for the transition rate from the larva stage to the adult stage at eight constant temperatures; whereas in Table 5.2, there are shown the sex ratio observed at six constant temperatures.

To model the seasonal drivers in the rate b(t) we use the function

(5.1) 
$$F(t) = \theta \left(\phi + \cos(2\pi t) + \sin(\psi t)\right).$$

This kind of function is used in [23] to modeling the biting rate of mosquitoes that transmit the malaria. Notice that F(t) is an almost periodic or periodic function depending on whether  $\frac{\psi}{2\pi}$  is an irrational number or a rational number, respectively.

In the following, we show the numerical simulations of the solutions of model (2.1) using the values of the parameters given in Tables 5.1 and 5.2 and the seasonal function given in (5.1) to model the birth rate b(t). It will be shown that solutions of the model go to a unique positive almost periodic solution, which is globally stable. Numerical simulations were performed using Mathematica 12.3.

We analyze the behavior of the aquatic and adult stages for three different levels of the temperature when we assume that the female mosquitoes lay an almost periodic number of egg in each oviposition. In the first scenario, we show the population behavior at 25°C. For this temperature,  $\mu_L = 0.6200$ ,  $\nu = 0.0671$  and  $\mu = 0.0400$  (Table 5.1). We use the mean values for the parameters r = 0.5 and  $\theta = 15$  as in [9]. That is, the proportion of female mosquitoes is 0.5 and  $\theta = 15$  (egg/female/day) means that the mean number of eggs laid by a female mosquito per day that have emerged as larvae is 15. We use the arbitrary values K = 200000,  $\phi = 1$  and  $\psi = \sqrt{5}$ . Then,

(5.2) 
$$b(t) = 15 \left( 1 + 0.36 \cos(2\pi t) + 0.12 \sin(\sqrt{5}t) \right).$$

With the values of the parameters mentioned above,  $b_* = 7.839$  and  $\mathcal{N}_* = 50.929$ , then the conditions of Theorem 1 are satisfied. Figure 1 shows numerical solutions of model (2.1). In the long term, both populations go to the stable almost periodic solution.



**Figure 1:** Scenario 1. Time evolution of aquatic and mosquitoes population at 25°C. In case 1 (a) the initial aquatic population is 250000. In case 1 (b) the initial population of mosquitoes is given by 200000.

Figure 2 shows the solutions of model (2.1) for different initial conditions.



Figure 2: Behavior of the aquatic population and the mosquitoes population for different initial conditions in scenario 1. In case (a), the larvae population has associated the color red, green, orange and magenta for 250000, 200000, 300000, 100000 number of larvae at t = 0, respectively. Case (b) shows the behavior of A(t) for different initial conditions

200000, 380000, 2000, 200, in which the color for each solutions is red, green, orange, magenta, respectively.

Now, we show the behavior of the solutions of model (2.1) at two different temperatures. For the second scenario, the temperature is 15°C. The values of the parameters are r = 0.475,  $\mu_L = 0.96$ ,  $\nu = 0.0236$  and  $\mu =$ 0.0286. For the third scenario, the temperature is 35°C, r = 0.666,  $\mu_L =$ 0.99,  $\nu = 0.0515$  and  $\mu = 0.0556$ . Figure 3 shows both scenarios.



**Figure 3:** Figures 3 (a-b) show the evolution of aquatic and mosquitoes populations at 15°C (Scenario 2) while Figures 3 (c-d) show the evolution of aquatic and mosquitoes populations at 35° C (scenario 3).

Fig. 4 (a-b) show the solutions of model (2.1) when the number of eggs laid by a female mosquito per day that have emerged as larvae is modeled by an almost periodic function and a periodic function (for the periodic case  $\psi = 2\pi$ ). We show that at the beginning of the dynamics the size of the aquatic population are similar even although b(t) is modeled by an almost periodic function and by a periodic function. However, the blowup timing in the almost periodic case firstly occurs in comparison with the periodic ones. Figures 4 (c-d) show the case when b(t) has an annual behavior.



Figure 4: It is compared the almost periodic case and the periodic case with 25°C. The almost periodic case is shown in red color. The periodic scenario is illustrated by blue color. Cases (a-b) are associated to b(t) with a daily behavior while cases (c-d) are associated to b(t) with an annual behavior.

## 6. Discussion

Epidemic outbreaks of many infectious diseases are affected by seasonal and social drivers such as rainfall, temperature and human activities as immunization schedules and school terms [20, 2]. Particularly, those infectious diseases that are transmitted by vectors; for example, malaria, dengue, leishmaniasis among others. Therefore, it is of paramount importance to know how the population dynamics of these vectors is, in order to control the disease they transmit. In this direction, seasonal models are useful tools to describe the incidence of diseases transmitted by vectors.

In seasonal models, some parameters are assumed to be exactly periodic. Sometimes, those parameters share a common period. However, the transmission patters could be almost periodic due to the diversity of seasonal drivers. Therefore, it is of paramount importance to modeling the population dynamics of vectors transmitting diseases using almost periodic models to control the incidence of those diseases.

In this work, we analyze an almost periodic model to describe the time evolution of the larvae and adult stage of mosquitoes population. To do this, we combine the age structure of mosquitoes with seasonal drivers in the fertility rate. Simulations of the solutions of model (2.1) show the existence of one stable almost periodic solution, validating the theoretical results.

In particular, Figure 3 shows that the solutions associated to the temperature of 35°C tend to a unique almost periodic solution in less time than the solutions associated to the temperature of 15°C when an almost periodic behavior is assumed in the number of eggs laid by a female mosquito. Also, the population of mosquitoes in both stages are very similar at 15°C and 35°C. However, by comparing the evolution of mosquitoes in both stages at different temperatures, it can be observed that population of mosquitoes at 25°C reaches a higher population density; see Figure 2 and 3.

Simulations of the solutions of model (2.1) show two different behaviors by assuming a daily almost periodic function. In the former case, sustained oscillations with a large amplitude occur in the larva stage, L. In the second case the amplitude in the population of mosquitoes, A, is so faint that it can hardly be perceived. In contrast, if an annual behavior in the almost periodic functions is assumed, the population shows sustained oscillations with a large amplitude; see Figure 4.

Also, Figure 4 shows that the larvae population has an asynchronous behavior when the almost periodic population dynamics is approximated by a periodic dynamics. Then, the density of larvae can be underestimated or overestimated which can lead to the decision makers to design wrong control strategies for the vector population. This calculation mistake can be catastrophic for the susceptible population because a disease outbreak can occur even though the control intervention was implemented.

In summary, the population densities of many species that transmit infectious diseases can be underestimated or overestimated if almost periodic dynamics is approximated by a periodic dynamics. This mistake can result in the design of wrong control strategies. Then, an epidemic outbreak may occur even though the breeding habitat-targeted mosquito control strategies were applied since the loss of synchronicity in the reproduction period was neglected.

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