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Common multiples of paths and stars with crowns

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Abstract

A graph G is a common multiple of two graphs H_1 and H_2 if there exists a decomposition of G into edge-disjoint copies of H_1 and also a decomposition of G into edge-disjoint copies of H_2. If G is a common multiple of H_1 and H_2, and G has q edges, then we call G a (q, H_1, H_2) graph. Our paper deals with the following question: Given two graphs H_1 and H_2, for which values of q does there exist a (q, H_1, H_2) graph? when H_1 is either a path or a star with 3 or 4 edges and H_2 is a crown.

Keywords: *Graph Decomposition, Common Multiples of Graphs, Path, Star, Crown.*

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1. Introduction

All graphs considered here are simple, finite, and undirected unless otherwise noted. Let $|V(G)|$ and $e(G)$ denote, respectively, the order of a graph G and the size of G, that is, the number of edges in G. The degree of a vertex u of G, denoted by $deg(u)$ is the number of edges incident to u in G.

We use the usual notation K_n to refer to the complete graph on n vertices and $K_{m,n}$, the complete bipartite graph with vertex partitions of cardinality m and n. A *k-path*, denoted by P_k, is a path with k vertices (is a path of length $k-1$); a *k-star*, denoted by S_k, is the complete bipartite graph $K_{1,k}$; a *k-cycle*, denoted by C_k, is a cycle of length k.

For an integer $n \geq 3$, the *crown* S_n^0, is the graph with vertex set $\{a_1, a_2, ..., a_n, b_1, b_2, ..., b_n\}$ and edge set $\{a_i b_j : 1 \leq i, j \leq n, i \neq j\}$ (see Figure 1 for S_4^0). Equivalently, the crown S_n^0 is the graph $K_{n,n} - I$, where $K_{n,n} - I$ denotes $K_{n,n}$ with 1-factor removed (or, obtained by deleting a perfect matching from the complete bipartite graph $K_{n,n}$).

Let G and H be two graphs. A *decomposition* of G is a set of edge-disjoint subgraphs of G whose union is G. An *H-decomposition* of G is a decomposition of G into copies of H. If G has an *H-decomposition*, we say that G is *H-decomposable* or H *divides* G and write $H|G$.

Given two graphs H_1 and H_2, one may ask for a graph G that is a *common multiple* of H_1 and H_2 in the sense that both H_1 and H_2 divide G. Several authors have investigated the problem of finding *least common multiples* of graphs; that is, graphs of minimum size which are both H_1 and H_2-decomposable. The problem was introduced by Chartrand et al. in [3] and they showed that every two nonempty graphs have a least common multiple. The least common multiple of two graphs may not be unique. The size of a least common multiple of two graphs H_1 and H_2 is denoted by $lcm(H_1, H_2)$. Also if q_1 and q_2 are two natural numbers, their number theoretic *lcm* is denoted by $lcm(q_1, q_2)$ as usual. Clearly, for two graphs H_1 and H_2, $lcm(H_1, H_2) \geq lcm(e(H_1), e(H_2))$. The problem of finding the size of least common multiples of graphs has been studied for several pairs of graphs: cycles and stars [3, 16], paths and complete graphs [11], pairs of cycles [8], and pairs of cubes [1]. Pairs of graphs having a unique least common multiple were investigated in [4] and least common multiples of digraphs were considered in [5].

If G is a common multiple of H_1 and H_2, and G has q edges, then we call G a (q, H_1, H_2) graph. An obvious necessary condition for the existence of a (q, H_1, H_2) graph is that $e(H_1)|q$ and $e(H_2)|q$. Obviously this necessary condition is not sufficient. For example, there is no $(15, K_3, K_6)$ graph as there is no K_3-decomposition of K_6. Hence a natural question is: *given two graphs H_1 and H_2, for which values of q, does a (q, H_1, H_2) graph exist?* Adams, Bryant, and Maenhaut [2] gave a complete solution to this problem in the case where H_1 is the 4-cycle and H_2 is a complete graph; Bryant and Maenhaut [7] gave a complete solution to this problem in the case where H_1 is the complete graph K_3 and H_2 is a complete graph. A complete solution to this problem in the case where H_1 is a path and H_2 is a star, is investigated in [6]. In [13, 14], the authors dealt with the common multiples of paths, stars, and cycles with complete graphs and complete bipartite graphs respectively.

In this paper, we establish the necessary and sufficient condition for the existence of a (q, P_4, S_n^0) graph, a (q, P_5, S_n^0) graph, a (q, S_3, S_n^0) graph, and a (q, S_4, S_n^0) graph. The graph theoretic concepts described here are suggested by their number theoretic counterparts.

2. Preliminaries

In this section, we collect some needed terminologies and notations and present some results which are useful for our discussions. The complete graph with vertex set $\{v_1, v_2, \ldots, v_m\}$ will be denoted by $[v_1, v_2, \ldots, v_m]$, m-cycle C_m with vertex set $\{v_1, v_2, \ldots, v_m\}$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, $\ldots$, $\{v_m, v_1\}$ will be denoted by $(v_1, v_2, \ldots, v_m)$, m-path P_m with vertex set $\{v_1, v_2, \ldots, v_m\}$ and edges $\{v_1, v_2\}$, $\{v_2, v_3\}$,$\ldots$, $\{v_{m-1}, v_m\}$ will be denoted by $\langle v_1, v_2, \ldots, v_m\rangle$ and m-star S_m with centre v_0 and end vertices $\{v_1, v_2, \ldots, v_m\}$ will be denoted by $[v_0; v_1, v_2, \ldots, v_m]$. The crown S_n^0 defined in the first section will be denoted by $(\{a_1, a_2, \ldots, a_n\}, \{b_1, b_2, \ldots, b_n\})$. Also note that the crown S_n^0 is defined only for $n \geq 3$.

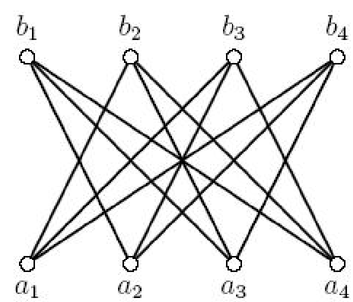

Figure 1: S_4^0

If G and H are graphs, and H is a subgraph of G, then the graph obtained by removing the edges of H from G will be denoted by $G - H$. If G_1 and G_2 are graphs, then the *union* of G_1 and G_2, denoted by $G_1 \cup G_2$, is the graph with vertex set $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and edge set $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$. (We shall only be considering the union of edge-disjoint graphs, but $V(G_1) \cap V(G_2) \neq \phi$.)

Theorem 1. *[15] For $n \geq 3, k \geq 1$, S_n^0 is P_{k+1}-decomposable if and only if $n(n-1) \equiv 0 \pmod{k}$ and*

$$k \leq \begin{cases} 2n-3 & \text{if } n \text{ is odd} \\ n-1 & \text{if } n \text{ is even.} \end{cases}$$

Theorem 2. *[10] For $k \geq 1, n \geq 3$, there exists a S_k-decomposition of S_n^0 if and only if $k \leq n-1$ and $n(n-1) \equiv 0 \pmod{k}$.*

Theorem 3. *[12] Let k, m, and n be positive integers. There exists a P_{k+1}-decomposition of $K_{m,n}$ if and only if $mn \equiv 0 \pmod{k}$ and one of cases in Table 2.1 occurs:*

Table 2.1: Necessary and Sufficient Conditions for P_{k+1}-Decomposition of $K_{m,n}$

Case	k	m	n	*Characterization*
1	*even*	*even*	*even*	$k \leq 2m, k \leq 2n$, *not both equalities*
2	*even*	*even*	*odd*	$k \leq 2m-2, k \leq 2n$
3	*even*	*odd*	*even*	$k \leq 2m, k \leq 2n-2$
4	*odd*	*even*	*even*	$k \leq 2m-1, k \leq 2n-1$
5	*odd*	*even*	*odd*	$k \leq 2m-1, k \leq n$
6	*odd*	*odd*	*even*	$k \leq m, k \leq 2n-1$
7	*odd*	*odd*	*odd*	$k \leq m, k \leq n$

We will use the following theorem on the least common multiple of two bipartite graphs, by O. Favaron and C. M. Mynhardt.

Theorem 4. *[8] If F and G are bipartite, then $lcm(F, G) \leq e(F)e(G)$, where equality holds if $gcd(e(F), e(G)) = 1$.*

Before entering the path decomposition, star decomposition, and crown decomposition, we consider the decomposition of bipartite graphs in [9]. Suppose that G is a bipartite graph with bipartition (X, Y), where $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$. Let $M(G)$ be the $m \times n$ matrix $(e_{i,j})$ where

$$e_{i,j} = \begin{cases} 1 & \text{if } x_i \text{ is adjacent to } y_j \\ 0 & \text{otherwise.} \end{cases}$$

Furthermore, if H is a subgraph of G, we use $M(H)$ to denote the $m \times n$ matrix $(e_{i,j})$ where

$$e_{i,j} = \begin{cases} 1 & \text{if } x_i \text{ is adjacent to } y_j \text{ in H} \\ 0 & \text{otherwise.} \end{cases}$$

With these notations, it is easy to see that

$$M(S_n^0) = \begin{bmatrix} 0 & 1 & \cdots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}$$

and if the edges of G can be decomposed into subgraphs $H_1, H_2, \ldots, H_t$, then

$$M(G) = M(H_1) + M(H_2) + \ldots + M(H_t).$$

3. Common Multiples of P_4 and S_n^0

In this section, we determine, for all positive integers $n \geq 3$, the set of integers q for which there exists a common multiple of P_4(4-path) and S_n^0 having precisely q edges.

Theorem 5. There exists a graph with q edges that is both P_4-decomposable and S_n^0-decomposable if and only if $q \equiv 0 \pmod{n(n-1)}$ and $q \equiv 0 \pmod 3$.

Proof. If there exists a (q, P_4, S_n^0) graph, then we require that 3 divides q and that $n(n-1)$ divides q. So necessary condition is obvious.

To show that the stated necessary condition is sufficient we consider two cases and construct the (q, S_3, S_n^0) graphs required to prove this theorem.

Case 1. $n \equiv 0, 1 \pmod 3$.
In this case $3|n(n-1)$. So $P_4|S_n^0$, by Theorem 1 and hence there exists a (q, P_4, S_n^0) graph G for all $q \equiv 0 \pmod{n(n-1)}$. Take G to be $\frac{q}{n(n-1)}$ vertex-disjoint copies of S_n^0, where each copy of S_n^0 in G is P_4-decomposable.

Case 2. $n \equiv 2 \pmod 3$.
Since P_4 and S_n^0 are bipartite and $gcd(3, n(n-1)) = 1$, $lcm(P_4, S_n^0) = 3n(n-1)$, by Theorem 4.
In this case, we can take a (q, P_4, S_n^0) graph G as the union of the following three edge-disjoint copies of S_n^0:

$$(\{a_1, a_2, a_3, ..., a_n\}, \{b_1, b_2, b_3, ..., b_n\})$$
$$(\{b_1, b_2, b_3, ..., b_n\}, \{c_1, c_2, c_3, ..., c_n\})$$
$$(\{a_1, a_2, a_3, ..., a_n\}, \{d_1, d_2, d_3, ..., d_n\}).$$

G can be decomposed into $n(n-1)$ copies of P_4, $\{\langle d_i, a_j, b_i, c_j\rangle, i \neq j, 1 \leq i, j \leq n\}$.
As an illustration of this case, we can see three copies of S_5^0 in $G = (60, P_4, S_5^0)$(see Figure 2) and G can be decomposed into 20 copies of P_4: $\{\langle d_i, a_j, b_i, c_j\rangle, i, j = 1, 2, 3, 4, 5, i \neq j\}$.

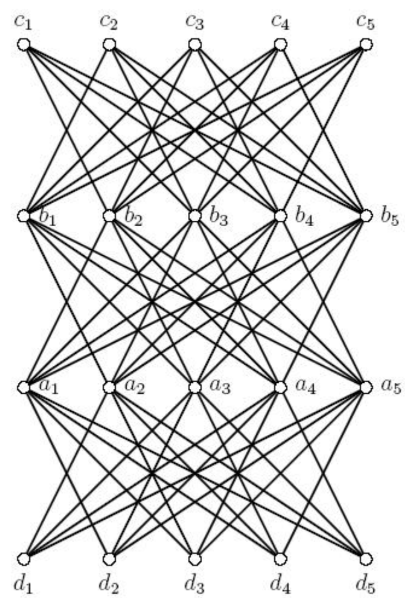

Figure 2: $G = (60, P_4, S_5^0)$

Therefore there exists a $(kn(n-1), P_4, S_n^0)$ graph for all $k \equiv 0 \pmod 3$.
□

4. Common Multiples of P_5 and S_n^0

In this section, we determine, for all positive integers $n \geq 3$, the set of integers q for which there exists a common multiple of P_5(5-path) and S_n^0 having precisely q edges.

Theorem 6. There exists a graph with q edges that is both P_5-decomposable and S_n^0-decomposable if and only if $q \equiv 0 \pmod{n(n-1)}$ and $q \equiv 0 \pmod 4$.

Proof. If there exists a (q, P_5, S_n^0) graph, then we require that 4 divides q and that $n(n-1)$ divides q. So the necessary condition is obvious.
(Sufficiency) **Case 1.** $n \equiv 0, 1 \pmod 4$**.**
In this case $4|n(n-1)$. So $P_5|S_n^0$, for all $n \geq 4$, by Theorem 1, and hence there exists a (q, P_5, S_n^0) graph G for all $q \equiv 0 \pmod{n(n-1)}$. Take G to be $\frac{q}{n(n-1)}$ vertex-disjoint copies of S_n^0.

Case 2. $n = 3$**.**
To construct a $(12, P_5, S_3^0)$ graph G, we let G be the union of the following two edge-disjoint copies of S_3^0:

$$(\{a_1, a_2, a_3\}, \{b_1, b_2, b_3\})$$

$$(\{c_1, c_2, a_3\}, \{d_1, b_2, b_3\}).$$

Then $G = \langle a_2, b_3, a_1, b_2, c_2\rangle \cup \langle a_2, b_1, a_3, b_2, c_1\rangle \cup \langle c_2, d_1, a_3, b_3, c_1\rangle$, the union of three edge-disjoint copies of P_5.
All other required graphs of sizes $q = 12k, k \geq 2$ can be constructed by the vertex-disjoint union of an appropriate number of copies of $(12, P_5, S_3^0)$.

Case 3. $n \equiv 2, 3 \pmod 4, n \neq 3$.
To construct a $(2n(n-1), P_5, S_n^0)$ graph G, we let G be the union of the following two edge-disjoint copies of S_n^0:

$$(\{a_1, a_2, a_3, \ldots, a_n\}, \{b_1, b_2, b_3, \ldots, b_n\})$$

$$(\{c_1, c_2, c_3, \ldots, c_{n-1}, a_n\}, \{d_1, d_2, d_3, \ldots, d_{n-2}, b_n, a_{n-1}\}).$$

Then $M(G) = 2M(S^0_{n-2}) + 2M(K_{2,n-2}) + 2M(K_{n-2,2}) + M(P_5)$.
So

$$G = 2S^0_{n-2} \cup 2K_{2,n-2} \cup 2K_{n-2,2} \cup \langle b_{n-1}, a_n, b_n, a_{n-1}, c_{n-1}\rangle.$$

Since $4|(n-2)(n-3), P_5|S^0_{n-2}$ (by Theorem 1). By Theorem 3, P_5 divides both $K_{2,n-2}$ and $K_{n-2,2}$ and hence P_5 divides G. Therefore there exists a $(2kn(n-1), P_5, S^0_n)$ graph for all $k \geq 1$. □

5. Common Multiples of S_3 and S^0_n

In this section, we determine, for all positive integers $n \geq 3$, the set of integers q for which there exists a common multiple of S_3(3-star) and S^0_n having precisely q edges.

Theorem 7. There exists a graph with q edges that is both S_3-decomposable and S^0_n-decomposable if and only if

1. $q \equiv 0 \pmod{n(n-1)}$ and $q \equiv 0 \pmod 3$
2. $q \neq 6$ when $n = 3$.

Proof. The given conditions are necessary for the following reasons. If there exists a (q, S_3, S^0_n) graph, then we require that 3 divides q and that $n(n-1)$ divides q. Condition (1) follows immediately from this and will be referred to as the obvious necessary condition.

If n = 3, then S^0_3 is isomorphic to C_6 and $S_3 \nmid C_6$. So $q \neq 6$.

To show that the stated necessary conditions are sufficient we consider each in turn and construct the required (q, S_3, S^0_n) graphs.

Case 1. $n = 3$**.**
For a $(12, S_3, S^0_3)$ graph G, consider $G = K_{2,2,2}$, the octahedral graph (see Figure 3) and it is S_3-decomposable and S^0_3-decomposable.

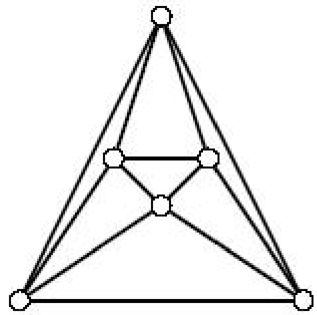

Figure 3: The octahedral graph $K_{2,2,2}$

To construct an $(18, S_3, S_3^0)$ graph G, we let G be the union of the following three edge-disjoint copies of S_3^0:

$$(\{a_1, a_2, a_3\}, \{b_1, b_2, b_3\})$$

$$(\{a_1, a_2, a_3\}, \{c_1, c_2, c_3\})$$

$$(\{a_1, a_2, a_3\}, \{d_1, d_2, d_3\}).$$

An S_3-decomposition of G is given by the following six edge-disjoint copies of S_3:

$$\{[a_i; b_j, c_j, d_j] : i, j = 1, 2, 3, i \neq j\}.$$

and all other required graphs of sizes $q = 6k, k \geq 4$ can be constructed by the vertex disjoint union of an appropriate number of copies of $(12, S_3, S_3^0)$ and $(18, S_3, S_3^0)$.

Case 2. $n \equiv 0, 1 \pmod 3), n \geq 4$**.**
In this case $3|n(n-1)$. So $S_3|S_n^0$, by Theorem 2, and hence there exists a (q, S_3, S_n^0) graph G for all $q \equiv 0 \pmod{n(n-1)}$. Take G to be $\frac{q}{n(n-1)}$ vertex-disjoint copies of S_n^0.

Case 3. $n \equiv 2 \pmod 3$**.**
Since S_3 and S_n^0 are bipartite and $gcd(3, n(n-1)) = 1, lcm(S_3, S_n^0) = 3n(n-1)$, by Theorem 4. In this case, we can take a (q, S_3, S_n^0) graph G as the union of the following three edge-disjoint copies of S_n^0:

$$(\{a_1, a_2, a_3, \ldots, a_n\}, \{b_1, b_2, b_3, \ldots, b_n\})$$

$$(\{a_1, a_2, a_3, \ldots, a_n\}, \{c_1, c_2, c_3, \ldots, c_n\})$$
$$(\{a_1, a_2, a_3, \ldots, a_n\}, \{d_1, d_2, d_3, \ldots, d_n\}).$$

G can be decomposed into $n(n-1)$ copies of S_3 as $(n-1)$ copies of S_3 centered at each vertex $a_i, 1 \leq i \leq n$.

Therefore there exists a $(kn(n-1), S_3, S_n^0)$ graph for all $k \equiv 0 \pmod 3$. Thus sufficient conditions (1) and (2) were obtained. □

6. Common Multiples of S_4 and S_n^0

In this section, we determine, for all positive integers $n \geq 3$, the set of integers q for which there exists a common multiple of S_4(4-star) and S_n^0 having precisely q edges.

Theorem 8. There exists a graph with q edges that is both S_4-decomposable and S_n^0-decomposable if and only if

1. $q \equiv 0 \pmod{n(n-1)}$ and $q \equiv 0 \pmod 4$
2. $q \neq 12$ when $n = 4$.

Proof. If there exists a (q, S_4, S_n^0) graph, then we require that 4 divides q and that $n(n-1)$ divides q. Condition (1) follows immediately from this and will be referred to as the obvious necessary condition.

If $n = 4$, then $S_4 \not| S_4^0$, by Theorem 2. So $q \neq 12$.
To show that the stated necessary conditions are sufficient we consider four cases and construct the required (q, S_4, S_n^0) graphs.

Case 1. $n \equiv 0, 1 \pmod 4, n \geq 5$.
In this case $4|n(n-1)$. So $S_4|S_n^0$, for all $n \geq 5$, by Theorem 2, and hence there exists a (q, S_4, S_n^0) graph G for all $q \equiv 0 \pmod{n(n-1)}$. Take G to be $\frac{q}{n(n-1)}$ vertex-disjoint copies of S_n^0.

Case 2. $n = 4$.
We have $e(S_4) = 4$ and $e(S_4^0) = 12$.
For a $(24, S_4, S_4^0)$ graph G,
consider $G = [1, 2, 3, 4, 5, 6, 7, 8] - \{(1, 3), (5, 7), (2, 4), (6, 8)\}$, the graph $K_8 - I$.

An S_4^0-decomposition of G is given by the following two edge-disjoint copies of S_4^0:

$$(\{1,3,5,7\},\{2,4,6,8\})$$
$$(\{1,3,6,8\},\{4,2,7,5\}).$$

An S_4-decomposition of G is given by the following six edge-disjoint copies of S_4:

$$[1;2,5,6,8] \qquad [2;5,6,7,8] \qquad [3;2,4,6,8]$$
$$[4;1,6,7,8] \qquad [5;3,4,6,8] \qquad [7;1,3,6,8].$$

To construct a $(36, S_4, S_4^0)$ graph G, we let G be the union of the following three edge-disjoint copies of S_4^0:

$$(\{a_1,a_2,a_3,a_4\},\{b_1,b_2,b_3,b_4\})$$
$$(\{a_3,a_4,c_1,c_2\},\{b_4,d_1,d_2,d_3\})$$
$$(\{d_1,b_1,d_2,d_3\},\{a_1,a_2,c_3,c_4\}).$$

An S_4-decomposition of G is given by the following nine edge-disjoint copies of S_4:

$$[a_1;b_1,b_2,b_3,b_4] \qquad [a_2;b_1,b_3,b_4,d_1] \qquad [a_3;b_2,d_1,d_2,d_3]$$
$$[a_4;b_2,b_3,d_2,d_3] \qquad [b_1;a_3,a_4,c_3,c_4] \qquad [b_4;a_3,a_4,c_1,c_2]$$
$$[d_1;c_1,c_2,c_3,c_4] \qquad [d_2;a_1,a_2,c_2,c_4] \qquad [d_3;a_1,a_2,c_1,c_3].$$

Case 3. $n \equiv 2 \pmod 4$**.**
In this case, $n = 4p+2$, p is a positive integer and $n(n-1) \equiv 2 \pmod 4$. To construct a $(2n(n-1), S_4, S_n^0)$ graph G, we let G be the union of the following two edge-disjoint copies of S_n^0:

$$(\{a_1,a_2,a_3,\ldots,a_n\},\{b_1,b_2,b_3,\ldots,b_n\})$$
$$(\{a_1,a_2,a_3,\ldots,a_n\},\{c_1,c_2,c_3,\ldots,c_n\}).$$

In G, $deg(a_i) = 2n-2 = 8p+2$,
$deg(b_i) = n-1 = 4p+1$,
$deg(c_i) = n-1 = 4p+1, 1 \le i \le n$ and hence

$$M(G) = npM(S_4) + npM(S_4) + 2pM(S_4) + M(S_4).$$

Then G can be decomposed into $\frac{n(n-1)}{2}$ copies of S_4 as follows:
p copies of S_4 centered at each $b_i, 1 \leq i \leq n$,
p copies of S_4 centered at each $c_i, 1 \leq i \leq n$,
$2p$ copies of S_4 centered at a_1,
and one $S_4 = [a_n; b_1, b_2, c_1, c_2]$, centered at a_n.
Therefore there exists a $(2kn(n-1), S_4, S_n^0)$ graph for all $k \geq 1$.

Case 4. $n \equiv 3 \pmod 4$.
Then $n(n-1) \equiv 2 \pmod 4$.
To construct a $(2n(n-1), S_4, S_n^0)$ graph G, we let G as the union of the following two edge-disjoint copies of S_n^0:

$$(\{a_1, a_2, a_3, \ldots, a_n\}, \{b_1, b_2, b_3, \ldots, b_n\})$$
$$(\{a_1, a_2, a_3, \ldots, a_n\}, \{c_1, c_2, c_3, \ldots, c_n\}).$$

Every edge of G is adjacent to one of the vertices $a_i, 1 \leq i \leq n$. Also $deg(a_i) = 2(n-1)$ and $4|2(n-1)$.
So G can be decomposed into $\frac{n(n-1)}{2}$ copies of
$S_4 \left(\frac{(n-1)}{2}\right.$ copies of S_4 centered at each vertex $\left. a_i, 1 \leq i \leq n\right)$.
Therefore there exists a $(2kn(n-1), S_4, S_n^0)$ graph for all $k \geq 1$. $\square$

7. Conclusion

In this paper, we focus on the decomposition of graphs into paths, stars, and crowns with special emphasis on common multiples of graphs. It would be of interest to find the graphs of size q, which is a common multiple of the crown S_n^0 and the paths P_4, P_5 or the stars S_3, S_4. This study is interesting from the number theoretical point of view.

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