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Computation of wiener polynomial and index of line subdivision friendship and line subdivision bifriendship graphs using matlab program

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Abstract

A topological index is a branch of chemical graph theory that is vital to analyzing the physio-chemical characteristics of chemical compound structures divided into a degree-based molecular structure such as Zagreb indices, a distance-based molecular structure such as Wiener index, and a mixed such as Gutman index. In this paper, some definitions, results, and examples of Wiener polynomial and index for subdivision graph of friendship, and bifriendship graphs, line subdivision graph of friendship, and bifriendship graphs were introduced. Moreover, we used the MATLAB program to calculate the Wiener polynomial and index of these graphs and refer to some applications.

Keywords: *Line graph, subdivision graph, friendship graph, bifriendship graph, Wiener polynomial, Wiener index.*

MSC (2020): *05C07, 05C12, 05C09, 05C10, 05C05, 05C31*

1. Introduction

There are several applications for graphs in various applied sciences such as organic chemistry, biology (DNA, RNA), electronics, networks, businesses, and etc. In practical applications, topological indices are among the best applications to recognize physical properties, chemical reactions, and biological activities. For the past 40 years, chemical graph theory, considered an important branch of both computational chemistry and graph theory, that has attracted much attention, and the results obtained in this field have been applied to many chemical and pharmaceutical engineering applications [19, 33, 37]. Chemical graphs are models of molecules in which atoms are represented by vertices and chemical bonds by edges of a graph. Topological indices are important attributes to analyze the Physic-chemical characteristics of chemical compound structures, such as degree-based molecular structures, distance-based molecular structures, and mixed [26, 28, 17, 20].

A finite **graph** is a pair G = (V, E) consisting of a non-empty set V is called vertices, pointe, or nodes together with unordered pairs set of vertices is called edge, denoted by $|V|$ the **order** of vertices and $|E|$ the **size** of edges. The **degree** of vertex $v \in V$ dented by $d(v)$ is the number of edges incident on v. The **friendship graph** [21, 30] $F(n)_3$ of n triangles with common (center) vertex $v \in V(F(n)_3)$ has order $|V(F(n)_3)| = 2n+1$ and size $|E(F(n)_3)| = 3n$, see Figure 1.1.

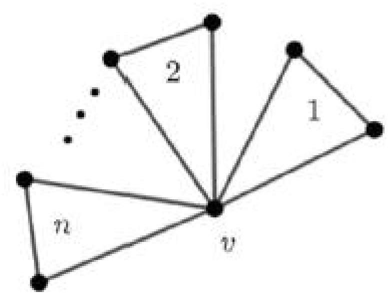

Figure 1.1: friendship graph $F(n)_3$

A **Bifriendship graph** [22] $BF(n,m)_3$ has two friendship graphs $F(n)_3$, $F(m)_3$ with n and m triangles joint the common (center) two vertices by an edge, see Figure 1.2, has $|V(BF(n,m)_3)| = 2(n+m+1)$ vertices and $|E(BF(n,m)_3)| = 3(n+m)+1$ edges.

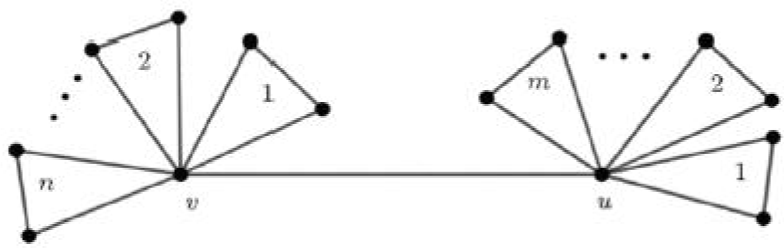

Figure 1.2: bifriendship graph $BF\,(n,m)_3$

An edge subdivision operation: The edge subdivision operation for an edge $\{u,v\} \in E$ is the deletion of $\{u,v\}$ from G and the addition of two edges $\{u,w\}$ and $\{w,v\}$ along with the new vertex w. A graph that has been derived from G by a sequence of edge subdivision operations is called a subdivision graph of G [38].

A **Subdivision graph** [39, 16] of a graph G is a graph with adding vertex of degree two for any edge of G. A **Line graph** [34, 31] of G L(G) is a graph with vertices as the edges of G and the two vertices of $L(G)$ are adjacency if and only if the two edges have common in G. A graph G is **connected** if every two vertices are joint by **path** denoted by $u{-}v$ and the length of the path denoted by $p(u{,}v)$ and the shortest path between two vertices is called a **distance**, denoted by d(u, v), define as $d\,(u,v) = min\{P\,(u,v) : u,v \in V\,(G)\}$ and the **diameter** of G denoted by $\delta\,(G)$, define as $\delta\,(G) = max\{d\,(u,v) : u,v \in V\,(G)\}$. The **Wiener polynomial** [13, 14] of a graph G define as

$$W\,(G;x) = \sum_{k=0}^{\delta(G)} d\,(G,k)\,x^k \ , \tag{1.1}$$

where $d\,(G,k)$ is the number of pairs with k a par, $k = 0,1,2,\ldots,\ \delta\,(G)$.

Obviously, $d\,(G,0) = |V|$, $d\,(G,1) = |E|$, $\sum_{k=0}^{\delta} d\,(G,k) = \binom{|V|+1}{2}$, and

$$W\,(v;G;x) = \sum_{k=0} d(v,G,k)x^k, \tag{1.2}$$

where $d(v, G, k)$ is the number of vertices that is distance k from the vertex $v \in V(G)$.
The Wiener index, $W(G)$ of G is the total distance in G, that is, $W(G) = \sum_{\{u,v\}} d(u, v)$, where the summation is taken over all pairs of distinct vertices in G. It is clear that

$$W(G) = \frac{d}{dx} W(G; x)_{|x=1}, \tag{1.3}$$

Now, let G_1 and G_2 be disjoint connected graphs, and let, $v \in V(G_1)$ and $u \in V(G_2)$. The graph $G_1 : G_2$ is obtained from G_1 and G_2 by introducing a new edge joining the vertices v and u. The Wiener polynomial of graph $G_1 : G_2$ given by Gutman [24, 23] in the following theorem.

Theorem 1.1: $W(G_1 : G_2; x) = W(G_1; x) + W(G_2; x) + xW(v; G_1; x)$ $W(u; G_2; x)$.
Walid in [38] supervised Ali generalizing Theorem (1) by introducing two new edges with common vertex joining the vertices v and u, in the following theorem.

Theorem 1.2: $W(G_1 : G_2; x) = W(G_1; x) + W(G_2; x) + x(W(v; G_1; x))$

$$+W(u; G_2; x) + x^2 W(v; G_1; x)\ W(u; G_2; x) + 1.$$

Our purpose, in this paper, gives definitions, results, and examples of Wiener polynomial and index for subdivision graphs of friendship and bifriendship graphs and line graph subdivision graphs of friendship and bifriendship graphs. We refer the interested reader to [12, 29]. Any graph in this paper is a simple graph.

2. Wiener Polynomial and Index of Subdivision Friendship Graph and Line Subdivision Friendship Graph

In this part, we compute the Wiener polynomial and index for subdivision graphs of friendship denoted by $SF(n)_3$.

Theorem 2.1: The Wiener polynomial and index for subdivision graphs of friendship graph are the following

$$W(SF(n)_3; x) = 5n + 1 + 6nx + 2n(n+2)x^2 + n(4n-1)x^3 + 4n(n-1)x^4$$

$+2n\,(n-1)\,x^5+\frac{1}{2}n(n-1)x^6$, and

$$W(SF\,(n)_3)=3n(15n-6).$$

Proof: By definition of subdivision graph of friendship graph $SF\,(n)_3$ given in Figure 2.1 of n triangles have vertices $|V(SF\,(n)_3|=5n+1=d(SF\,(n)_3,0)$ and edges $|E(SF\,(n)_3)|=6n=d(SF\,(n)_3,1)$, and $d\,(SF\,(n)_3,2)=$

$$\begin{cases} 6n \quad thenumber\ of\ pairs\ of\ n\ triangles\ with\ a\ length\ of\ two, \\ \\ 2n\,(n-1)\ \ the\ number\ of\ pairs\ of\ n\ triangles\ with\ common\ vertex\ v \\ of\ length\ 2, \end{cases}$$

Therefore, $d\,(SF\,(n)_3,2)=6n+2n\,(n-1)=2n(n+2)$, so $d\,(SF\,(n)_3,3)=$

$$\begin{cases} 3n\ the \quad number\ of\ pairs\ of\ n\ triangles\ with\ a\ length\ of\ three, \\ \\ 4n\,(n-1)\ \ the\ number\ of\ pairs\ of\ n\ triangles\ with\ common\ vertex\ v\ of \\ length\ 3. \end{cases}$$

Therefore, $d\,(SF\,(n)_3,3)=3n+4n\,(n-1)=n(4n-1)$, Similar can be found $d\,(SF\,(n)_3,k)\,;\ k=4,5,6.$ Hence

$$d\,(SF\,(n)_3,k)=\begin{cases} 4n\,(n-1) & the\ number\ of\ pairs\ for\ k=4, \\ 2n\,(n-1) & the\ number\ of\ pairs\ for\ k=5, \\ \frac{1}{2}n\,(n-1) & the\ number\ of\ pairs\ for\ k=6. \end{cases}$$

From these arguments, the result follows $W\,(SF\,(n)_3)=\frac{d}{dx}W\,(SF\,(n)_3;x)_{|x=1}=45n^2-18n=3n(15n-6)$.

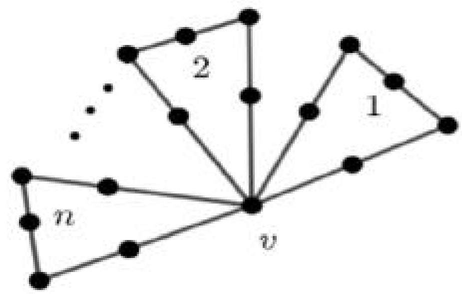

Figure 2.1: subdivision friendship graph $SF(n)_3$

Example: For $n=1$ the friendship graph $F(1)_3$ is the triangle and subdivision graph of friendship graph $SF(1)_3$ is the cycle with six vertices, see Figure 2.2, using Theorem (2.1), the Wiener polynomial and the Wiener index are given as the following $W(SF(1)_3;x) = 6+6x+6x^2+3x^3$, $W(SF(1)_3) = 27$. For $n=2$, the friendship graph $F(2)_3$ is the two triangles, and the subdivision graph of the friendship graph $SF(2)_3$ is the two cycles with a common vertex, using Theorem (2.1), the Wiener polynomial and the Wiener index are given as the following:

$$W(SF(2)_3;x) = 11+12x+16x^2+14x^3+8x^4+4x^5+x^6,$$

$$W(SF(2)_3) = 144.$$

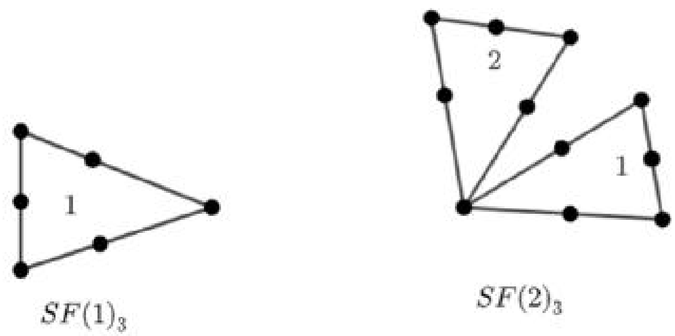

Figure 2.2: subdivision graphs $SF\,(1)_3 and \;\; SF\,(2)_3$

Now, we can constrict the line subdivision graph of friendship graph from subdivision graph of friendship graph for n=1,2, as following (see 2.3)

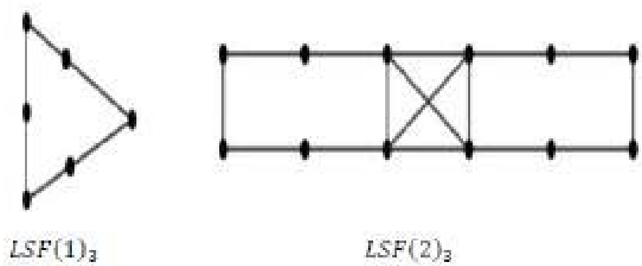

Figure 2.3: line subdivision graphs $LSF\,(1)_3$ *and* $\;\; LSF\,(2)_3$

In general, the constrict the line subdivision graph of friendship graph $LSF\,(n)_3$ has complete graph $k_{2\ n}$ and n cycles C_6, common with two vertices and an edge. See Figure 2.4, we get the following result.

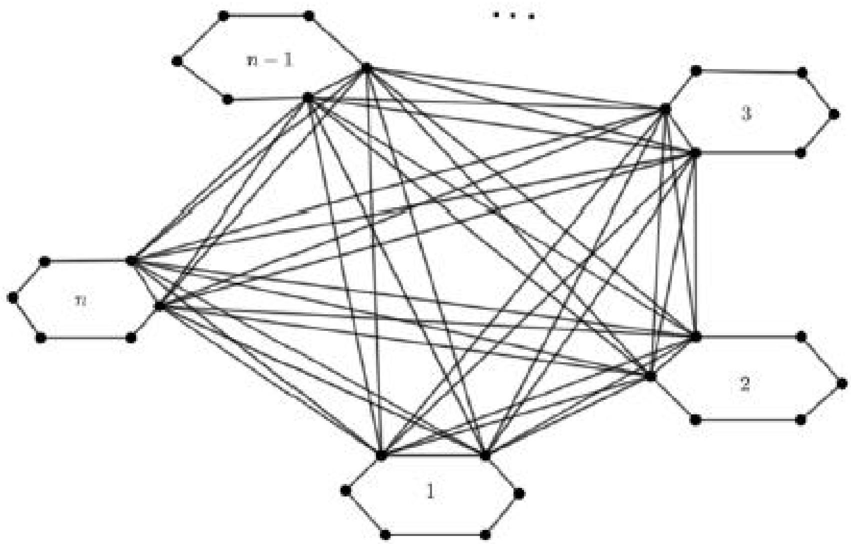

Figure 2.4: line subdivision friendship graph $LSF\,(n)_3$

Theorem 2.2: The Wiener polynomial and index for line subdivision graphs of friendship graph are given as the following:

$$W(LSF\,(n)_3; x) = 6n \ + 2n\,(n+2)\,x + 4n\,(n-1)\,x^2 + 3n\,(2n-1)\,x^3$$
$$+4n\,(n-1)\,x^4 + 2n(n-1)x^5,$$

and

$$W(LSF\,(n)_3) = 27n(2n-1).$$

Proof: Clear that constrict the line subdivision graph of friendship graph $LSF\,(n)_3$, see Figure 2.4 and Theorem (2.1) we get

$$d(LS\,(F\,(n)_3, 0) = |V(\ LSF\,(n)_3)| = 6n = d(S(F\,(n)_3, 1),$$

$$d(LS\,(F\,(n)_3, 1) = |E(\ LSF\,(n)_3)| = 2n(n+2) = d(S(F\,(n)_3, 2),$$

$d(LS\,(F\,(n)_3, 4) = \ 4n(n-1) = d(S(F\,(n)_3, 4)$, and

$$d(LS\,(F\,(n)_3, 5) = \ 2n(n-1) = d(S(F\,(n)_3, 5).$$

$$d\left(SF\left(n\right)_3, 3\right) = \begin{cases} 3n \quad \textit{the number of pairs of } n \textit{ cycles with} \\ \qquad\qquad \textit{alength of three}, \\ \\ 6n\left(n-1\right) \quad \textit{the number of pairs of } n \textit{ cycles with} \\ \qquad\qquad \textit{common } k_{2n} \textit{ and length of } 3. \end{cases}$$

Therefore, $d\left(LSF\left(n\right)_3, 3\right) = 3n + 6n\left(n-1\right) = 3n(2n-1)$,and

$$\begin{aligned} d\left(LSF(n)_3; 2\right) &= \frac{1}{2}6n\left(6n-1\right) - d\left(LSF(n)_3, 0\right) - d\left(LSF(n)_3, 1\right) \\ &\quad - \sum_{k=3}^{5} d\left(LSF(n)_3, k\right) \\ &= \frac{1}{2}6n\left(6n-1\right) - 6n - 2n^2 - 4n - 6n^2 + 3n - 4n^2 + 4n - 2n^2 + 2n \\ &= 4n\left(n-1\right). \end{aligned}$$

From these claim the result follows, and the Wiener index is given as the following

$$W(LSF(n)_3) = \frac{d}{dx} W(LSF(n)3; x)\big|_{x=1} = 54n^2 - 39n.$$

Example: For n=1, the Wiener polynomial and index for line subdivision graphs of friendship graph $LSF\left(1\right)_3$ are the same, the Wiener polynomial and indices for subdivision graph of friendship graph $SF\left(1\right)_3$, see Figure2.3. For n=2, the Wiener polynomial and index for line subdivision graphs of friendship graph $LSF\left(2\right)_3$, see Figure 2.3, are the following $W(LSF\left(2\right)_3; x) = 12 + 16x + 8x^2 + 18x^3 + 8x^4 + 4x^5$, and $W(SF\left(2\right)_3) = 138$.

3. Wiener Polynomial and Index of Subdivision Bifriendship Graph and Line Subdivision Bifriendship Graph

We can look the Figure3.1 of the subdivision graph of bifriendship graph $S(BF\left(n, m\right)_3)$ by two graphs $S(F\left(n\ \right)_3)$ and $S(F\left(m\right)_3)$ with adding the two edges uw, wv with a common vertex w, we get the graph $S\left(F\left(n\right)\right)$: $S\left(F\left(m\right)\right)$, which is the same graph $S\left(BF\left(n, m\right)\right)$ with $uw, wv \in E(\ S(BF\left(n, m\right)_3))$, applied Theorem (1.2) in the introduction and by Theorem (2.1)

$$\begin{aligned} W(S(BF\,(n,m)_3);x) &= W(S(F\,(n)_3) : S(F\,(m)_3);x) \\ &= \ W(S(F\,(n)_3);x)\ \ + W(S(F\,(m)_3);x) \\ &\quad + x\left[W(u, S(F\,(n)_3)\,;x) + W(v, S(F\,(m)_3)\ ;x)\right] \\ &\quad +x^2\left[W(u, S(F\,(n)_3)\,;x) W(v, S(F\,(m)_3)\ ;x)\right] + 1. \qquad \# \end{aligned}$$

where,

$$W(S(BF\,(n)_3);x) = 5n{+}1{+}6nx{+}2n\,(n+2)\,x^2{+}n\,(4n-1)\,x^3{+}4n\,(n-1)\,x^4$$

$$+2n\,(n-1)\,x^5 + \frac{1}{2}n(n-1)x^6,$$

$$W(S(BF\,(m)_3);x) = 5m + 1 + 6mx + 2m\,(m+2)\,x^2 + m\,(4m-1)\,x^3$$

$$+4m\,(m-1)\,x^4 + 2m\,(m-1)\,x^5 + \frac{1}{2}m(m-1)x^6,$$

$$W(u, S(F\,(n)_3)\ ;x) = 1 + 2nx + 2n\ x^2 + n\ x^3,$$

$$W(v, S(F\,(m)_3)\ ;x) = 1 + 2mx + 2mx^2 + mx^3,$$

$$x[W(u, S(F\,(n)_3)\ ;x) + W(v, S(F\,(m)_3)\ ;x)]$$

$$= 2x + 2(n+m)x^2 + 2\,(n+m)\,x^3 + (n+m)\,x^4,$$

and

$$x^2\left[W(u, S(F\,(n)_3)\,;x) W(v, S(F\,(m)_3)\ ;x)\right] + 1 = 1 + x^2 + 2(n+m)x^3$$

$$+2\,(n + 2nm + m)\,x^4 + (n + 8nm + m)\,x^5 + 8nmx^6 + 4nmx^7 + nmx^8.$$

Substitution in formula #, we get the following theorem.

Theorem 3.1: The Wiener polynomial of subdivision graph of bifriendship graph $S(BF\,((n,m)_3)$ is

$$W(S(BF\,(n,m)_3);x) = 3 + 5\,(n+m) + 2\,(3n + 3m + 1)\,x$$

$$+ (2n\,(n+3) + 2m\,(m+3) + 1)\,x^2 + [n(4n+3) + m(4m+3)]x^3$$

$$+[n(4n-1) + m(4m-1) + 4mn]x^4 + \left[\binom{2n}{2} + \binom{2m}{2} + 8nmx^5\right]$$

$$+\left[\binom{n}{2} + \ \binom{m}{2} + 8nmx^6\right] + 4nmx^7 + nmx^8,$$

and the Wiener index is:

$$W\left(S(BF\left(n,m\right)_3)\right) = \frac{d}{dx} W\left(S(BF\left(n,m\right)_3);x\right)_{|x=1}$$

$$= [15n(3n+1) + 15m(3m+1)] + 140mn + 4.$$

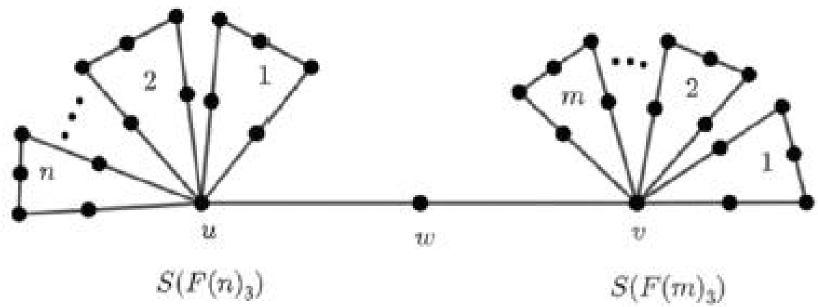

Figure 3.1: subdivision graph of bifriendship graph $S(BF\left(n,m\right)_3)$

Example: Take n=2 and m=3 in Theorem (3.1), we get $W(S(BF\left(2,3\right)_3);x) = 28+32x+57x^2+67x^3+71x^4+69x^5+52x^6+24x^7+6x^8$, and

$$W(S(BF\left(2,3\right)_3)) = 1413.$$

Corollary 3.2: The Wiener polynomial of $S(BF\left(n,n\right)_3)$ graph is

$$W(S(BF\left(n,n\right)_3);x)=10n+3+2\left(6n+1\right)x+\left(4n\left(n+3\right)+1\right)x^2+2n(4n+3)x^3$$
$$+2n\left(n-16\right)x^4+2n\left(6n-1\right)x^5+n\left(9n-1\right)x^6+4n^2x^7+n^2x^8,$$

and Wiener index of $S(BF\left(n,n\right)_3)$ graph is

$$W(S(BF\left(n,n\right)_3)) = 10n(23n+3)+4.$$

Proof: Take n=m in Theorem (3.1) and Figure 3.1, the result follows.

Example: Let$n$=2, see Figure 3.2. The Wiener polynomial of $S(BF\left(n,n\right)_3)$ graph is

$$W(S(BF\left(2,2\right)_3);x)=23+26x+41x^2+44x^3+44x^4+44x^5+34x^6+16x^7+4x^8,$$

and the Wiener index of $S(BF\,(2,2)_3)$ graph is

$$W(S(BF\,(2,2)_3)) = 984.$$

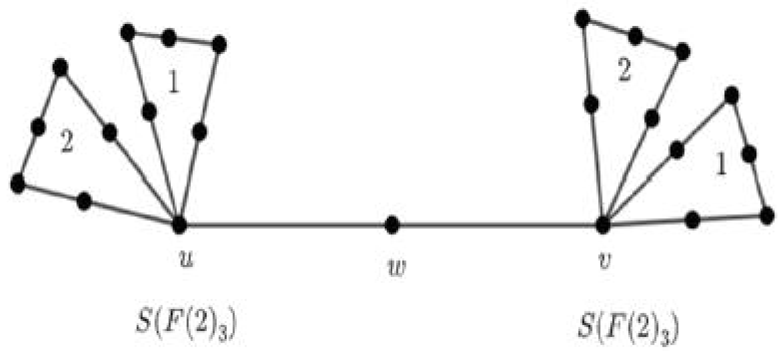

Figure 3.2: subdivision graph of bifriendship graph $S(BF\,(2,2)_3)$

Now, we can look the Figure3.3 of the line graph subdivision graph of bifriendship graph $LS(BF\,(n,m)_3)$ by two graphs $LS(F\,(n\;)_3)$ and $LS(F\,(m)_3)$ with adding an edge uv, where $u \in V\,(LS\,(F\,(n)))\,, v \in V\,(LS\,(F\,(m)))$, we get the graph $LS\,(F\,(n)) : LS\,(F\,(m))$, which is the same graph $LS\,(BF\,(n,m))$ with $vu \in E(L\;S(BF\,(n,m)_3))$, applied Theorem (1.1) in the introduction, and by Corollary (3.2), for $n = m$, we get the following Lemma.

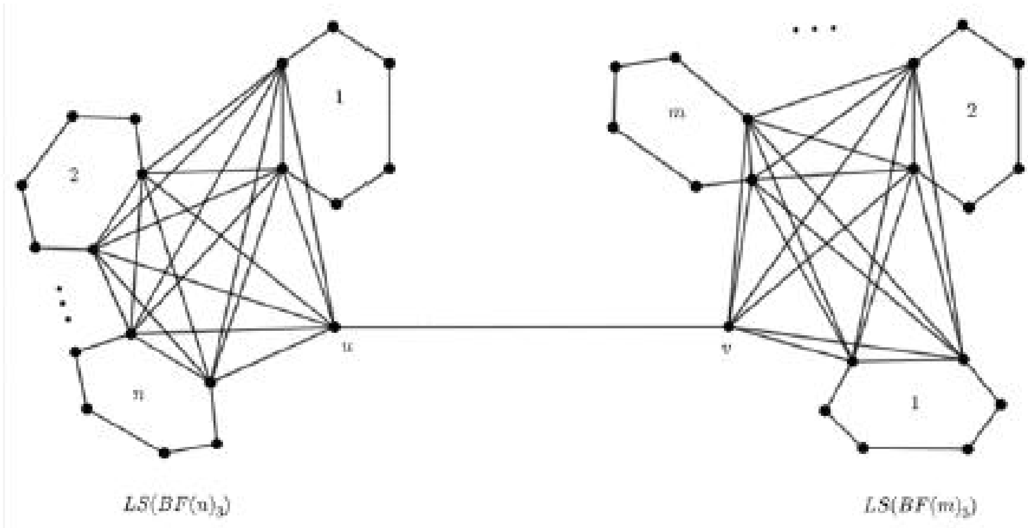

Figure 3.3: line subdivision graph of bifriendship graph $LS(BF\,(n,m)_3)$

Lemma 3.3: The Wiener polynomial of $L\ S(BF\,(n,n)_3)$ graph is

$W(LS(BF\,(n,n)_3);x) =\ 2(6n{+}1){+}(4n\,(n+3)+1)\,x{+}32nx^2{+}4n\,(n+6)\,x^3$

$+4n(4n-1)\quad x^4 + 4n\,(4n-1)\,x^5 + 8n^2x^6 + 4n^2x^7$, and the Wiener index of $L\ S(BF\,(n,n)_3)$ graph is

$$W(LS(BF\,(n,n)_3)) = 4n\,(59n+28)+1$$

Proof: $W(LS(BF\,(n,n)_3);x) = W(LS(BF\,(n)_3) : LS(BF\,(n)_3);x)$

$$= W(LS(BF\,(n)_3);x) + W(LS(BF\,(n)_3);x)$$

$$+\,x\,[W(u, LS(BF\,(n)_3)\,;x)\ W(v, LS(BF\,(n)_3)\ ;x)] \tag{3.1}$$

where, $W(LS(BF\,(n)_3);x) + W(LS(BF\,(n)_3);x) = 2W(LS(BF\,(n)_3);x)$

$$= 2(6n+1\ \ + 2n\,(n+3)\,x + 14n\ x^2$$

$$+10nx^3 + 4n\,(n-1)\,x^4 + 2n(n-1)x^5),$$

$x\,[W(u, LS(BF\,(n)_3)\,;x)\ W(v, LS(BF\,(n)_3)\ ;x)] = x{+}4nx^2{+}4n\,(n+1)\,x^3$

$$+4n\,(2n+1)\,x^4+12n^2x^5+8n^2x^6+4n^2x^7.$$

Substitution in formula (3.1), the Wiener polynomial of $L\ S(BF\,(n,n)_3)$ graph in Lemma (3.3), and the Wiener index of $L\ S(BF\,(n,n)_3)$ graph is given as the following

$$W\,(S(BF\,(n,n)_3))=\frac{d}{dx}W\,(S(BF\,(n,n)_3);x)_{|x=1}$$

$$=236n^2+112n+1=4n\,(59n+28)+1.$$

Example: Let n=2 in Lemma (2.3), see Figure 3.4, the Wiener polynomial of $L\ S(BF\,(2,2)_3)$ graph is
$W(LS(BF\,(2,2)_3);x)=26+41x+64x^2+64x^3+56x^4+56x^5+32x^6+16x^7$, and the Wiener index of $L\ S(BF\,(2,2)_3)$ graph is

$$W\,(S(BF\,(2,2)_3))=8\,(28+118)+1=1169.$$

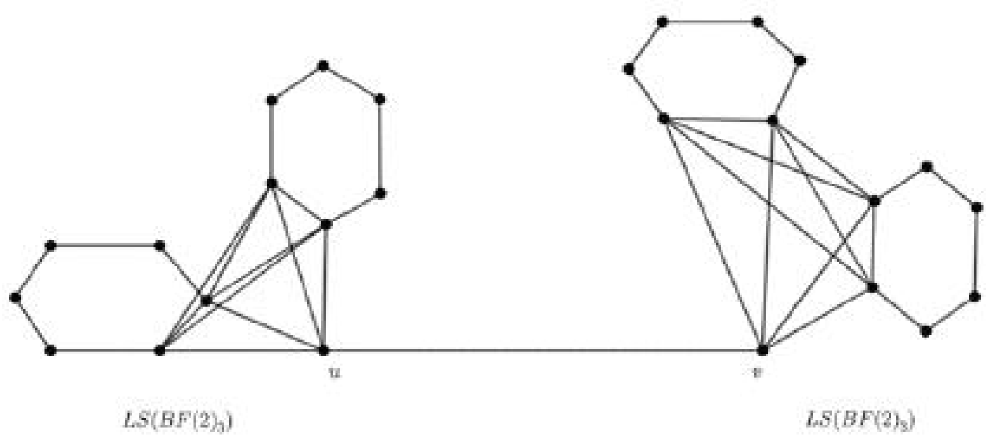

Figure 3.4: line subdivision graph of bifriendship graph $LS(BF\,(2,2)_3)$

Theorem 3.4: The Wiener polynomial of $L\ S(BF\,(n,m)_3)$ graph for $n\neq m$ is

$$W(LS(BF\,(n,m)_3);x)\ \ =\ \ 2(3(n+m)+1)+(2n\,(n+3)+2m\,(m+3)+1)\,x$$

$$+16(n+m)x^2+4(3\,(n+m)+nm)x^3$$
$$+2\,(n\,(2n-1)+m\,(2m-1)+4nm)\;\;x^4$$
$$+2(n\,(\;n-1)+m\,(m-1)+6nm)x^5$$
$$+8nmx^6+4nmx^7,$$

and the Wiener index of $L\ S(BF\,(n,m)_3)$ graph is

$$W(LS(BF\,(n,m)_3))=28n\,(n+2)+28m\,(m+2)+180nm+1.$$

Proof: $W(LS(BF\,(n,m)_3);x)=W(LS(BF\,(n)_3):LS(BF\,(m)_3);x)$

$$=\;W(LS(BF\,(n)_3);x)+W(LS(BF\,(m)_3);x)$$

$$+\,x\,[W(v.LS(BF\,(n)_3)\,;x)\;W(u,LS(BF\,(m)_3)\;\;;x)] \tag{3.2}$$

where, $W(LS(BF\,(n)_3);x)+W(LS(BF\,(\;m)_3);x)=6\,(n+m)+2$
$+(2\text{n}(n+3)+2m\,(m+3))x+14(n+m)\quad+10\,(n+m)\,x^3$
$+\,(4n\,(n-1)+4m\,(m-1))\,x^4+(2n\,(n-1)+2m\,(m-1))\,x^5,$
$x\,[W(u,LS(BF\,(n)_3)\,;x)\;W(v,LS(BF\,(m)_3)\;\;;x)]=x+2(n+m)x^2$
$+\,2(n+2nm+m)x^3+2\;\;(\;n+4nm+m)\,x^4+12nmx^5+8nmx^6+4nmx^7.$

Substitution in formula (3.2), the Wiener polynomial of $LS(BF\,(n,m)_3)$ graph in Theorem (3.4), and the Wiener index of $L\ S(BF\,(n,m)_3)$ graph is given as the following

$$W\,(S(BF\,(n,m)_3))=\frac{d}{dx}W\,(S(BF\,(n,m)_3);x)_{|x=1}$$

$$=28n\,(n+2)+28m\,(m+2)+180nm+1.$$

Example: Let n=3, m=2 in Theorem (3.4), the Wiener polynomial of $L\ S(BF\,(3,2)_3)$ graph is
$W(LS(BF\,(3,2)_3);x)=32+57x+80x^2+84x^3+90x^4+88x^5+48x^6+24x^7,$
and
the Wiener index of $L\ S(BF\,(3,2)_3)$ graph is

$$W\,(S(BF\,(3,2)_3))=84\,(5)+56(4)+1=645.$$

4. Build Algorithms to Calculate the Wiener Polynomial and Index

In this section, we show two programs that build an algorithm to calculate the Wiener polynomial and index of subdivision friendship graph, line subdivision friendship graph, subdivision bifriendship graph, and line subdivision bifriendship graph using MATLAB PROGRAM.
In these programs we will use the following shortcuts:
L: line, S: subdivision, F: friendship, and BF: bifriendship

4.1 The Wiener polynomial algorithm of subdivision friendship graph, line subdivision friendship graph subdivision bifriendship graph, and line subdivision bifriendship graph,

```
clear;
clc;
syms x;
n=input('Enter number of vertices: n= ');
m=input('Enter number of vertices: m= ');

a0=(5*n+1);
a1=(6*n);
a2=(2*n*(n+2));
a3=(n*(4*n-1));
a4=(4*n*(n-1));
a5=(2*n*(n-1));
a6=(0.5*n*(n-1));

b0=6*n; b1=2*n*(n+2);
b2=4*n*(n-1);
b3=3*n*(2*n-1);
b4=4*n*(n-1);
b5=2*n*(n-1);

c0=3+5*(n+m);
c1=2*(3*n+3*m+1);
c2=2*n*(n+3)+2*m*(m+3)+1;
c3=n*(4*n+3)+m*(4*m+3);
c4=n*(4*n-1)+m*(4*m-1)+4*n*m;
c5=n*(2*n-1)+m*(2*m-1)+8*n*m;
```

```
c6=(0.5)*n*(n-1)+(0.5)*m*(m-1)+8*n*m;
c7=4*n*m;
c8=n*m;

d0=2*(3*(n+m)+1);
d1=2*n*(n+3)+2*m*(m+3)+1;
d2=16*(n+m);
d3=4*(3*(n+m)+n*m);
d4=2*(n*(2*n-1)+m*(2*m-1)+4*n*m);
d5=2*(n*(n-1)+m*(m-1)+6*n*m);
d6=8*n*m;
d7=4*n*m;

fprintf('WSF(n;x)=,a0,a1,a2,a3,a4,a5,a6);

fprintf('WLSF(n;x)=b1,b2,b3,b4,b5);

fprintf('WSBF(n;x)=+
fprintf('WLSBF(n;x)=+
```

4.2 The Wiener index algorithm of subdivision friendship graph, line subdivision friendship graph subdivision bifriendship graph, and line subdivision bifriendship graph,

```
clear;
clc;

n=input('Enter number of vertices: n= ');
m=input('Enter number of vertices: m= ');

WSF(n)=3*n*(15*n-6);
WLSF(n)=27*n*(2*n-1);

WSBF(n,m)=15*n*(3*n+1)+15*m*(3*m+1)+140*n*m+4;
WLSBF(n,m)=28*n*(n+2)+28*m*(m+2)+180*n*m+1;

fprintf('Wiener index of SF graph W(SF(n))=
fprintf('Wiener index of LSF graph W(LSF(n))= WLSF(n));
```

```
fprintf('Wiener index of SBF graph W(SBF(n,m))= WSBF(n,m));

fprintf('Wiener index of LSBF graph W(LSBF(n,m))= WLSBF(n,m));
```

Table 4.1: Data analysis of Wiener index for subdivision graph and line subdivision graph of friendship graph and bifriendship graph when ($n = 3, 4, \ldots, 1000$).

n-cycles	$W(SF(n)_3)$	$W(LSF(n)_3)$	$W(SBF(n)_3)$	$W(LSBF(n)_3)$
3	351	405	2164	2461
4	648	756	3804	4225
5	1035	1215	5904	6461
6	1512	1782	8464	9169
7	2079	2457	11484	12349
8	2736	3240	14964	16001
9	3483	4131	18904	20125
10	4320	5130	23304	24721
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1000	44982000	53973000	230030004	236112001

Table 4.2: Data analysis of Wiener index for subdivision graph of bifriendship graph when ($n = 3, 4, \ldots, 1000$) and ($m = 3, 4, \ldots, 1000$).

(n, m)	3	4	5	6	7	$\cdots$	1000
3	2164	2914	3754	4684	5704	$\cdots$	45435454
4	2914	3804	4784	5854	7014	$\cdots$	45575784
5	3754	4784	5904	7114	8414	$\cdots$	45716204
6	4684	5854	7114	8464	9904	$\cdots$	45856714
7	5704	7014	8414	9904	11484	$\cdots$	45997314
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1000	45435454	45575784	45716204	45856714	45997314	$\cdots$	230030004

Table 4.3: Data analysis of Wiener index for line subdivision graph of bifriendship graph when ($n = 3, 4, \ldots, 1000$) and ($m = 3, 4, \ldots, 1000$).

(n, m)	3	4	5	6	7	$\cdots$	1000
3	2461	3253	4101	5005	5965	$\cdots$	28596421
4	3253	4225	5253	6337	7477	$\cdots$	28776673
5	4101	5253	6461	7725	9045	$\cdots$	28956981
6	5005	6337	7725	9169	10669	$\cdots$	29137345
7	5965	7477	9045	10669	12349	$\cdots$	29317765
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
1000	28596421	28776673	28956981	29137345	29317765	$\cdots$	236112001

5. Applications

Wiener polynomial and Wiener index for subdivision graph of friendship and bifriendship graphs and line graph subdivision graph of friendship and bifriendship graphs can be applied to many practical graphs [27, 22, 25, 11, 24, 35, 15, 36, 32, 18] such as Dutch windmill graph, paracactus chain graphs, Oxide network and chain silicate, molecular structure of Ethane, molecular graph of Ethane and Para-line graph of Ethane, Polyphony Chains, dicoronylene, biphenylene, and V-Phenylenic nanosheet.

6. Conclusion

In this paper, the expressions for some distance-based molecular structures and such as Wiener polynomial and Wiener index for subdivision graph of friendship and bifriendship graphs and line graph subdivision graph of friendship and bifriendship graphs have been derived. Also, we used the MATLAB program to calculate the Wiener index of subdivision friendship graph, line subdivision friendship graph, subdivision bifriendship graph, and line subdivision bifriendship graph.

Data Availability
The data used to support the findings of this work are cited at relevant places within the text as references.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

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