



Decomposition dimension of corona product of some classes of graphs

Reji T.

Government College Chittur, India

and

Ruby R.

Government College Chittur, India

Received : May 2022. Accepted : August 2022

Abstract

For an ordered k -decomposition $\mathcal{D} = \{G_1, G_2, \dots, G_k\}$ of a connected graph $G = (V, E)$, the \mathcal{D} -representation of an edge e is the k -tuple $\gamma(e/\mathcal{D}) = (d(e, G_1), d(e, G_2), \dots, d(e, G_k))$, where $d(e, G_i)$ represents the distance from e to G_i . A decomposition \mathcal{D} is resolving if every two distinct edges of G have distinct representations. The minimum k for which G has a resolving k -decomposition is its decomposition dimension $\text{dec}(G)$. In this paper, the decomposition dimension of corona product of the path P_n and cycle C_n with the complete graphs K_1 and K_2 are determined.

Key words: *Decomposition dimension, Corona product, Path, Cycle.*

Mathematical Subject Classification Codes: *05C38, 05C70*

1. Introduction

Let $G = (V, E)$ be a finite undirected connected graph without loops or multiple edges. A decomposition of a graph G is a collection of subgraphs of G , none of which have isolated vertices, whose edge sets provide a partition of $E(G)$. A decomposition of G into k subgraphs is a k -decomposition of G . A decomposition $\mathcal{D} = \{G_1, G_2, \dots, G_k\}$ is ordered if the ordering (G_1, G_2, \dots, G_k) has been imposed on \mathcal{D} . If each subgraph G_i of \mathcal{D} is isomorphic to a graph H , then \mathcal{D} is said to be an H -decomposition of G .

For edges $e, f \in E(G)$, the distance $d(e, f)$ between e and f is the minimum non negative integer k for which there exists a sequence $e = e_0, e_1, e_2, \dots, e_k = f$ of edges of G such that e_i and e_{i+1} are adjacent for $i = 0, 1, \dots, k - 1$. For an edge e of G and a subgraph F of G , $d(e, F) = \min\{d(e, f), f \in E(F)\}$. Let $\mathcal{D} = \{G_1, G_2, \dots, G_k\}$ be an ordered k -decomposition of G . The \mathcal{D} -representation of an edge e is the k -tuple $\gamma(e/\mathcal{D}) = (d(e, G_1), d(e, G_2), \dots, d(e, G_k))$, where $d(e, G_i)$ represents the distance from e to G_i . We call \mathcal{D} a resolving k -decomposition if for any pair of edges e and f , there exists some index i such that $d(e, G_i) \neq d(f, G_i)$. The minimum k for which G has a resolving k -decomposition is its decomposition dimension $dec(G)$. These concepts were introduced by Chartrand et.al in [1]. It is further studied in [2,3,8].

The concepts of resolving set and minimum resolving set have appeared in the literature previously. Slater introduced and studied these ideas with a different terminology 'locating set' in [9]. Harary and Melter [4] discovered these concepts independently. Later these concepts were rediscovered by Johnson in [5]. Chartrand et.al [1] proved that $dec(G) \geq 3$ for all connected graphs G that are not paths and for a tree T of order n and diameter d , $dec(T) \leq n - d + 1$. M. Hagita, A. Kundgen and D. B. West [3] used probabilistic methods to obtain upper bounds for decomposition dimension of complete graphs and regular graphs. H. Enomoto and T. Nakamigawa [2] established a lower bound for decomposition dimension of graphs using the maximum degree of G . They proved that for any graph G , $dec(G) \geq \lceil \log_2 \Delta(G) \rceil + 1$. Reji T. and Ruby R. studied about decomposition dimension of cartesian product of graphs in [6].

The corona product, $G_1 \odot G_2$ of two graphs G_1 (with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2 , and then joining the i th vertex of G_1 with an edge to every vertex in the i th copy of G_2 .

Metric dimension and partition dimension, which distinguishes the vertices of a graph using distance, of corona product of graphs are studied in [7,10].

2. Main Results

Define $\alpha_i^+ : \mathbf{R}^n \rightarrow \mathbf{R}^n$ by $\alpha_i^+(x_1, \dots, x_i, \dots, x_n) = (x_1, \dots, x_i + 1, \dots, x_n)$ and $\alpha_i^- : \mathbf{R}^n \rightarrow \mathbf{R}^n$ by $\alpha_i^-(x_1, \dots, x_i, \dots, x_n) = (x_1, \dots, x_i - 1, \dots, x_n)$

Theorem 1. $dec(P_n \odot K_1) = \begin{cases} 2 & \text{if } n = 2 \\ 3 & \text{if } n \geq 3 \end{cases}$

Proof. Case 1: $n = 2$

The corona product of the path P_2 and the complete graph K_1 , $P_2 \odot K_1$ is the path P_4 . Hence $dec(P_2 \odot K_1) = 2$.

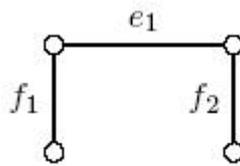


Figure 1. $P_2 \odot K_1$.

Case 2: $n \geq 3$

The corona product of the path P_n and the complete graph K_1 , $P_n \odot K_1$ is also known as the n -centipede graph. Let v_1, v_2, \dots, v_n be the n vertices and e_1, e_2, \dots, e_{n-1} be the $n - 1$ edges of the path P_n . Label the edges joining the vertex v_i in P_n and K_1 as $f_i, 1 \leq i \leq n$.

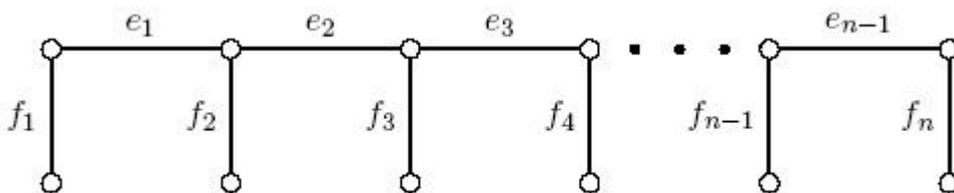


Figure 2. $P_n \odot K_1$.

Consider the decomposition $\mathcal{D} = \{G_1, G_2, G_3\}$ of $P_n \odot K_1$ where $E(G_1) = \{f_1\}$, $E(G_2) = \{f_n\}$ and $E(G_3)$ consists of all other edges of $P_n \odot K_1$. Then $\gamma(f_1/\mathcal{D}) = (0, n, 1)$, $\gamma(f_n/\mathcal{D}) = (n, 0, 1)$, $\gamma(f_i/\mathcal{D}) = (i, n + 1 - i, 0)$, $2 \leq i \leq n - 1$ and $\gamma(e_i/\mathcal{D}) = (i, n - i, 0)$, $1 \leq i \leq n - 1$. Thus \mathcal{D} is a resolving decomposition of $P_n \odot K_1$. So $dec(P_n \odot K_1) \leq 3$. Since $P_n \odot K_1$ is not a path $dec(P_n \odot K_1) \geq 3$. Hence $dec(P_n \odot K_1) = 3$. \square

Theorem 2. $dec(P_2 \odot K_2) = 3$ and $dec(P_n \odot K_2) \leq 4$, if $n \geq 3$

Proof. Case 1: $n = 2$

Consider the graph $P_2 \odot K_2$. Let v_1, v_2 be the vertices of the path P_2 and e_1 be the edge joining v_1 and v_2 in P_2 . For $i = 1, 2$ label the edges joining the vertex v_i in P_2 and K_2 as f_i, g_i and let h_i be the edge in K_2 adjacent to the edges f_i and g_i .

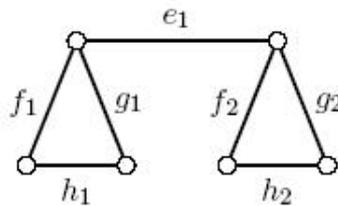


Figure 3. $P_2 \odot K_2$.

Consider the decomposition $\mathcal{D} = \{G_1, G_2, G_3\}$ of $P_2 \odot K_2$ where $E(G_1) = \{g_1\}$, $E(G_2) = \{g_2\}$ and $E(G_3)$ consists of all other edges of $P_2 \odot K_2$. Then $\gamma(g_1/\mathcal{D}) = (0, 2, 1)$, $\gamma(g_2/\mathcal{D}) = (2, 0, 1)$, $\gamma(f_1/\mathcal{D}) = (1, 2, 0)$, $\gamma(f_2/\mathcal{D}) = (2, 1, 0)$, $\gamma(h_1/\mathcal{D}) = (1, 3, 0)$, $\gamma(h_2/\mathcal{D}) = (3, 1, 0)$, $\gamma(e_1/\mathcal{D}) = (1, 1, 0)$. Thus \mathcal{D} is a resolving decomposition of $P_2 \odot K_2$. So $dec(P_2 \odot K_2) \leq 3$. Since $P_2 \odot K_2$ is not a path, $dec(P_2 \odot K_2) \geq 3$. Hence $dec(P_2 \odot K_2) = 3$.

Case 2: $n \geq 3$

Consider the corona product of the path P_n and the complete graph K_2 , $P_n \odot K_2$. Let v_1, v_2, \dots, v_n be the n vertices and e_1, e_2, \dots, e_{n-1} be the $n - 1$ edges of the path P_n . For $i = 1, 2, \dots, n$ label the edges joining the vertex v_i in P_n and K_2 as f_i, g_i and let h_i be the edge in K_2 adjacent to the edges f_i and g_i .

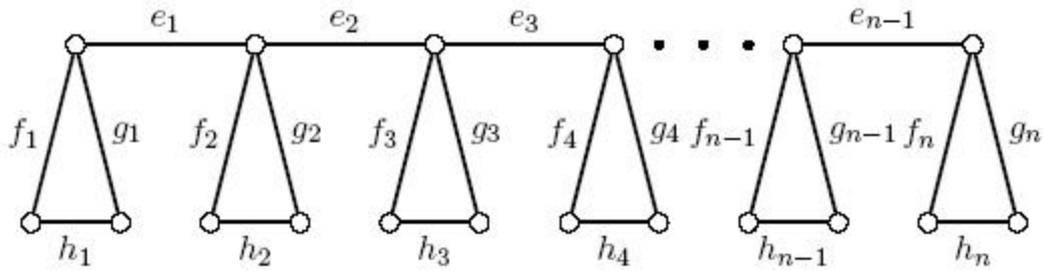


Figure 4. $P_n \odot K_2$.

Since $P_n \odot K_2$ is not a path, $dec(P_n \odot K_2) \geq 3$. Consider the decomposition $\mathcal{D} = \{G_1, G_2, G_3, G_4\}$ of $P_n \odot K_2$ where $E(G_1) = \{g_1\}$, $E(G_2) = \{g_2, g_3, \dots, g_{n-1}\}$, $E(G_3) = \{g_n\}$ and $E(G_4)$ consists of all other edges of $P_n \odot K_2$.

Then $\gamma(g_1/\mathcal{D}) = (0, 2, n, 1)$, $\gamma(g_n/\mathcal{D}) = (n, 2, 0, 1)$, $\gamma(f_1/\mathcal{D}) = (1, 2, n, 0)$, $\gamma(f_n/\mathcal{D}) = (n, 2, 1, 0)$, $\gamma(h_1/\mathcal{D}) = (1, 3, n + 1, 0)$, $\gamma(h_n/\mathcal{D}) = (n + 1, 3, 1, 0)$, $\gamma(e_i/\mathcal{D}) = (i, 1, n - i, 0)$, $1 \leq i \leq n - 1$. For $2 \leq i \leq n - 1$, $\gamma(g_i/\mathcal{D}) = (i, 0, n + 1 - i, 1)$, $\gamma(f_i/\mathcal{D}) = (i, 1, n + 1 - i, 0)$, $\gamma(h_i/\mathcal{D}) = (i + 1, 1, n + 2 - i, 0)$. Thus \mathcal{D} is a resolving decomposition of $P_n \odot K_2$. So $dec(P_n \odot K_2) \leq 4$. \square

Theorem 3. $dec(C_n \odot K_1) = 3$

Proof. Consider the corona product of the cycle C_n and the complete graph K_1 , $C_n \odot K_1$. Let v_1, v_2, \dots, v_n be the n vertices of the path C_n and e_1, e_2, \dots, e_n be the n edges of the cycle C_n . Label the edges joining the vertex v_i in C_n and K_1 as f_i , $1 \leq i \leq n$.

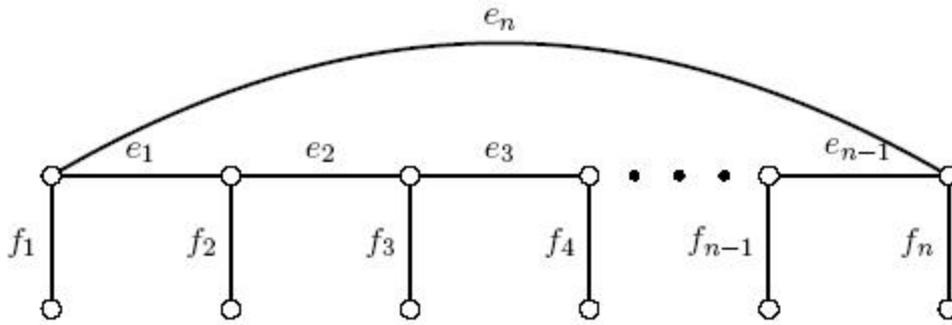


Figure 5. $C_n \odot K_1$.

Let $n \geq 3$ be any positive integer. Then $n = 3k-1$ or $3k$ or $3k+1$, where $k = 1, 2, \dots$. Consider the decomposition $\mathcal{D} = \{G_1, G_2, G_3\}$ of $C_n \odot K_1$.

Case 1: $n = 3k - 1$

Let $E(G_1) = \{f_1, f_n, f_{n-1}, \dots, f_{n-k+3}\}$, $E(G_2) = \{f_2, f_3, \dots, f_{k+1}\}$ and $E(G_3)$ consists of all other edges of $C_n \odot K_1$. Then

$$\gamma(f_i/\mathcal{D}) = \begin{cases} (0, 2, 1) & \text{if } i = 1 \\ (i, 0, 1) & \text{if } 2 \leq i \leq k + 1 \\ (k + 1, 2, 0) & \text{if } i = k + 2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(f_{i-1})) & \text{if } k + 3 \leq i \leq n - k + 2 \\ (0, k, 1) & \text{if } i = n - k + 3 \\ \alpha_2^-(\gamma(f_{i-1})) & \text{if } n - k + 4 \leq i \leq n \end{cases}$$

$$\gamma(e_i/\mathcal{D}) = \begin{cases} (i, 1, 0) & \text{if } 1 \leq i \leq k + 1 \\ (k, 2, 0) & \text{if } i = k + 2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(e_{i-1})) & \text{if } k + 3 \leq i \leq n - k + 2 \\ (1, k - 1, 0) & \text{if } i = n - k + 3 \\ \alpha_2^-(\gamma(e_{i-1})) & \text{if } n - k + 4 \leq i \leq n \end{cases}$$

Case 2: $n = 3k$ or $3k + 1$

Let $E(G_1) = \{f_1, f_n, f_{n-1}, \dots, f_{n-k+2}\}$, $E(G_2) = \{f_2, f_3, \dots, f_{k+1}\}$ and

$E(G_3)$ consists of all other edges of $C_n \odot K_1$.

When $n = 3k$

$$\gamma(f_i/\mathcal{D}) = \begin{cases} (0, 2, 1) & \text{if } i = 1 \\ (i, 0, 1) & \text{if } 2 \leq i \leq k + 1 \\ (k + 1, 2, 0) & \text{if } i = k + 2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(f_{i-1})) & \text{if } k + 3 \leq i \leq n - k + 1 \\ (0, k + 1, 1) & \text{if } i = n - k + 2 \\ \alpha_2^-(\gamma(f_{i-1})) & \text{if } n - k + 3 \leq i \leq n \end{cases}$$

$$\gamma(e_i/\mathcal{D}) = \begin{cases} (i, 1, 0) & \text{if } 1 \leq i \leq k + 1 \\ (k, 2, 0) & \text{if } i = k + 2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(e_{i-1})) & \text{if } k + 3 \leq i \leq n - k + 1 \\ (1, k, 0) & \text{if } i = n - k + 2 \\ \alpha_2^-(\gamma(e_{i-1})) & \text{if } n - k + 3 \leq i \leq n \end{cases}$$

When $n = 3k + 1$

$$\gamma(f_i/\mathcal{D}) = \begin{cases} (0, 2, 1) & \text{if } i = 1 \\ (i, 0, 1) & \text{if } 2 \leq i \leq k + 1 \\ (k + 2, 2, 0) & \text{if } i = k + 2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(f_{i-1})) & \text{if } k + 3 \leq i \leq n - k + 1 \\ (0, k + 1, 1) & \text{if } i = n - k + 2 \\ \alpha_2^-(\gamma(f_{i-1})) & \text{if } n - k + 3 \leq i \leq n \end{cases}$$

$$\gamma(e_i/\mathcal{D}) = \begin{cases} (i, 1, 0) & \text{if } 1 \leq i \leq k + 1 \\ (k + 1, 2, 0) & \text{if } i = k + 2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(e_{i-1})) & \text{if } k + 3 \leq i \leq n - k \\ (1, k + 1, 0) & \text{if } i = n - k + 1 \\ \alpha_2^-(\gamma(e_{i-1})) & \text{if } n - k + 2 \leq i \leq n \end{cases}$$

Thus \mathcal{D} is a resolving decomposition of $C_n \odot K_1$. So $dec(C_n \odot K_1) \leq 3$. Since $C_n \odot K_1$ is not a path $dec(C_n \odot K_1) \geq 3$. Hence $dec(C_n \odot K_1) = 3$. \square

Theorem 4. $dec(C_n \odot K_2) \leq 4$

Proof. Consider the corona product of the cycle C_n and the complete graph K_2 , $C_n \odot K_2$. Let v_1, v_2, \dots, v_n be the n vertices of the path C_n and e_1, e_2, \dots, e_n be the n edges of the cycle C_n . For $i = 1, 2, \dots, n$ label the edges joining the vertex v_i in C_n and K_2 as f_i, g_i and let h_i be the edge in K_2 adjacent to the edges f_i and g_i .

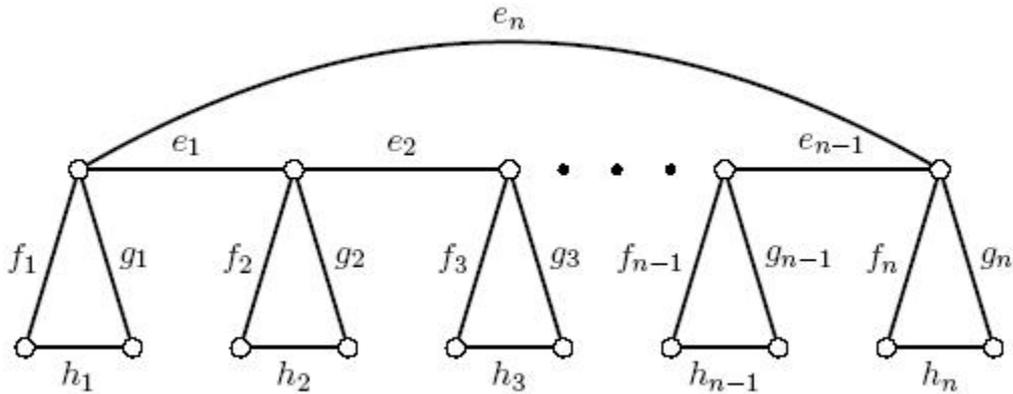


Figure 6. $C_n \odot K_2$.

Let n be any positive integer. By division algorithm there exists positive integers q, r such that $n = 3q + r$ where $r = 0$ or 1 or 2 . Since $C_n \odot K_2$ is not a path, $dec(C_n \odot K_2) \geq 3$. Consider the decomposition $\mathcal{D} = \{G_1, G_2, G_3, G_4\}$ of $C_n \odot K_2$.

Case 1: $n = 3q$

Let $E(G_1) = \{g_1, g_2, \dots, g_q\}$, $E(G_2) = \{g_{q+1}, g_{q+2}, \dots, g_{2q}\}$, $E(G_3) = \{g_{2q+1}, g_{2q+2}, \dots, g_n\}$ and $E(G_4)$ consists of all other edges of $C_n \odot K_2$. Then

$$\gamma(f_i/\mathcal{D}) = \begin{cases} (1, q+2-i, i+1, 0) & \text{if } 1 \leq i \leq q \\ (2, 1, q+1, 0) & \text{if } i = q+1 \\ (\alpha_1^+ \circ \alpha_3^-)(\gamma(f_{i-1})) & \text{if } q+2 \leq i \leq 2q \\ (q+1, 2, 1, 0) & \text{if } i = 2q+1 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(f_{i-1})) & \text{if } 2q+2 \leq i \leq n \end{cases}$$

$$\gamma(e_i/\mathcal{D}) = \begin{cases} (1, q+1-i, i+1, 0) & \text{if } 1 \leq i \leq q \\ (\alpha_1^+ \circ \alpha_3^-)(\gamma(e_{i-1})) & \text{if } q+1 \leq i \leq 2q \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(e_{i-1})) & \text{if } 2q+1 \leq i \leq n \end{cases}$$

$$\gamma(h_i/\mathcal{D}) = \begin{cases} (\alpha_2^+ \circ \alpha_3^+)(\gamma(f_i)) & \text{if } 1 \leq i \leq q \\ (\alpha_1^+ \circ \alpha_3^+)(\gamma(f_i)) & \text{if } q+1 \leq i \leq 2q \\ (\alpha_1^+ \circ \alpha_2^+)(\gamma(f_i)) & \text{if } 2q+1 \leq i \leq n \end{cases}$$

$\gamma(g_i/\mathcal{D}), 1 \leq i \leq n$ is obtained by replacing 1 and 0 in corresponding $\gamma(f_i/\mathcal{D})$ by 0 and 1.

Case 2: $n = 3q + 1$

Let $E(G_1) = \{g_1, g_n, \dots, g_{q+1}\}$, $E(G_2) = \{g_{q+2}, g_{q+2}, \dots, g_{2q+1}\}$, $E(G_3) = \{g_{2q+2}, g_{2q+3}, \dots, g_n\}$ and $E(G_4)$ consists of all other edges of $C_n \odot K_2$. Then

$$\gamma(f_i/\mathcal{D}) = \begin{cases} (1, q+3-i, i+1, 0) & \text{if } 1 \leq i \leq q+1 \\ (2, 1, q+1, 0) & \text{if } i = q+2 \\ (\alpha_1^+ \circ \alpha_3^-)(\gamma(f_{i-1})) & \text{if } q+3 \leq i \leq 2q+1 \\ (q+1, 2, 1, 0) & \text{if } i = 2q+2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(f_{i-1})) & \text{if } 2q+3 \leq i \leq n \end{cases}$$

$$\gamma(e_i/\mathcal{D}) = \begin{cases} (1, q+2-i, i+1, 0) & \text{if } 1 \leq i \leq q \\ (1, 1, q+1, 0) & \text{if } i = q+1 \\ (\alpha_1^+ \circ \alpha_3^-)(\gamma(e_{i-1})) & \text{if } q+2 \leq i \leq 2q+1 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(e_{i-1})) & \text{if } 2q+2 \leq i \leq n \end{cases}$$

$$\gamma(h_i/\mathcal{D}) = \begin{cases} (\alpha_2^+ \circ \alpha_3^+)(\gamma(f_i)) & \text{if } 1 \leq i \leq q+1 \\ (\alpha_1^+ \circ \alpha_3^+)(\gamma(f_i)) & \text{if } q+2 \leq i \leq 2q+1 \\ (\alpha_1^+ \circ \alpha_2^+)(\gamma(f_i)) & \text{if } 2q+2 \leq i \leq n \end{cases}$$

$\gamma(g_i/\mathcal{D}), 1 \leq i \leq n$ is obtained by replacing 1 and 0 in corresponding $\gamma(f_i/\mathcal{D})$ by 0 and 1.

Case 3: $n = 3q + 2$

Let $E(G_1) = \{g_1, g_n, \dots, g_{q+1}\}$, $E(G_2) = \{g_{q+2}, g_{q+3}, \dots, g_{2q+2}\}$, $E(G_3) = \{g_{2q+3}, g_{2q+4}, \dots, g_n\}$ and $E(G_4)$ consists of all other edges of $C_n \odot K_2$. Then

$$\gamma(f_i/\mathcal{D}) = \begin{cases} (1, q+3-i, i+1, 0) & \text{if } 1 \leq i \leq q+1 \\ (2, 1, q+2, 0) & \text{if } i = q+2 \\ (\alpha_1^+ \circ \alpha_3^-)(\gamma(f_{i-1})) & \text{if } q+3 \leq i \leq 2q+2 \\ (q+1, 2, 1, 0) & \text{if } i = 2q+3 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(f_{i-1})) & \text{if } 2q+4 \leq i \leq n \end{cases}$$

$$\gamma(e_i/\mathcal{D}) = \begin{cases} (1, q+2-i, i+1, 0) & \text{if } 1 \leq i \leq q+1 \\ (\alpha_1^+ \circ \alpha_3^-)(\gamma(e_{i-1})) & \text{if } q+2 \leq i \leq 2q+1 \\ (q+1, 1, 1, 0) & \text{if } i = 2q+2 \\ (\alpha_1^- \circ \alpha_2^+)(\gamma(e_{i-1})) & \text{if } 2q+3 \leq i \leq n \end{cases}$$

$$\gamma(h_i/\mathcal{D}) = \begin{cases} (\alpha_2^+ \circ \alpha_3^+)(\gamma(f_i)) & \text{if } 1 \leq i \leq q+1 \\ (\alpha_1^+ \circ \alpha_3^+)(\gamma(f_i)) & \text{if } q+2 \leq i \leq 2q+2 \\ (\alpha_1^+ \circ \alpha_2^+)(\gamma(f_i)) & \text{if } 2q+3 \leq i \leq n \end{cases}$$

$\gamma(g_i/\mathcal{D})$, $1 \leq i \leq n$ is obtained by replacing 1 and 0 in corresponding $\gamma(f_i/\mathcal{D})$ by 0 and 1.

Thus \mathcal{D} is a resolving decomposition of $C_n \odot K_2$. So $\text{dec}(C_n \odot K_2) \leq 4$.
□

References

- [1] G. Chartrand, D. Erwin, M. Raines and P. Zhang, "The decomposition dimension of graphs", *Graphs and Combinatorics*, vol. 17, pp. 599-605, 2001. doi: 10.1007/PL00007252
- [2] H. Enomoto and T. Nakamigawa, "On the decomposition dimension of trees", *Discrete Mathematics*, vol. 252, pp. 219-225, 2002. doi: 10.1016/S0012-365X(01)00454-X
- [3] M. Hagita, A. Kundgen and D. B. West, "Probabilistic methods for decomposition dimension of graphs", *Graphs and Combinatorics*, vol. 19, pp. 493-503, 2003. [On line]. Available: <https://bit.ly/3RHITD4>
- [4] F. Harary and R. A. Melter, "On the metric dimension of a graph", *Ars Combinatoria*, vol. 15, pp. 191-195, 1976.

- [5] M. A. Johnson, "Structure-activity maps for visualizing the graph variables arising in drug design", *Journal of Biopharmaceutical Statistics*, vol. 3, pp. 203-236, 1993. doi: 10.1080/10543409308835060
- [6] T. Reji and R. Ruby, "Decomposition dimension of cartesian product of some graphs", *Discrete Mathematics Algorithms and Applications*, 2022, doi: 10.1142/S1793830922501154
- [7] J. A. Rodríguez-Velázquez, I. G. Yero and D. Kuziak, "The partition dimension of corona product graphs", *arXiv:1010.5144v*, 2010.
- [8] V. Saenpholphat, P. Zhang, "Connected Resolving Decompositions in Graphs", *Mathematica Bohemica*, vol. 128, pp. 121-136, 2003. doi: 10.21136/mb.2003.134033
- [9] P. J. Slater, "Dominating and reference sets in graphs", *Journal of Mathematical Physics*, vol. 22, pp. 445-455, 1988.
- [10] I. G. Yero, D. Kuziak, J. A. Rodríguez-Velázquez, "On the metric dimension of corona product graphs", *Computers and Mathematics with Applications*, vol. 61, pp. 2793-2798, 2011. doi: 10.1016/j.camwa.2011.03.046

Reji T.

Department of Mathematics
Government College Chittur
Palakkad,
Kerala,
India-678104
India
e-mail: rejiaran@gmail.com

and

Ruby R.

Department of Mathematics
Government College Chittur
Palakkad,
Kerala,
India-678104
India
e-mail: rubymathpkd@gmail.com
Corresponding author