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On local edge antimagic chromatic number of graphs

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Abstract

Let $G = (V, E)$ be a graph of order p and size q having no isolated vertices. A bijection $f : V \to \{1, 2, 3, ..., p\}$ is called a local edge antimagic labeling if for any two adjacent edges $e = uv$ and $e^{'} = vw$ of G, we have $w(e) \neq w(e^{'})$, where the edge weight $w(e = uv) = f(u)+f(v)$ and $w(e^{'}) = f(v)+f(w)$. A graph G is local edge antimagic if G has a local edge antimagic labeling. The local edge antimagic chromatic number $\chi^{'}_{lea}(G)$ is defined to be the minimum number of colors taken over all colorings of G induced by local edge antimagic labelings of G. In this paper, we determine the local edge antimagic chromatic number for a friendship graph, wheel graph, fan graph, helm graph, flower graph, and closed helm.

Keywords: *Local edge antimagic labeling, Local edge antimagic chromatic number, Friendship graph, Wheel graph, Fan graph, Helm graph, Flower graph.*

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1. Introduction

A graph $G = (V, E)$ is a finite, undirected graph with neither loops nor multiple edges. Let $|V| = p$ and $|E| = q$ be the order and size of G. For graph-theoretic terminology; we refer to Chartrand and Lesniak [1].

Hartsfield and Ringel's [2] introduced the concept of antimagic labeling of a graph. Let $f : E \to \{1, 2, \ldots, |E|\}$ be a bijection. For each vertex $u \in V(G)$, the weight $w(u) = \sum_{e \in E(u)} f(e)$, where $E(u)$ is the set of edges incident to u. If $w(u) \neq w(v)$ for any two distinct vertices $u, v \in V(G)$, then f is called an antimagic labeling of G. A graph G is called antimagic if G has an antimagic labeling. For further reference see [5, 7, 8, 9].

In 2017, Arumugam et al.[3] introduced a new labeling local antimagic labeling and parameter local antimagic chromatic number using the concepts of antimagic labeling and vertex coloring. They defined as a bijection $f : E \to \{1, 2, \ldots, |E|\}$ is called local antimagic labeling if for all $uv \in E$ we have $w(u) \neq w(v)$, where $w(u) = \sum_{e \in E(u)} f(e)$. A graph G is local antimagic if G has a local antimagic labeling. The local antimagic chromatic number is defined to be the minimum number of colors taken overall coloring of G induced by local antimagic labeling of G, and they proved some basic results. For further reference see [5, 6, 10].

In 2017, Agustin et al. [4] introduced the concept of local edge antimagic chromatic number of graphs motivated by local antimagic chromatic number. It is defined as a bijection $f : V(G) \to \{1, 2, \ldots, p\}$ is called a local edge antimagic labeling if for any two adjacent edges $e = uv$ and $e^{'} = vw$ of G we have $w(e) \neq w(e')$, where $w(e) = f(u) + f(v)$ and $w(e^{'}) = f(v) + f(w)$. A graph G is local edge antimagic if G has a local edge antimagic labeling. The local edge antimagic chromatic number $\chi^{'}_{lea}(G)$ is defined to be the minimum number of colors taken overall coloring of G induced by local edge antimagic labeling of G. They obtained a trivial lower bound and proved the following results.

Theorem 1.1. [4] If $\Delta(G)$ is maximum degree of G, then we have $\chi^{'}_{lea}(G) \geq \Delta(G)$.

Theorem 1.2. [4] For $n \geq 3$, the local edge antimagic chromatic number of P_n is $\chi^{'}_{lea}(P_n) = 2$.

Theorem 1.3. [4] For $n \geq 3$, the local edge antimagic chromatic number of C_n is $\chi^{'}_{lea}(C_n) = 3$.

The *friendship graph* F_n is a set of n triangles having a common central vertex and otherwise disjoint.

Theorem 1.4. [4] For $n \geq 3$, the local edge antimagic chromatic number of F_n is $\chi'_{lea}(F_n) = 2n + 1$.

Theorem 1.5. [4] For $n \geq 3$, the local edge antimagic chromatic number of W_n is $\chi'_{lea}(W_n) = n + 2$.

Theorem 1.6. [4] For $n \geq 3$, the local edge antimagic chromatic number of K_n is $\chi'_{lea}(K_n) = 2n - 3$.

In this paper, we determine the local edge antimagic chromatic number for wheel related graphs.

2. Local edge Chromatic Number of Wheel related graphs

This section shows that the local edge antimagic chromatic number for the friendship graph F_n and wheel graph W_n.

These results show that the result given in Agustin et al.[4] are not correct.

Theorem 2.1. *For the friendship graph F_n, we have*

$$\chi'_{lea}(F_n) = \begin{cases} 3 & \text{if } \; n = 1, \\ 2n & \text{if } \; n \geq 2. \end{cases}$$

Proof. Let $V(F_n) = \{c, u_i, v_i, 1 \leq i \leq n\}$ and $E(F_n) = \{cu_i, cv_i, u_iv_i, 1 \leq i \leq n\}$ be the vertex set and edge set of F_n. Then $|V(F_n)| = 2n + 1$ and $|E(F_n)| = 3n$. If $n = 1$ then $F_1 \cong C_3$ and by Theorem 1.3[4], it follows, we get $\chi'_{lea}(F_1) = 3$. For $n \geq 2$, define a bijection $f_1 : V(F_n) \to \{1, 2, 3, ..., 2n + 1\}$ by

$f_1(c) \quad = 2n$

$f_1(u_i) \quad = 2i - 1, 1 \leq i \leq n$

$$f_1(v_i) = \begin{cases} 2n + 1 & \text{if } \; i = 1, \\ 2n + 2 - 2i & \text{if } \; 2 \leq i \leq n. \end{cases}$$

Then the edge weights of F_n are

$$w_1(cu_i) \quad = 2n + 2i - 1, 1 \leq i \leq n \;\; w_1(cv_i) = \begin{cases} 4n + 1 & \text{if } \; i = 1, \\ 4n + 2 - 2i & \text{if } \; 2 \leq i \leq n. \end{cases}$$

$$w_1(u_iv_i) = \begin{cases} 2n + 2 & \text{if } \; i = 1, \\ 2n + 1 & \text{if } \; 2 \leq i \leq n. \end{cases}$$

The weights of the edges $\{cu_i, 1 \le i \le n, cv_1, cv_i, 2 \le i \le n\}$, under the labeling f_1, constitute the sets $\{2n+1, 2n+3, 2n+5, \ldots, 4n-1\}, \{4n+1\}, \{4n-2, 4n-4, 4n-6, \ldots, 2n+2\}$ and rest of the edge weights of F_n, under the labeling f_1, constitute the set $\{2n+2, 2n+1\}$. Hence these sets consist of $2n$ weights (colors) and for any two adjacent edges are received different colors. Therefore, f_1 induces a proper edge coloring of F_n and hence $\chi'_{lea}(F_n) \le 2n$. Since $\Delta(F_n) = 2n$, it follows, we get $\chi'_{lea}(F_n) \ge 2n$. Thus $\chi'_{lea}(F_n) = 2n$. □

Theorem 2.2. *For the wheel graph W_n on $n+1$ vertices, we have*

$$\chi'_{lea}(W_n) = \begin{cases} 5 & \text{if} \quad n = 3, 4 \\ n & \text{if} \quad n \ge 5. \end{cases}$$

Proof. Let $V(W_n) = \{c, v_i, 1 \le i \le n\}$ and $E(W_n) = \{cv_i, 1 \le i \le n\} \cup \{v_i v_{i+1}, 1 \le i \le n-1\} \cup \{v_n v_1\}$ be the vertex set and edge set of W_n. Then $|V(W_n)| = n+1$ and $|E(W_n)| = 2n$.

Case-1: $n = 3, 4$
If $n = 3$ then $W_3 \cong K_4$ and by Theorem 1.6[4], it follows, we get $\chi'_{lea}(W_3) = 5$. For $n = 4$, we assume that $\chi'_{lea}(W_4) = 4$. Then there exists a local edge antimagic labeling f with 4-colors (edge weights) w_1, w_2, w_3 and w_4. Clearly, the incident edges of the central vertex c are received the colors $w(cv_i) = w_i, 1 \le i \le 4$ and hence the edges $e_1 = v_1v_2, e_2 = v_2v_3, e_3 = v_3v_4$, and $e_4 = v_4v_1$ are must recevied the colors from the set $\{w_1, w_2, w_3, w_4\}$. Therefore, every color $w_i, 1 \le i \le 4$ occurs exactly two times and hence $\sum\limits_{i=1}^{5} deg(v_i)f(v_i) = 2\sum\limits_{i=1}^{4} w_i$, which implies that $3\left[\frac{5\times 6}{2} - f(c)\right] + 4f(c) = 2\sum\limits_{i=1}^{4} w_i$. This implies $f(c) = 2\sum\limits_{i=1}^{4} w_i - 45$. Hence $f(c)$ =1 or 3 or 5. If $f(c) = 1$ then there is no edge $e_i, 1 \le i \le 4$ received the edge weight 3 or 4, which is a contradiction. If $f(c) = 3$ then $f(v_i) \in \{1, 2, 4, 5\}$, $1 \le i \le 4$ and $w(cv_i) \in \{4, 5, 7, 8\}$. Hence there is no edge $e_i, 1 \le i \le 4$ received the edge weight 4, which is a contradiction. If $f(c) = 5$ then $f(v_i) \in \{1, 2, 3, 4\}$ and $w(cv_i) \in \{6, 7, 8, 9\}$. Hence there is no edge $e_i, 1 \le i \le 4$ received the edge weight 9, which is a contradiction. Thus $\chi'_{lea}(W_4) \ge 5$.

Now, define a labeling $f_2 : V(W_4) \to \{1, 2, 3, 4, 5\}$ by $f_2(c) = 4, f_2(v_1) = 1, f_2(v_2) = 5, f_2(v_3) = 2, f_2(v_4) = 3$. Then the edge weights are $w_2(cv_1) = 5, w_2(cv_2) = 9, w_2(cv_3) = 6, w_2(cv_4) = 7, w_2(v_1v_2) = 6, w_2(v_2v_3) = 7, w_2(v_3v_4) =$

$5, w_2(v_4v_1) = 4$. Thus $\chi'_{lea}(W_4) \leq 5$. Hence $\chi'_{lea}(W_4) = 5$.

Case-2: $n \geq 5$
We define a bijection $f_3 : V(W_n) \to \{1, 2, 3, ..., n+1\}$ by

$$f_3(c) = \begin{cases} \frac{n+1}{2} & \text{if } n \text{ is odd,} \\ \frac{n+4}{2} & \text{if } n \text{ is even.} \end{cases}$$

$$f_3(v_i) = \begin{cases} \frac{i+1}{2} & \text{if } i \text{ is odd and } i \neq n \\ n+2-\frac{i}{2} & \text{if } i \text{ is even and } i \neq n-2, n \\ \frac{n+3}{2} & \text{if } n \text{ is odd and } i = n \\ \frac{n+2}{2} & \text{if } n \text{ is even and } i = n-2 \\ \frac{n+6}{2} & \text{if } n \text{ is even and } i = n. \end{cases}$$

Then the edge weights of W_n are

$$w_3(cv_i) = \begin{cases} \frac{n+2+i}{2} & \text{if } n \text{ is odd and } i \text{ is odd } 1 \leq i \leq n-2 \\ \frac{3n+5-i}{2} & \text{if } n \text{ is odd and } i \text{ is even } 2 \leq i \leq n-1 \\ n+2 & \text{if } n \text{ is odd and } i = n \\ \frac{n+5+i}{2} & \text{if } n \text{ is even and } i \text{ is odd } 1 \leq i \leq n-1 \\ \frac{3n+8-i}{2} & \text{if } n \text{ is even and } i \text{ is even } 2 \leq i \leq n-4 \\ n+3 & \text{if } n \text{ is even and } i = n-2 \\ n+5 & \text{if } n \text{ is even and } i = n. \end{cases}$$

$$w_3(v_iv_{i+1}) = \begin{cases} n+2 & \text{if } n \text{ is odd and } i \text{ is odd } 1 \leq i \leq n-2 \\ n+2 & \text{if } n \text{ is even and } i \text{ is odd } 1 \leq i \leq n-5 \\ n+3 & \text{if } n \text{ is odd and } i \text{ is even } 2 \leq i \leq n-3 \\ n+3 & \text{if } n \text{ is even and } i \text{ is even } 2 \leq i \leq n-4. \end{cases}$$

$$\begin{aligned} w_3(v_{n-1}v_n) &= n+4, \text{if } n \text{ is odd} \\ w_3(v_nv_1) &= \tfrac{n+5}{2}, \text{if } n \text{ is odd} \\ w_3(v_{n-3}v_{n-2}) &= n, \text{if } n \text{ is even} \\ w_3(v_{n-2}v_{n-1}) &= n+1, \text{if } n \text{ is even} \\ w_3(v_{n-1}v_n) &= n+3, \text{if } n \text{ is even} \\ w_3(v_nv_1) &= \tfrac{n+8}{2}, \text{if } n \text{ is even.} \end{aligned}$$

For n is odd, the weights of the edges $\{cv_i, i \neq n \text{ is odd}, cv_n, cv_i, i \geq 2 \text{ is even}\}$, under the labeling f_3, constitute the sets $\{\frac{n+1}{2}+1, \frac{n+1}{2}+2, \frac{n+1}{2}+$

$3,\ldots,n+1\},\{n+2\},\{n+2+\frac{n+1}{2}-1,n+2+\frac{n+1}{2}-2,n+2+\frac{n+1}{2}-3,\ldots,n+2+\frac{n+1}{2}-\frac{n}{2}\}$ and rest of the edge weights of W_n, under the labeling f_3, constitute the set $\{n+2,n+3,n+4,\frac{n+5}{2}\}$. For n is even, the weights of the edges $\{cv_i, i \text{ is odd}, cv_{n-2}, cv_n, cv_i, i \neq n-2, n \text{ is even}\}$, under the labeling f_3, constitute the sets $\{\frac{n}{2}+3,\frac{n}{2}+4,\ldots,n+2\},\{n+3\},\{n+5\},\{n+4+\frac{n}{2}-1,n+4+\frac{n}{2}-2,\ldots,n+6\}$ and rest of the edges weights of W_n, under the labeling f_3, constitute the set $\{\frac{n}{2}+4,n,n+1,n+2,n+3\}$. Hence, these sets consist of n weights(colors) and for any two adjacent edges are received different colors. Therefore, f_3 induces a proper edge coloring of W_n and hence $\chi'_{lea}(W_n) \leq n$. Since $\Delta(W_n)=n$ and by Theorem 1.1[4], it follows, we get $\chi'_{lea}(W_n) \geq n$. Thus $\chi'_{lea}(W_n)=n$. □

A *fan graph* $T_n, n \geq 2$ is a graph obtained by joining all vertices of path P_n to a further vertex, called the *central vertex*.

Theorem 2.3. *For the fan graph T_n on $n+1$ vertices, we have*

$$\chi'_{lea}(T_n) = \begin{cases} n+1 & \text{if } n=2,3 \\ n & \text{if } \ n \geq 4. \end{cases}$$

Proof. Let $V(T_n)=\{c,v_i,1 \leq i \leq n\}$ and $E(T_n)=\{cv_i,1 \leq i \leq n\} \cup \{v_iv_{i+1},1 \leq i \leq n-1\}$ be the vertex set and edge set of T_n. Then $|V(T_n)|=n+1$ and $|E(T_n)|=2n-1$.

Case-1: $n=2,3$.
Since $T_2 \cong K_3$, and by Theorem 1.6[4], we get $\chi'_{lea}(T_2)=3$. For $n=3$, suppose $\chi'_{lea}(T_3)=3$, then there exists a local edge antimagic labeling f with 3-colors (edge weights) w_1, w_2 and w_3. Let $V(T_3)=\{c,v_1,v_2,v_3\}$ and $E(T_3)=\{cv_1,cv_2,cv_3,v_1v_2,v_2v_3\}$ be the vertex set and edge set of T_3. Since $\Delta(T_3)=3$, it follows, the incident edges of the central vertex c are received the colors w_1, w_2 and w_3 and hence the edges v_1v_2 and v_2v_3 are must received the colors w_3 and w_1. Therefore, the colors w_1 and w_3 are used two times and w_2 used only one time. Since $3 \leq w(e) \leq 7, e \in E(T_3)$, it follows, a weight 5 only two possibles sets of two elements $\{1,4\}$ and $\{2,3\}$ and all other weights $3,4,6$ and 7 are only one possible set of two elements. Therefore, $w_1=5$ or $w_3=5$. Suppose $w_1=5$. Then $f(c)=1$ or 4, $f(v_1)=4$ or 1 and hence $f(v_2),f(v_3) \in \{2,3\}$ and $w(v_1v_2) \in \{6,7\}$ or $\{3,4\}$. Thus an edge v_1v_2 with weight $w(v_1v_2) \neq w_1$, which is a contradiction. If $w_3=5$ then $f(c)=2$ or 3, $f(v_1)=3$ or 2 and hence $f(v_2),f(v_3) \in \{1,4\}$ and

$w(v_1v_2) \in \{4,7\}$ or $\{3,6\}$. Thus an edge v_1v_2 with weight $w(v_1v_2) \neq w_3$, which is a contradiction. Thus $\chi'_{lea}(T_3) \geq 4$.

Now, we define the labeling $f_4 : V(T_3) \to \{1,2,3,4\}$ by $f_4(c) = 3, f_4(v_1) = 1, f_4(v_2) = 4, f_4(v_3) = 2$. Then the edge weight of T_3 are $w_4(cv_1) = 4, w_4(cv_2) = 7, w_4(cv_3) = 5, w_4(v_1v_2) = 5, w_4(v_2v_3) = 6$. Thus $\chi'_{lea}(T_3) \leq 4$. Hence $\chi'_{lea}(T_3) = 4$.

Case-2: $n \geq 4$.
We define a bijection $f_5 : V(T_n) \to \{1,2,3,...,n+1\}$ by

$$f_5(c) = n$$

$$f_5(v_i) = \begin{cases} \frac{i+1}{2} & \text{if } i \text{ is odd}, 1 \leq i \leq n \\ n+1 & \text{if } i = 2 \\ n+1-\frac{i}{2} & \text{if } i \text{ is even}, 4 \leq i \leq n. \end{cases}$$

Then the edge weights of T_n are

$$w_5(cv_i) = \begin{cases} n+\frac{i+1}{2} & \text{if } i \text{ is odd}, 1 \leq i \leq n \\ 2n+1 & \text{if } i = 2 \\ 2n+1-\frac{i}{2} & \text{if } i \text{ is even}, 4 \leq i \leq n. \end{cases}$$

$$w_5(v_iv_{i+1}) = \begin{cases} n+1 & \text{if } i \text{ is odd}, 3 \leq i \leq n \\ n+3 & \text{if } i = 2 \\ n+2 & \text{if } i = 1 \text{ and } i \text{ is even}, 4 \leq i \leq n. \end{cases}$$

The weights of the edges $\{cv_i, i \text{ is odd} \cup cv_2, cv_i, i \geq 4 \text{ is even}\}$, under the labeling f_5, constitute the sets $\{n+1, n+2, n+3, \dots, n+\frac{n+1}{2}\}, \{2n+1\}, \{2n-1, 2n-2, \dots, 2n+1-\frac{n}{2}\}$ and rest of the edges weights of T_n, under the labeling f_5, constitute the set $\{n+1, n+3, n+2\}$. Hence, these sets consist of n weights(colors) and for any two adjacent edges are received different colors. Therefore, f_5 induces a proper edge coloring of T_n and hence $\chi'_{lea}(T_n) \leq n$. Since $\Delta(T_n) = n$ and by Theorem 1.1[4], it follows, we get $\chi'_{lea}(T_n) \geq n$. Thus $\chi'_{lea}(T_n) = n$. □

The *helm graph* H_n is a graph obtained from the wheel graph by adjoining a pendant edge at each node of the cycle.

P1:Procedure for obtaining the vertices u_1, v_2, v_3 and c labels of H_3 graph

Let $V(H_3) = \{c, v_i, u_i, 1 \leq i \leq 3\}$ and $E(H_3) = \{cv_i, v_iu_i, 1 \leq i \leq 3\} \cup \{v_1v_2, v_2v_3, v_3v_1\}$ be the vertex set and edge set of H_3. Then $|V(H_3)| = 7$.

Let S_1 be the set of all possible four weights w_1, w_2, w_3 and ,w_4. Clearly, $5 \leq w \leq 11$, where $w \in \{w_1, w_2, w_3, w_4\}$. Let $[n]$ denote the set of all positive integers less than or equal to n.

Step 1: Let $s \in S_1$ and $f(v_1) = x, 1 \leq x \leq 7$. Then we construct a 7×4 subtraction table using $f(v) = w_i - x$,$1 \leq x \leq 7$, $1 \leq i \leq 4$, $v \in \{u_1, v_2, v_3, c\}$

Step 2: If $f(v) \leq 0, f(v) = f(v_1)$ and $f(v) \geq 8$ then remove the corresponding row labels from the 7×4 subtraction table. The remaining row labels are received by the vertices u_1, v_2, v_3 and c. Clearly, $f(v) \in \{1, 2, 3, 4, 5, 6, 7\}, v \in \{u_1, v_2, v_3, c\}$.

Step 3: The edges $e_1 = v_1u_1, e_2 = v_1v_2, e_3 = v_1v_3$ and $e_4 = v_1c$ with their weights $w(e_i) \in \{w_1, w_2, w_3, w_4\}$. Use the vertices labels which are obtained from Step 2 to form a weight $w = w(e = uu') = f(u) + f(u'), u, u' \in \{u_1, v_2, v_3, c\}, e \in \{e_1, e_2, e_3, e_4\}$.

Step 4: If $w(e) \notin \{w_1, w_2, w_3, w_4\}$ for some e then $\chi'_{lea}(H_3) \neq 4$. Otherwise, $\chi'_{lea}(H_3) = 4$ provided the edges $e'_i = v_iu_i$ with their weights $w(e'_i) \in \{w_1, w_2, w_3, w_4\}$ for all $i = 2, 3$. These edge weights are obtained from the vertices labels $f(u_i) = [7] - \{f(c), f(v_1), f(v_2), f(v_3)\}$.

Theorem 2.4. For the helm graph H_3, we have $\chi'_{lea}(H_3) = 5$.

Proof. Let $V(H_3) = \{c, v_i, u_i, 1 \leq i \leq 3\}$ and $E(H_3) = \{cv_i, v_iu_i, 1 \leq i \leq 3\} \cup \{v_1v_2, v_2v_3, v_3v_1\}$ be the vertex set and edge set of H_3. Then $|V(H_3)| = 7$ and $|E(H_3)| = 9$. Suppose $\chi'_{lea}(H_3) = 4$. Then there exists a local edge antimagic labeling f with 4-colors (edge weights) w_1, w_2, w_3 and w_4. Since $\Delta(H_3) = 4$, it follows, the incident edges of the central vertex c received the colors w_1, w_2, w_3 and w_4. The minimum and maximum possible edge weights are 5 and 11. Let S_1 be set of all possible four edge weights set from the set $\{5, 6, 7, 8, 9, 10, 11\}$. Then there are 35 possible such sets are given as follows:

$S_1 = \{\{5,6,7,8\},\{5,6,7,9\},\{5,6,7,10\},\{5,6,7,11\},\{5,6,8,9\},\{5,6,8,10\},$
$\{5,6,8,11),\{5,6,9,10\},\{5,6,9,11\},\{5,6,10,11\},\{5,7,8,9\},\{5,7,8,10\},$
$\{5,7,8,11\},\{5,7,9,10\},\{5,7,9,11\},\{5,7,10,11\},\{5,8,9,10\},\{5,8,9,11\},$
$\{5,8,10,11\},\{5,9,10,11\},\{6,7,8,9\},\{6,7,8,10\},\{6,7,8,11\},\{6,7,9,10\},$
$\{6,7,9,11\},\{6,7,10,11\},\{6,8,9,10\},\{6,8,9,11\},\{6,8,10,11\},\{6,9,10,11\},$
$\{7,8,9,10\},\{7,8,9,11\},\{7,8,10,11\},\{7,9,10,11\},\{8,9,10,11\}\}$.

We apply the above procedure P1 and obtain the vertices u_1, v_2, v_3 and c labels of H_3. Let $e_1' = cv_2, e_2' = cv_3$ and $e_3' = v_2v_3$. Then form all possible edge weights $w(e_i'), i = 1,2,3$ from the labels $\{f(u_1), f(v_2), f(v_3), f(c)\}$. Clearly, at least one of the edge weight $w' \notin \{w_1, w_2, w_3, w_4\} \in S_1$, which is a contradiction. Thus $\chi'_{lea}(H_3) \geq 5$.

Now, we define a labeling $f_6 : V(H_3) \to \{1,2,3,4,5,6,7\}$ by $f_6(c) = 3, f_6(u_1) = 4, f_6(u_2) = 5, f_6(u_3) = 6, f_6(v_1) = 7, f_6(v_2) = 2, f_6(v_3) = 1$. Then the edge weight of H_3 are $w_6(cu_1) = 7, w_6(cu_2) = 8, w_6(cu_3) = 9, w_6(u_1u_2) = 9, w_6(u_2u_3) = 11, w_6(u_3u_1) = 10, w_6(u_1v_1) = 11, w_6(u_2v_2) = 7, w_6(u_3v_3) = 7$. Thus $\chi'_{lea}(H_3) \leq 5$. Hence $\chi'_{lea}(H_3) = 5$. □

$P2$:**Procedure for obtaining the vertices v_1, v_2, v_3 and v_4 labels of H_4 graph**
Let $V(H_4) = \{c, v_i, u_i, 1 \leq i \leq 4\}$ $E(H_4) = \{cv_i, v_iu_i, 1 \leq i \leq 4\} \cup \{v_iv_{i+1}, 1 \leq i \leq 3\} \cup \{v_1v_4\}$ be the vertex set and edge set of H_4. Then $|V(H_4)| = 9$.

Let S_2 be the set of all possible four weights set $\{w_1, w_2, w_3, w_4\}$. Clearly, $7 \leq w \leq 13$, where $w \in \{w_1, w_2, w_3, w_4\}$.

Step 1: Let $s \in S_2$ and $f(c) = x, 1 \leq x \leq 9$. Then we construct a 9×4 subtraction table using $f(v_i) = w_i - x$,$1 \leq i \leq 4$.

Step 2: If $f(v_i) \leq 0, f(v_i) = f(c)$ and $f(v_i) \geq 10$ then remove the corresponding row labels from the 9×4 subtraction table. The remaining row labels are received by the vertices v_1, v_2, v_3 and v_4. Clearly, $f(v_i) \in \{1,2,3,4,5,6,7,8,9\}, i = 1,2,3,4$.

Step 3: The edges $e_1 = v_1v_2, e_2 = v_2v_3, e_3 = v_3v_4$ and $e_4 = v_4v_1$ with their weights $w(e_i) \in \{w_1, w_2, w_3, w_4\}$. Use the vertices labels which are obtained from Step 2 to form a weight $w(e = uv) = f(u) + f(v), u, v \in \{v_1, v_2, v_3, v_4\}, e \in \{e_1, e_2, e_3, e_4\}$.

Step 4: If $w(e) \notin \{w_1, w_2, w_3, w_4\}$ for some e then $\chi'_{lea}(H_4) \neq 4$. Otherwise, $\chi'_{lea}(H_4) = 4$ provided the pendant edges $e'_i = v_i u_i, i = 1, 2, 3, 4$ with their weights $w(e'_i) \in \{w_1, w_2, w_3, w_4\}$ for all $i = 1, 2, 3, 4$. These pendant edge weights are obtained from the pendant vertices labels $f(u_i) = [9] - \{f(c), f(v_i), 1 \leq i \leq 4\}$.

Theorem 2.5. *For the helm graph H_4, we have $\chi'_{lea}(H_4) = 5$.*

Proof. Let $V(H_4) = \{c, v_i, u_i, 1 \leq i \leq 4\}$ and $E(H_4) = \{cv_i, v_i u_i, 1 \leq i \leq 4\} \cup \{v_i v_{i+1}, 1 \leq i \leq 3\} \cup \{v_1 v_4\}$ be the vertex set and edge set of H_4. Then $|V(H_4)| = 9$ and $|E(H_4)| = 12$. Suppose $\chi'_{lea}(H_4) = 4$. Then there exists a local edge antimagic labeling f with 4-colors w_1, w_2, w_3 and w_4. Every color $w_i, 1 \leq i \leq 4$ must assigned to three nonadjacent edges of H_4. So, every edge $e = uv$ with weight $w(e)$ has at least 3 possibles two elements sets. The minimum and maximum possible edge weights are 7 and 13. Let S_2 be the collection of all possible 4 edge weights from the set $\{7, 8, 9, 10, 11, 12, 13\}$. Then there are 35 possible such sets are given as follows:

$S_2 = \{\{7,8,9,10\}, \{7,8,9,11\}, \{7,8,9,12\}, \{7,8,9,13\}, \{7,8,10,11\},$
$\{7,8,10,12\}, \{7,8,10,13\}, \{7,8,11,12\}, \{7,8,11,13\}, \{7,8,12,13\},$
$\{7,9,10,11\}, \{7,9,10,12\}, \{7,9,10,13\}, \{7,9,11,12\}, \{7,9,11,13\},$
$\{7,9,12,13\}, \{7,10,11,12\}, \{7,10,11,13\}, \{7,10,12,13\}, \{7,11,12,13\},$
$\{8,9,10,11\}, \{8,9,10,12\}, \{8,9,10,13\}, \{8,9,11,12\}, \{8,9,11,13\},$
$\{8,9,12,13\}, \{8,10,11,12\}, \{8,10,11,13\}, \{8,10,12,13\}, \{8,11,12,13\},$
$\{9,10,11,12\}, \{9,10,11,13\}, \{9,10,12,13\}, \{9,11,12,13\}, \{10,11,12,13\}\}.$

We apply the above procedure $P2$ and obtain the vertices v_1, v_2, v_3 and v_4 labels of H_4. Let $e'_1 = v_1 v_2, e'_2 = v_2 v_3, e'_3 = v_3 v_4$ and $e'_4 = v_1 v_4$. Then form all possible edge weights $w(e'_i), i = 1, 2, 3, 4$ from the labels $\{f(v_1), f(v_2), f(v_3), f(v_4), f(c)\}$. Clearly, at least one of the edge weight $w' \notin \{w_1, w_2, w_3, w_4\}$, where $w' \in \{w(e'_1), w(e'_2), w(e'_3), w(e'_4)\}$, which is a contradiction. Thus $\chi'_{lea}(H_4) \geq 5$.

Now, define the labeling $f_7 : V(H_4) \to \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ by $f_7(c) = 5, f_7(u_1) = 7, f_7(u_2) = 4, f_7(u_3) = 6, f_7(u_4) = 3, f_7(v_1) = 1, f_7(v_2) = 8, f_7(v_3) = 2, f_7(v_4) = 9$. Then the edge weights are $w_7(cu_1) = 12, w_7(cu_2) = 9, w_7(cu_3) = 11, w_7(cu_4) = 8, w_7(u_1 u_2) = 11, w_7(u_2 u_3) = 10, w_7(u_3 u_4) = 9, w_7(u_4 u_1) = 10, w_7(u_1 v_1) = 8, w_7(u_2 v_2) = 12, w_7(u_3 v_3) = 8, w_7(u_4 v_4) = 12$. Thus $\chi'_{lea}(H_4) \leq 5$. Hence $\chi'_{lea}(H_4) = 5$. $\square$

Theorem 2.6. *For the helm graph $H_n, n \geq 5$, we have $\chi'_{lea}(H_n) = n$.*

Proof. Let $V(H_n) = \{c, u_i, v_i, 1 \leq i \leq n\}$ and $E(H_n) = \{cu_i, u_iv_i, 1 \leq i \leq n\} \cup \{u_iu_{i+1}, 1 \leq i \leq n-1\} \cup \{u_nu_1\}$. Then $|V(H_n)| = 2n+1$ and $|E(H_n)| = 3n$. Define a bijection $f_8 : V(H_n) \to \{1, 2, \ldots, 2n+1\}$ by

$$f_8(c) = n+1$$

$$f_8(u_i) = \begin{cases} \frac{n+2+i}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{3n+5-i}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-1 \\ n+2 & \text{if } n \text{ is odd, } i = n \\ \frac{3n+3-i}{2} & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ n & \text{if } n \text{ is even, } i = 2 \\ \frac{n-2+i}{2} & \text{if } n \text{ is even and } i \text{ is even, } 4 \leq i \leq n \end{cases}$$

$$f_8(v_i) = \begin{cases} \frac{4n+3-i}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{i}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-1 \\ \frac{n+1}{2} & \text{if } n \text{ is odd, } i = n \\ \frac{i+1}{2} & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ \frac{3n+4}{2} & \text{if } n \text{ is even and } i = 2 \\ \frac{4n+6-i}{2} & \text{if } n \text{ is even and } i \text{ is even, } 4 \leq i \leq n \end{cases}$$

The edge weights of H_n are

$$w_8(cu_i) = \begin{cases} \frac{3n+4+i}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{5n+7-i}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-1 \\ 2n+3 & \text{if } n \text{ is odd, } i = n \\ \frac{5n+5-i}{2} & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ 2n+1 & \text{if } n \text{ is even, } i = 2 \\ \frac{3n+i}{2} & \text{if } n \text{ is even and } i \text{ is even, } 4 \leq i \leq n \end{cases}$$

$$w_8(u_iv_i) = \begin{cases} \frac{5n+5}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{3n+5}{2} & \text{if } n \text{ is odd and } i \text{ is even, } i = n \text{ and } 2 \leq i \leq n-1 \\ \frac{3n+4}{2} & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ \frac{5n+4}{2} & \text{if } n \text{ is even and } i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$w_8(u_iu_{i+1}) = \begin{cases} 2n+3 & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ 2n+4 & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-3 \\ \frac{5n+2}{2} & \text{if } n \text{ is even and } i = 1 \\ \frac{5n}{2} & \text{if } n \text{ is even and } i = 2 \\ 2n+1 & \text{if } n \text{ is even and } i \text{ is odd, } 3 \leq i \leq n-1 \\ 2n & \text{if } n \text{ is even and } i \text{ is even, } 4 \leq i \leq n-2 \end{cases}$$

$$\begin{aligned} w_8(u_{n-1}u_n) &= 2n+5, \text{if } n \text{ is odd,} \\ w_8(u_nu_1) &= \tfrac{3n+7}{2}, \text{if } n \text{ is odd,} \\ w_8(u_nu_1) &= \tfrac{5n}{2}, \text{if } n \text{ is even.} \end{aligned}$$

For n is odd, the weights of the edges $\{cu_i, i \text{ is odd}, cu_n, cu_i, i \text{ is even}\}$, under the labeling f_8, constitute the sets $\{n+2+\frac{n+1}{2}, n+2+\frac{n+3}{2}, n+2+\frac{n+5}{2}, \ldots, 2n+1\}, \{2n+3\}, \{2n+3+\frac{n-1}{2}, 2n+3+\frac{n-3}{2}, 2n+3+\frac{n-5}{2}, \ldots, 2n+4\}$. For n is even, the weights of the edges $\{cu_i, i \text{ is odd}, cu_2, cu_i, i \geq 4 \text{ is even}\}$, under the labeling f_8, constitute the sets $\{2n+2+\frac{n}{2}, 2n+2+\frac{n-2}{2}, 2n+2+\frac{n-4}{2}, \ldots, 2n+4, 2n+3\}, \{2n+1\}, \{2n+\frac{n}{2}+2, 2n+\frac{n}{2}+3, 2n+\frac{n}{2}+4, \ldots, 2n\}$ and rest of the edge weights of H_n, under the labeling f_8, are belongs to the weight of $w(cu_i), 1 \leq i \leq n$. Hence these sets consist of n colors and for any two adjacent edges are received different colors. Therefore f_8 induces a proper edge coloring of H_n and hence $\chi'_{lea}(H_n) \leq n$. Since $\Delta(H_n) = n$, it follows, we get $\chi'_{lea}(H_n) \geq n$. Thus $\chi'_{lea}(H_n) = n$. □

A flower graph Fl_n is a graph obtained from a helm H_n by joining every pendant vertex to the central vertex.

Theorem 2.7. *For the flower graph Fl_n, we have $\chi'_{lea}(Fl_n) = 2n, n \geq 3$.*

Proof. Let $V(Fl_n) = \{c, u_i, v_i, 1 \leq i \leq n\}$ and $E(Fl_n) = \{cu_i, cv_i, u_iv_i, 1 \leq i \leq n\} \cup \{u_iu_{i+1}, 1 \leq i \leq n-1\} \cup \{u_nu_1\}$. Then $|V(Fl_n)| = 2n+1$ and $|E(Fl_n)| = 4n$.

For $n = 3$, we define a labeling $f_9 : V(Fl_3) \to \{1, 2, 3, 4, 5, 6, 7\}$ by $f_9(c) = 3, f_9(u_1) = 1, f_9(u_2) = 4, f_9(u_3) = 6, f_9(v_1) = 7, f_9(v_2) = 5, f_9(v_3) = 2$. Then the edge weights of Fl_3 are $w_9(cu_1) = 4, w_9(cu_2) = 7, w_9(cu_3) = 9, w_9(cv_1) = 10, w_9(cv_2) = 8, w_9(cv_3) = 5, w_9(u_1u_2) = 5, w_9(u_2u_3) = 10, w_9(u_3u_1) = 7, w_9(u_1v_1) = 8, w_9(u_2v_2) = 9, w_9(u_3v_3) = 8$. Therefore, $\chi'_{lea}(Fl_3) \leq 6$. Since $\Delta(Fl_3) = 6$ and by Theorem 1.1[4], it follows, we get $\chi'_{lea}(Fl_3) \geq 6$. Hence $\chi'_{lea}(Fl_3) = 6$. For $n \geq 4$, we define a bijection $f_{10} : V(Fl_n) \to \{1, 2, \ldots, 2n+1\}$ by

$$f_{10}(c) = \begin{cases} \frac{3n+3}{2} & \text{if } n \text{ is odd} \\ \frac{3n+2}{2} & \text{if } n \text{ is even} \end{cases}$$

$$f_{10}(u_i) = \begin{cases} \frac{4n+3-i}{2} & \text{if } n \text{ is odd and } i \text{ is odd}, 1 \leq i \leq n-2 \\ \frac{n+1+i}{2} & \text{if } n \text{ is odd and } i \text{ is even}, 2 \leq i \leq n-1 \\ \frac{3n+1}{2} & \text{if } n \text{ is odd}, i = n \\ \frac{4n+3-i}{2} & \text{if } n \text{ is even and } i \text{ is odd}, 1 \leq i \leq n-1 \\ \frac{2n+2-i}{2} & \text{if } n \text{ is even and } i \text{ is even}, 2 \leq i \leq n \end{cases}$$

$$f_{10}(v_i) = \begin{cases} \frac{i+1}{2} & \text{if } n \text{ is odd and } i \text{ is odd}, 1 \leq i \leq n \\ \frac{2n+i}{2} & \text{if } n \text{ is odd and } i \text{ is even}, 2 \leq i \leq n-1 \\ \frac{i+1}{2} & \text{if } n \text{ is even and } i \text{ is odd}, 1 \leq i \leq n-1 \\ \frac{2n+i}{2} & \text{if } n \text{ is even and } i \text{ is even}, 2 \leq i \leq n \end{cases}$$

Then the edge weights of Fl_n are

$$w_{10}(cu_i) = \begin{cases} \frac{7n+6-i}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{4n+4+i}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-1 \\ 3n+2 & \text{if } n \text{ is odd, } i = n \\ \frac{7n+5-i}{2} & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ \frac{5n+4-i}{2} & \text{if } n \text{ is even and } i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$w_{10}(cv_i) = \begin{cases} \frac{3n+4+i}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n \\ \frac{5n+3+i}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-1 \\ \frac{3n+3+i}{2} & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ \frac{5n+2+i}{2} & \text{if } n \text{ is even and } i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$w_{10}(u_iv_i) = \begin{cases} 2n+2 & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{3n+2i+1}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-1 \\ 2n+1 & \text{if } n \text{ is odd, } i = n \\ 2n+2 & \text{if } n \text{ is even and } i \text{ is odd, } 1 \leq i \leq n-1 \\ 2n+1 & \text{if } n \text{ is even and } i \text{ is even, } 2 \leq i \leq n \end{cases}$$

$$w_{10}(u_iu_{i+1}) = \begin{cases} \frac{5n+5}{2} & \text{if } n \text{ is odd and } i \text{ is odd, } 1 \leq i \leq n-2 \\ \frac{5n+3}{2} & \text{if } n \text{ is odd and } i \text{ is even, } 2 \leq i \leq n-3 \\ 3n+2-i & \text{if } n \text{ is even, } 1 \leq i \leq n-1 \end{cases}$$

$$\begin{aligned} w_{10}(u_{n-1}u_n) &= \tfrac{5n+1}{2}, \text{if } n \text{ is odd,} \\ w_{10}(u_nu_1) &= \tfrac{7n+3}{2}, \text{if } n \text{ is odd,} \\ w_{10}(u_nu_1) &= \tfrac{5n+4}{2}, \text{if } n \text{ is even.} \end{aligned}$$

The weights of the edges $\{cu_i, cv_i, 1 \leq i \leq n\}$, under the labeling f_{10}, constitute the sets $\{w_{10}(cu_i)\}, \{w_{10}(cv_i)\}$ and rest of the edge weights of Fl_n, under the labeling f_{10}, constitute the sets $\{w_{10}(u_iv_i), w_{10}(u_iu_{i+1})\}$. Hence these sets consist of $2n$ colors and for any two adjacent edges are received different colors. Therefore, f_{10} induces a proper edge coloring of Fl_n and hence $\chi'_{lea}(Fl_n) \leq 2n$. Since $\Delta(Fl_n) = 2n$ and by Theorem 1.1[4], it follows, we get $\chi'_{lea}(Fl_n) \geq 2n$. Hence $\chi'_{lea}(Fl_n) = 2n$. □

The Closed Helm graph CH_n is obtained from H_n by adding edges $v_iv_{i+1}, 1 \leq i \leq n-1$ and v_nv_1.

Theorem 2.8. *For the closed helm graph CH_n, $n \geq 6$ and n is even, we have $\chi'_{lea}(CH_n) = n$.*

Proof. Let $V(CH_n) = \{c, u_i, v_i, 1 \leq i \leq n\}$ and $E(CH_n) = \{cu_i, u_iv_i, 1 \leq i \leq n\} \cup \{u_iu_{i+1}, v_iv_{i+1}, 1 \leq i \leq n-1\} \cup \{u_nu_1, v_nv_1\}$.Then $|V(CH_n)| = 2n+1$ and $|E(CH_n)| = 4n$. Now, we define a bijection $f_{11} : V(CH_n) \rightarrow$

$\{1, 2, \ldots, 2n+1\}$ by

$$f_{11}(c) = n+1$$
$$f_{11}(u_i) = \begin{cases} \frac{3n+3-i}{2} & \text{if } i \text{ is odd, } 1 \le i \le n-1 \\ n & \text{if } i = 2 \\ \frac{n-2+i}{2} & \text{if } i \text{ is even, } 4 \le i \le n \end{cases}$$
$$f_{11}(v_i) = \begin{cases} \frac{i+1}{2} & \text{if } i \text{ is odd, } 1 \le i \le n-1 \\ \frac{3n+4}{2} & \text{if } i = 2 \\ \frac{4n+6-i}{2} & \text{if } i \text{ is even, } 4 \le i \le n \end{cases}$$

Then the edge weights of CH_n are

$$w_{11}(cu_i) = \begin{cases} \frac{5n+5-i}{2} & \text{if } i \text{ is odd, } 1 \le i \le n \\ 2n+1 & \text{if } i = 2 \\ \frac{3n+i}{2} & i \text{ is even, } 4 \le i \le n \end{cases}$$
$$w_{11}(u_iu_{i+1}) = \begin{cases} \frac{5n+2}{2}, & \text{if } i = 1 \\ \frac{5n}{2}, & \text{if } i = 2 \\ 2n+1, & \text{if } i \text{ is odd, } 3 \le i \le n-1 \\ 2n, & \text{if } i \text{ is even, } 4 \le i \le n-2 \end{cases}$$
$$w_{11}(u_iv_i) = \begin{cases} \frac{3n+4}{2}, & \text{if } i \text{ is odd, } 1 \le i \le n-1 \\ \frac{5n+4}{2}, & \text{if } i \text{ is even, } 2 \le i \le n \end{cases}$$
$$w_{11}(u_nu_1) = \frac{5n}{2}$$
$$w_{11}(v_iv_{i+1}) = \begin{cases} \frac{3n+6}{2}, & i = 1 \\ \frac{3n+8}{2}, & i = 2 \\ 2n+3, & \text{if } i \text{ is odd, } 3 \le i \le n-1 \\ 2n+4, & \text{if } i \text{ is even, } 4 \le i \le n-2 \end{cases}$$
$$w_{11}(v_nv_1) = \frac{3n+8}{2}$$

The weights of the edges $\{cu_i, 1 \le i \le n\}$, under the labeling f_{11}, constitute the set $\{w_{11}(cu_i)\}$ and rest of the edges weights of CH_n, under the labeling f_{11}, constitute the set $\{w_{11}(u_iv_i), w_{11}(u_iu_{i+1}), w_{11}(v_iv_{i+1})\}$. Hence these sets consist of n colors and for any two adjacent edges are received different colors. Therefore, f_{11} induces a proper edge coloring of CH_n and hence $\chi'_{lea}(CH_n) \le n$. Since $\Delta(CH_n) = n$ and by Theorem 1.1[4], it follows, we get $\chi'_{lea}(CH_n) \ge n$. Hence $\chi'_{lea}(CH_n) = n$. □

3. Conclusion

In this paper, we obtained the local edge chromatic number for a friendship graph, wheel graph, fan graph, helm graph, flower graph, and closed helm graph CH_n, where n is even. The problem of determining the local edge chromatic number for remaining graphs is still open.

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