Proyecciones Journal of Mathematics
Vol. 41, Nº 5, pp. 1213-1228, October 2022.
Universidad Católica del Norte
Antofagasta - Chile

10.22199/issn.0717-6279-5302

On even-odd meanness of super subdivision of some graphs

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Received : January 2022. Accepted : May 2022

Abstract

Graph Labeling is a significant area of graph theory that is used in a variety of applications like coding hypothesis, x-beam crystallography, radar, cosmology, circuit design, correspondence network tending to, and database administration. This study provides a general overview of graph naming in heterogeneous fields, however it primarily focuses on graph subdivision. The even vertex odd meanness of super subdivide of various graphs is discussed in this study. The graphs generated by super subdivided of path, cycle, comb, crown, and planar grid are even-odd mean graphs, according to our proof.

Mathematics Subject Classification: *05C78.*

Keywords: *Labeling, Even-odd meanness, bigraph, Super subdivision.*

1. Introduction

In this work, all diagrams are simple, finite, linked, and undirected. Throughout this paper $G(V, E)$ refers to the graph with p vertices (nodes) and q edges (links). Let $K_{s,t}$ be a complete bipartite graph (bigraph), then the two partition V_1 and V_2 are called s-vertices part and t-vertices part respectively for more terminology and notation we follow Harary [6]. Path on n nodes is symbolized by P_n and a cycle on n nodes is symbolized by C_n. If m number of pendant nodes are attached at each node of G, hence the generated graph obtained from G is the graph $G \odot mK_1$. When $m = 1$, $G \odot K_1$ is called the corona of G. the graph $P_n \odot K_1$ is referred to as a comb and the graph $C_n \odot K_1$ is referred to as crown. Suppose G_1, G_2 are any two diagrams with p_1 and p_2 nodes respectively.

Then the Cartesian product of G_1 and G_2 symbolized by $G_1 \bigotimes G_2$, is the graph with vertex set $V(G_1) \bigotimes V(G_2) = \{(u, v) : u \in V(G_1), v \in V(G_2)\}$ and the ordered pair (u, v) is adjoining to $(\acute{u}, \acute{v})$ either u equal to $\acute{u}$ and $(v, \acute{v}) \in E(G_2)$ or v equal to $\acute{v}$ and $(u, \acute{u}) \in E(G_1)$. For $n \geq 3$, the product $P_m \bigotimes P_n$ is referred to a planar grid, while at $n = 2$ is called ladder and symbolized by L_n.

In 1967 Rosa introduced β-valuation process [8]. This process was used to break down the entire graph into isomorphic subgraphs. After that, this β-valuation was recalled as graceful labeling by Golomb [5]. Many virous uses of graph labeling are studied by V. Yegnanaryanan and P. Vaidhyanathan [12]. A comprehensive graph labeling survey is available in Gallian [4]. Even vertex odd meanness of the graph was provided by R. Vasuki et al. [10]. In the same paper, they invaginated even vertex odd meanness of some standard graphs. In [9] Sethuraman et al. introduced the notion of super subdivision of diagram (graph), also they discussed gracefulness of arbitrary super subdivision of many graphs.

Definition 1.1. *[9] Suppose G is a graph. The G' is considered a super subdivision of graph G if G' is gained from G by substituting each uv of G with a bigraph $K_{2,t}$ by identifying u and v with the two vertices of 2-vertices part of $K_{2,t}$ after deleting the uv edge from G.*

Illustration 1.1. *In the following Figure 1, the graph $G = P_2$ and its super subdivision by $K_{2,7}$ is shown. Here we identify the vertices v_1 and v_2 with the two vertices u_1 and u_2 of 2-vertices part of $K_{2,7}$.*

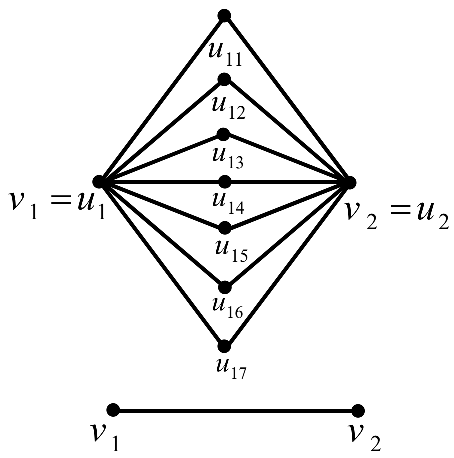

Figure 1. *The path P_2 and its super subdivision.*

Definition 1.2. *[10] Consider a graph $G(V, E)$ with p nodes and q links. If the nodes of a graph $G(V, E)$ can be labeled with even unequal integers from the set $\{0, 2, ..., 2q\}$, such that the link labels generated by the mean of the labels of the ends nodes are different odd numbers from the set $\{1, 3, ..., 2q - 1\}$. Then the graph G is considered to be an even node odd mean graph.*

A graph which allows an even-odd meanness is said to be an even-odd mean graphs [1,2,3,7,11]. Consider a and b are two integers, then we refer $[a, b]$ to be an interval of integers c, where $a \leq c \leq b$.

2. Constructing an even-odd Meanness

Theorem 2.1. *For any $n \geq 2$, $SS(P_n)$ is an even-odd mean graph.*

Proof. Consider u_i, $i \in [1, n]$ are the nodes of the path P_n, we replace each link $u_i u_{i+1}$, $i \in [1, n-1]$ by a bigraph $K_{2,t}$ for some $t \in N$. Suppose $u_{ij} (i \in [1, n-1], j \in [1, t])$ are the nodes which used for super subdivision. We notice that the graph $G = SS(P_n)$ has $nt - t + n$ nodes and $2nt - 2t$ links. The labeling of nodes and links is given as follows:
For $j \in [1, t]$.

$$\begin{aligned} \chi(u_i) &= 4ti - 4t, & i \in [1, n] \\ \chi(u_{ij}) &= 4ti + 4j - 4t - 2, & i \in [1, n-1]. \end{aligned}$$

Then, the resulting edge labeling χ^* is computed as follows:

$$\chi^*(u_iu_{ij}) = 4ti + 2j - 4t - 1, \qquad i \in [1, n-1]$$
$$\chi^*(u_{i+1}u_{ij}) = 4ti + 2j - 2t - 1, \quad i \in [1, n-1].$$

Hence, χ is an even-odd meanness of $SS(P_n)$. Thus $SS(P_n)$ is an even-odd mean graph. □

Illustration 2.1. *The super subdivision of P_5 and its an even-odd meanness are depicted in Figure 2, where $t = 5$.*

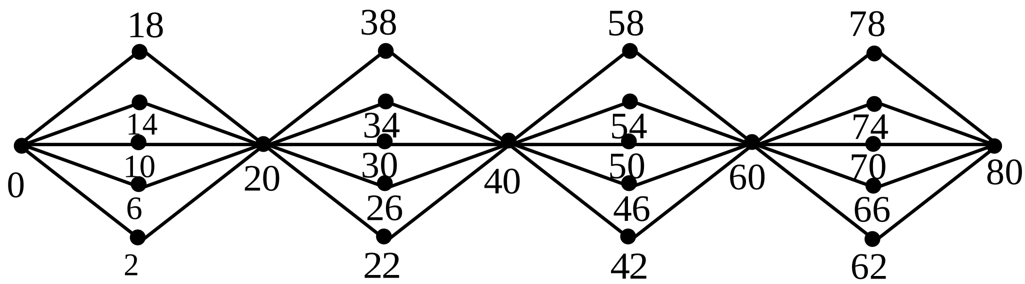

Figure 2. *Even-odd meanness of $SS(P_5)$.*

Theorem 2.2. *$SS(C_n)$ is an even-odd mean graph for $n \equiv 0(\text{mod } 4)$.*

Proof. Suppose C_n is a cycle of size n, where $n \equiv 0(\text{mod } 4)$ with vertices u_i, $i \in [1, n]$. Suppose that the edges $u_iu_{i+1}(i \in [1, n-1])$ and u_nu_1 are changed by a bigraph $K_{2,t}$. Let $u_{ij}(i \in [1, n-1], j \in [1, t])$ be the vertices that will be used in the super subdivision of C_n. Let $G = SS(C_n)$, then it obvious that $p = n + nt$ and $q = 2nt$.

The labeling of vertices is constructed in the following way:

For $j \in [1, t]$.

$$\chi(u_i) = \begin{cases} 4ti - 4t, & i \in [1, \frac{n}{2}] \\ \\ 4ti, & i \in [\frac{n}{2}+1, n]. \end{cases}$$

$$\chi(u_{ij}) = 4t(i-1) + 4j - 2, \qquad i \in [1, n].$$

Thus, the resulting edge labeling χ^* is computed by using the following:

$$\chi^*(u_i u_{ij}) = \begin{cases} 4ti + 2j - (4t+1), & i \in [1, \frac{n}{2}] \\ \\ 4ti + 2j - (2t+1), & i \in [\frac{n}{2}+1, n]. \end{cases}$$

$$\chi^*(u_{ij} u_{i+1}) = \begin{cases} 4ti + 2j - 4t - 1, & i \in [1, \frac{n}{2} - 1] \\ \\ 2tn + 2j - 1, & i = \frac{n}{2} \\ \\ 4ti + 2j - 1, & i \in [\frac{n}{2}+1, n-1]. \end{cases}$$

$$\chi^*(u_{nj} u_1) = 2nt + 2j - (2t+1).$$

Hence, χ is an even-odd meanness of $SS(C_n)$ for $n \equiv 0 (\mathrm{mod}\ 4)$. Then $SS(C_n)$ is an even-odd mean graph. $\square$

Illustration 2.2. *The super subdivision of C_8 and its an even-odd meanness are depicted in Figure 3, where $t = 5$.*

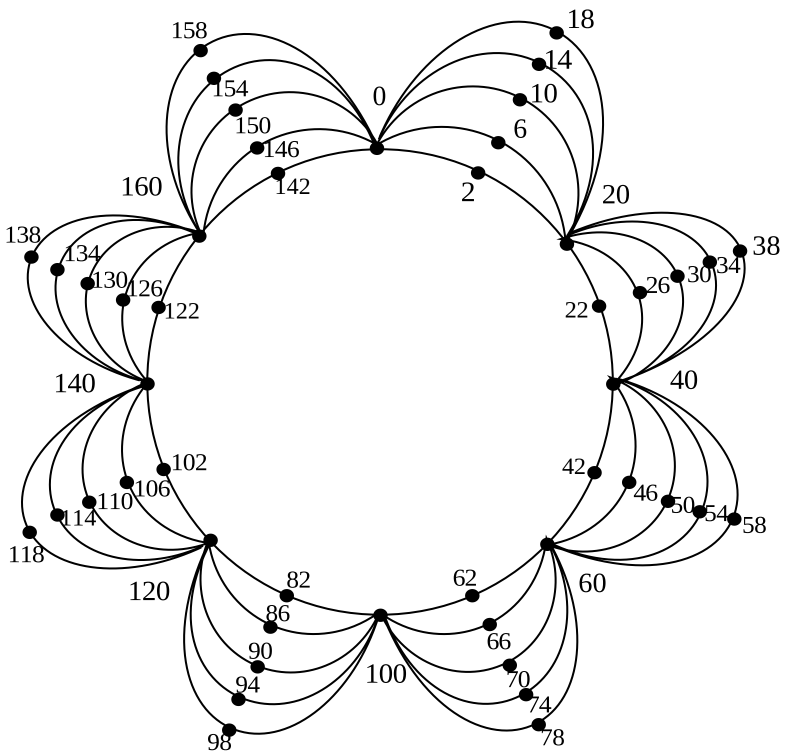

Figure 3. *Even-odd meanness of $SS(C_8)$*

Theorem 2.3. *For $n \geq 2$, $SS(P_n \odot K_1)$ is an even-odd mean graph.*

Proof. Suppose u_i, v_i, $i \in [1,n]$ are the vertices of $P_n \odot K_1$. Let e_i, $e_i^{'}$ denote to the edges u_iu_{i+1}, u_iv_i respectively. Let the edges e_i, $e_i^{'}(i \in [1,n-1])$, $e_n = u_nu_1$ and $e_n^{'}$ be replaced by a bigraph $K_{2,t}$ for certain $t \in N$. Let u_{ij}, w_{ij} and w_{nj} $(i \in [1,n-1], j \in [1,t])$ be the vertices which are used for super subdivision of the edges $e_i, e_i^{'}$ and $e_n^{'}$ respectively. Let $G = SS(P_n \odot K_1)$, then it obvious that $p = 2tn + (2n-t)$ and $q = 4nt - 2t$. Hence the labeling of vertices is constructed in the following way:
For $j \in [1,t]$.

$$\chi(u_i) = \begin{cases} 8ti - 4t, & i \in [1,n], \quad i \text{ is odd} \\ \\ 8ti - 8t, & i \in [1,n], \quad i \text{ is even .} \end{cases}$$

$$\chi(v_i) = \begin{cases} 8ti - 8t, & i \in [1,n], \quad i \text{ is odd} \\ \\ 8ti - 4t, & i \in [1,n], \quad i \text{ is even .} \end{cases}$$

$$\chi(u_{ij}) = \begin{cases} 8ti + 4j - (4t+2), & i \in [1,n-1], \quad i \text{ is odd} \\ \\ 8ti + 4j - 2, & i \in [1,n-1], \quad i \text{ is even} \end{cases}$$

$$\chi(w_{ij}) = \begin{cases} 4j - 2, & i = 1 \\ \\ 8ti + 4j - 12t - 2, & i \in [3,n], \quad i \text{ is odd} \\ \\ 8ti + 4j - 8t - 2, & i \in [2,n], \quad i \text{ is even .} \end{cases}$$

Then the resulting edge labeling χ^* is computed by using the following:

$$\chi^*(u_iu_{ij}) = 8ti + 2j - (4t+1) \qquad i \in [1,n].$$

$$\chi^*(u_{ij}u_{i+1}) = \begin{cases} 8ti + 2j - (2t+1), & i \in [1,n-1], \quad i \text{ is odd} \\ \\ 8ti + 2j + (2t-1), & i \in [1,n-1], \quad i \text{ is even .} \end{cases}$$

$$\chi^*(v_iw_{ij}) = \begin{cases} 2j - 1, & i = 1 \\ \\ 8ti + 2j - (10t+1), & i \in [2,n], \quad i \text{ is odd} \\ \\ 8ti + 2j - (6t+1), & i \in [2,n], \quad i \text{ is even} \end{cases}$$

$$\chi^*(u_iw_{ij}) = \begin{cases} 2t + 2j - 1, & i = 1 \\ \\ 8ti + 2j - (8t+1), & i \in [2,n]. \end{cases}$$

Thus, χ is an even-odd meanness of $SS(P_n \odot K_1)$. Hence $SS(P_n \odot K_1)$ is an even-odd mean graph. $\square$

Illustration 2.3. *The super subdivision of $P_6 \odot K_1$ and its even vertex odd meanness are depicted in Figure 4, where $t = 3$.*

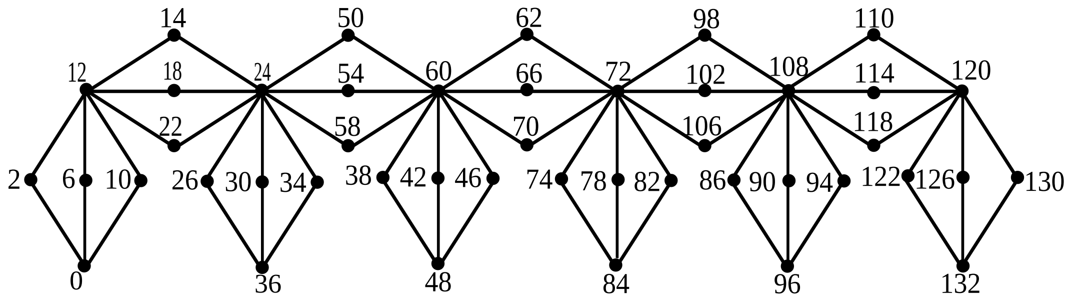

Figure 4. Even-odd meanness of $SS(P_6 \odot K_1)$.

Theorem 2.4. *For $n \equiv 0 (mod\ 4)$, $SS(C_n \odot K_1)$ is an even-odd mean graph.*

Proof. Let $C_n \odot K_1$ be the crown gained from a cycle $C_n : u_1 u_2 ... u_n\, u_1$ by connecting a pendant vertex v_i to u_i, for each $i, (i \in [1, n])$. Let $e_i = u_i u_{i+1} (i \in [1, n-1]), e_n = u_n u_1$ and $e_i^{'} = u_i v_i (i \in [1, n])$. Let G be the graph gained by super subdivision of $C_n \odot K_1$. That is, the edges $e_i, e_i^{'} (i \in [1, n-1])$,$e_n = u_n u_1$ and $e_n^{'} = u_n v_n$ are changed by a bigraph $K_{2,t}$ for certain fixed $t \in N$. Let u_{ij}, w_{ij} $(i \in [1, n], j \in [1, t])$ be the vertices which utilized for super subdivision. Let $G = SS(C_n \odot K_1)$, then it evident that $p = 2n + 2tn$ and $q = 4tn$. Define labeling $\chi : V(SS(C_n \odot K_1)) \to \{0, 2, ..., 2q-2, 2q = 8tn\}$ in the following way:

For $j \in [1, t]$.

$$\chi(u_i) = \begin{cases} 8ti - 4t, & i \in [1, \frac{n}{2}], \quad i \text{ is odd} \\ 8ti - 8t, & i \in [1, \frac{n}{2}], \quad i \text{ is even} \\ 8ti, & i \in [\frac{n}{2} + 1, n], \quad i \text{ is odd} \\ 8ti - 4t, & i \in [\frac{n}{2} + 1, n], \quad i \text{ is even} . \end{cases}$$

$$\chi(v_i) = \begin{cases} 8ti - 8t, & i \in [1, \frac{n}{2}], \quad i \text{ is odd} \\ 8ti - 4t, & i \in [1, \frac{n}{2}], \quad i \text{ is even} \\ 8ti - 4t, & i \in [\frac{n}{2}+1, n], \quad i \text{ is odd} \\ 8ti, & i \in [\frac{n}{2}+1, n], \quad i \text{ is even} . \end{cases}$$

$$\chi(u_{ij}) = \begin{cases} 8ti + 4j - (4t+2), & i \in [1, n-1], \quad i \text{ is odd} \\ 8ti + 4j - 2, & i \in [1, n-1], \quad i \text{ is even} \\ 8tn + 4j - (8t+2), & i = n. \end{cases}$$

$$\chi(w_{ij}) = \begin{cases} 4j - 2, & i = 1 \\ 8ti + 4j - (12t+2), & i \in [2, n-1], \quad i \text{ is odd} \\ 8ti + 4j - (8t+2), & i \in [2, n-1], \quad i \text{ is even} \\ 8tn + 4j - (4t+2), & i = n. \end{cases}$$

Hence the resulting edge labeling χ^* is computed by using the following:

$$\chi^*(u_i u_{ij}) = \begin{cases} 8ti + 2j - (4t+1), & i \in [1, \frac{n}{2}] \\ 8ti + 2j - (2t+1), & i \in [\frac{n}{2}+1, n-1] \\ 8tn + 2j - (6t+1), & i = n. \end{cases}$$

$$\chi^*(u_i w_{ij}) = \begin{cases} 2t + 2j - 1, & i = 1 \\ 8ti + 2j - (8t+1), & i \in [2, \frac{n}{2}] \\ 8ti + 2j - (6t+1), & i \in [\frac{n}{2}+1, n-1] \\ 8tn + 2j - (4t+1), & i = n. \end{cases}$$

$$\chi^*(v_i w_{ij}) = \begin{cases} 2j-1, & i = 1 \\ 8ti + 2j - (10t+1), & i \in [2, \frac{n}{2}], \quad i \text{ is odd} \\ 8ti + 2j - (6t+1), & i \in [2, \frac{n}{2}], \quad i \text{ is even} \\ 8ti + 2j - (8t+1), & i \in [\frac{n}{2}+1, n-1], \quad i \text{ is odd} \\ 8ti + 2j - (4t+1), & i \in [\frac{n}{2}+1, n-1], \quad i \text{ is even} \\ 8tn + 2j - (2t+1), & i = n. \end{cases}$$

$$\chi^*(u_{ij} u_{i+1}) = \begin{cases} 8ti + 2j - (2t+1), & i \in [1, \frac{n}{2}], \quad i \text{ is odd} \\ 8ti + 2j + 2t - 1, & i \in [2, \frac{n}{2} - 2], \quad i \text{ is even} \\ 4tn + 2j + 4t - 1, & i = \frac{n}{2} \\ 8ti + 2j - 1, & i \in [\frac{n}{2}+1, n], \quad i \text{ is odd} \\ 8ti + 2j + 4t - 1, & i \in [\frac{n}{2}+1, n-2], \quad i \text{ is even} \end{cases}$$

$$\chi^*(u_{nj} u_1) = 4tn + 2j - (2t+1).$$

Then, χ is an even-odd meanness of $SS(C_n \odot K_1)$. Thus $SS(C_n \odot K_1)$ is an even-odd mean graph. $\square$

Illustration 2.4. *The super subdivision of $SS(C_8 \odot K_1)$ and its even vertex odd meanness are depicted in Figure 5, where $t = 3$.*

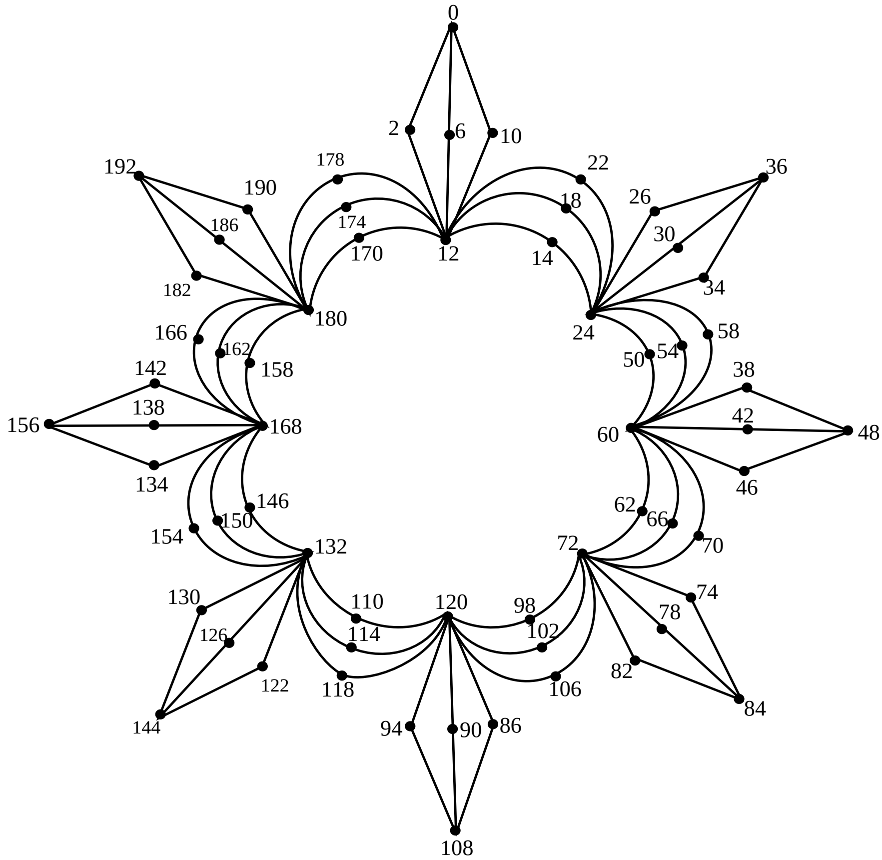

Figure 5. Even-odd meanness of $SS(C_8 \odot K_1)$.

Theorem 2.5. *For* $m, n \geq 2$, $SS(P_m \bigotimes P_n)$ *is an even-odd mean graph.*

Proof. Let $u_{ij}(i \in [1,m], j \in [1,n])$ be the vertices of planar grid $P_m \bigotimes P_n$. We know that the cardinality of $P_m \bigotimes P_n$ is mn and its size is $2mn - (m+n)$. Suppose G is the graph gained by super subdivided of $P_m \bigotimes P_n$ by a bigraph $K_{2,t}$, where $t \in N$. Let $e_{ij} = u_{ij}u_{i(j+1)}(i \in [1,m], j \in [1,n-1])$ and $e'_{ij} = u_{ij}u_{(i+1)j}(i \in [1,m-1], j \in [1,n])$ be the horizontal and vertical edges of $P_m \bigotimes P_n$ respectively. Let $v_{ij,k}(i \in [1,m], j \in [1,n-1], k \in [1,t])$ and $w_{ij,k}(i \in [1,m-1], j \in [1,n], k \in [1,t])$ be the vertices which are used for super subdivision of the edges e_{ij} and e'_{ij} respectively. We observe that the graph $G = SS(P_m \bigotimes P_n)$ has $mn(1+2t) - (m+n)t$ vertices and $4mnt - 2(m+n)t$ edges. We define labeling $\chi : V(SS(P_m \bigotimes P_n)) \rightarrow \{0, 2, ..., 2q = 8mnt - 4(m+n)t\}$ as follows:

For $k \in [1, t]$.

$$\chi(u_{ij}) = \begin{cases} 4tj - 4t, & i = 1, j \in [1, n] \\ 4t(2n-1)i + 4tj - & \\ -8nt, & i \in [3, m], j \in [1, n], i \text{ is odd} \\ 4t(2n-1)i - 4tj - & \\ -4t(n-1), & i \in [2, m], j \in [1, n], i \text{ is even} . \end{cases}$$

$$\chi(v_{ij,k}) = \begin{cases} 8tj + 4k - (4t+2), & i = 1, j \in [1, n-1] \\ -4tj + 4k + (12nt - 8t - 2), & i = 2, j \in [1, n-1] \\ 4t(2n-1)i + 8tj + 4k - & \\ -(12tn+2) \qquad , & i \in [3, m], j \in [1, n-1], i \text{ is odd} \\ 4t(2n-1)i - 8tj + 4k - & \\ -(4tn+2), & i \in [1, m], j \in [1, n-1], i \text{ is even} . \end{cases}$$

$$\chi(w_{ij,k}) = \begin{cases} 8tj + 4k - (8t+2), & i = 1, j \in [1, n] \\ 8tj + 4k + (12tn - 16t - 2), & i = 2, 1 \leq j \leq n \\ 6t(n+1)i - 8tj + 4k + & \\ +(10tn - 30t - 2), & i \in [3, m-1], j \in [1, n], i \text{ is odd} \\ 6t(n+1)i + 8tj + 4k + & \\ +(4tn - 48t - 2), & i \in [4, m-1], j \in [1, n], i \text{ is even} . \end{cases}$$

Then the resulting edge labeling χ^* is computed by using the following:

$$\chi^*(u_{ij}\ v_{ij,k}) = \begin{cases} 6tj + 2k - (4t+1), & i = 1, j \in [1, n-1] \\ -4tj + 2k + (12tn - 6t - 1), & i = 2, j \in [1, n-1] \\ 4t(2n-1)i + 6tj + 2k - & \\ \quad -(10tn + 1), & i \in [3, m], j \in [1, n-1], i \text{ is odd} \\ 4t(2n-1)i - 6tj + 2k - & \\ \quad -(4tn - 2t + 1), & i \in [4, m], j \in [1, n-1], i \text{ is even}\ . \end{cases}$$

$$\chi^*(v_{ij,k}\ u_{i(j+1)}) = \begin{cases} 6tj + 2k - (2t+1), & i = 1, j \in [1, n-1] \\ -4tj + 2k + (12tn - 8t - 1), & i = 2, j \in [1, n-1] \\ 4t(2n-1)i + 6tj + 2k + & \\ \quad +(2t - 10tn - 1), & i \in [3, m], j \in [1, n-1], i \text{ is odd} \\ 4t(2n-1)i - 6tj + 2k - & \\ \quad -(4tn + 1), & i \in [4, m], j \in [1, n-1], i \text{ is even}\ . \end{cases}$$

$$\chi^*(u_{ij}\ w_{ij,k}) = \begin{cases} 6tj + 2k - (6t+1), & i = 1, j \in [1, n] \\ 2tj + 2k + (12tn - 10t - 1), & i = 2, j \in [1, n] \\ t(7n+1)i - 2tj + 2k + & \\ \quad +(tn - 15t - 1), & i \in [3, m-1], j \in [1, n], i \text{is odd} \\ t(7n+1)i + 2tj + 2k - & \\ \quad -(22t + 1), & i \in [4, m-1], j \in [1, n], i \text{is even}\ . \end{cases}$$

$$\chi^*(w_{ij,k}\ u_{(i+1)j}) = \begin{cases} 2tj + 2k + (6tn - 6t - 1), & i = 1, j \in [1, n] \\ 6tj + 2k + (14tn - 14t - 1), & i = 2, j \in [1, n] \\ t(7n + 1)i - 6tj + 2k + \\ \quad +(7tn - 15t - 1), & i \in [3, m - 1], j \in [1, n], i \text{is odd} \\ t(7n + 1)i + 6tj + 2k + \\ \quad +(2tn - 26t - 1), & i \in [4, m - 1], j \in [1, n], i \text{is even} . \end{cases}$$

Hence, χ is an even-odd meanness of $SS(P_m \bigotimes P_n)$. Thus $SS(P_m \bigotimes P_n)$ is an even-odd mean graph. □

Illustration 2.5. *The super subdivision of $SS(P_4 \bigotimes P_5)$ and its even vertex odd meanness are depicted in Figure 6, where $t = 4$.*

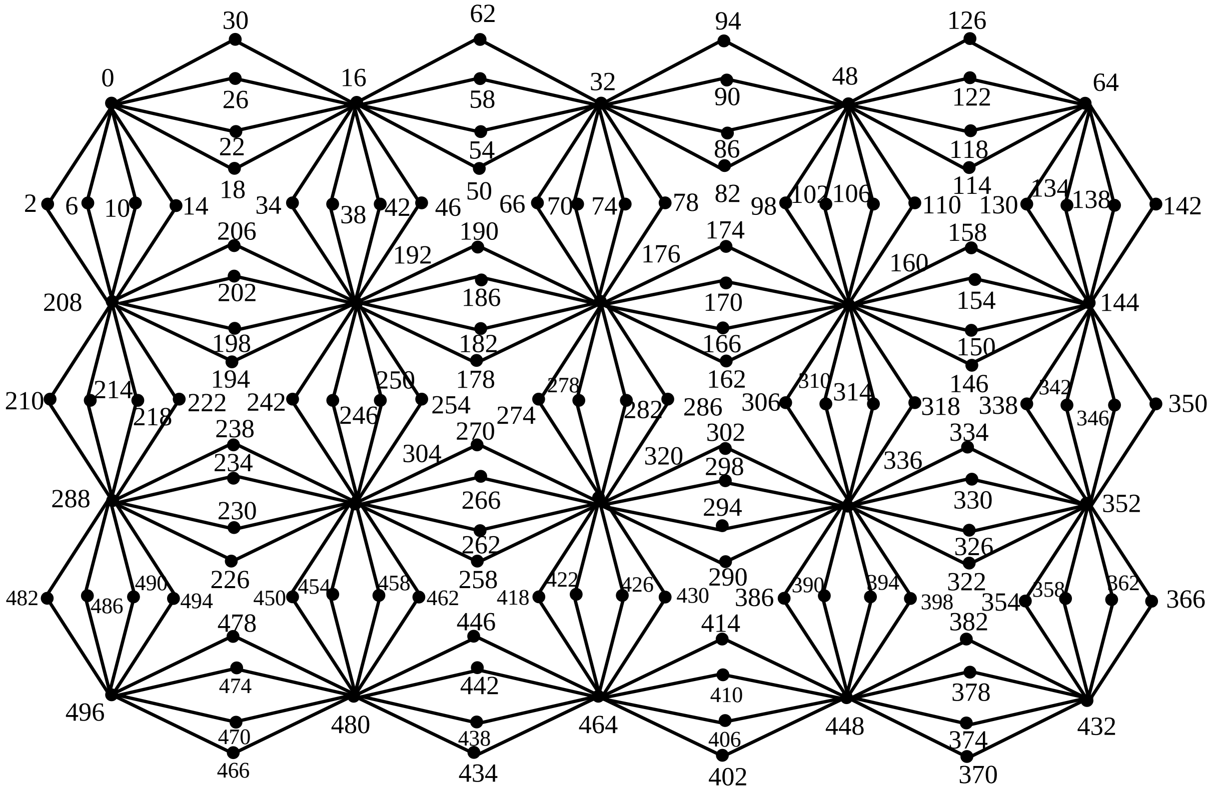

Figure 6. Even-odd meanness of $SS(P_4 \otimes P_5)$.

Corollary 2.1. *$SS(L_n)$ is an even-odd mean graph for all n.*

Proof. Since the ladder L_n is a $P_n \otimes P_2$ planar grid, by Theorem 2.5, the super subdivision of L_n is also an even-odd mean graph. $\square$

3. Conclusion

It's worth noting that the super subdivision graph $SS(G)$ may be obtained from any graph G for $t \geq 2$. We prove that path, cycle, comb, crown, and planar grid super subdivisions are even-odd mean graphs.

4. Funding Statement

There is no funding to declare for this research study.

References

[1] M. Basher, "Further results on even vertex odd mean graphs", *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 24, no. 1, pp. 93-117, 2021. doi: 10.1080/09720529.2019.1675301

[2] M. Basher, "Even vertex odd mean labeling of some cycle related graphs", *Journal of Discrete Mathematical Sciences and Cryptography,* 2021. doi: 10.1080/09720529.2020.1841969

[3] M. Basher, "On even vertex odd mean labeling of the calendula graphs", *Proyecciones (Antofagasta),* vol. 39, no. 6, pp. 1515-1535, 2020. doi: 10.22199/issn.0717-6279-2020-06-0091

[4] J. A. Gallian, "A dynamic survey of graph labeling", *The Electronic Journal of Combinatorics,* 20th ed. #DS6, 2017.

[5] S. Golomb, "How to number a graph", in *Graph theory and computing*, R. C. Reading, Ed. New York: Academic Press, 1972.

[6] F. Harary, *Graph Theory.* Reading (MA): Addison-Wesley, 1972.

[7] M. Kannan, R. Vikrama Prasad and R. Gopi, "Some Graph Operations Of Even Vertex Odd Mean Labeling Graphs", *International Journal of Applied Engineering Research,* vol. 12, no. 18, pp. 7749-7753, 2017.

[8] A. Rosa, *On certain valuations of the vertices of a graph, Theory of graphs* (International symposium, Rome, July 1966). New York: Gordon and Breach, 1967.

[9] G. Sethuraman and P. Selvaraju, "Gracefulness of Arbitrary of Super Subdivision of Graphs", *Indian Journal of Pure and Applied Mathematics,* vol. 32, no. 7, pp. 1059-1064, 2001.

[10] R. Vasuki, A. Nagarajan and S. Arockiaraj, "Even vertex odd mean labeling of graphs", *SUT Journal of Mathematics,* vol. 49, no. 2, pp. 79-92, 2013. doi: 10.55937/sut/1394108286

[11] R. Vikrama Prasad, M. Kannan and R. Gopi, "Some results on even vertex odd mean labeling graphs", *International Journal of Mechanical Engineering and Technology*, vol. 9, no. 2, pp. 615-621, 2018.

[12] V. Yegnanaryanan and P. Vaidhyanathan, "Some interesting Applications of graph labelings", *International Journal of Computational and Mathematical Sciences,* vol. 2, no. 5, pp. 1522-1531, 2012.

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