Proyecciones Journal of Mathematics
Vol. 42, N^o 3, pp. 651-661, June 2023.
Universidad Católica del Norte
Antofagasta - Chile

10.22199/issn.0717-6279-5220

Generation of anti-magic graphs from binary graph products

P. Ragukumar
Vellore Institute of Technology, India
Received : November 2021. Accepted : November 2022

Abstract

An anti-magic labeling of a graph G is a one-to-one correspondence between $E(G)$ and $\{1, 2, \cdots, |E|\}$ such that the vertex-sum for distinct vertices are different. Vertex-sum of a vertex $u \in V(G)$ is the sum of labels assigned to edges incident to the vertex u. It was conjectured by Hartsfield and Ringel that every tree other than K_2 has an anti-magic labeling. In this paper, we consider various binary graph products such as corona, edge corona and rooted products to generate anti-magic graphs. We prove that corona products of an anti-magic regular graph G with K_1 and K_2 are anti-magic. Further, we prove that rooted product of two anti-magic trees are anti-magic. Also, we prove that rooted product of an anti-magic graph with an anti-magic tree admits anti-magic labeling.

Mathematics Subject Classification: *05C78;05C05.*

Keywords: *Corona product, edge corona product,rooted product,anti-magic graphs.*

1. Introduction

All graphs considered in this paper are simple, finite and undirected. Terms that are not defined in this paper can be refered from book [13]. Let $G = (V, E)$ be a graph and $f : E \to \{1, 2, \cdots, |E|\}$ is a bijective function. For each vertex $u \in V(G)$, the vertex-sum $\varphi_f(u)$ at u is defined as $\varphi_f(u) = \sum\limits_{e \in E(u)} f(e)$, where $E(u)$ is the set of edges incident to u. If $\varphi_f(u) \neq \varphi_f(v)$ for any two distinct vertices u, v of G, then f is called an anti-magic labeling of G. A graph G is called anti-magic if G has an anti-magic labeling. The problem of anti-magic labeling of graphs was introduced by Hartsfield and Ringel [7]. They posed the following conjectures on anti-magic labeling of graphs.

Conjecture 1.[7] Every connected graph other than K_2 is anti-magic.

Conjecture 2.[7] Every tree other than K_2 is anti-magic.

In spite of much attention given by many researchers, both conjectures remain open. Alon et al.[1] proved that there is an absolute constant C such that graphs with minimum degree $\delta(G) \geq C\ log|V(G)|$ are anti-magic. Also they proved that all complete partite graphs except K_2 are anti-magic. Liang and Zhu [10] proved that cubic graphs are anti-magic. Cranston, Liang and Zhu [3] proved that odd degree regular graphs are anti-magic.

For Conjecture 2, J. Shang [12] proved that spiders are antimagic. Kaplan et al. [9] showed that trees without vertices of degree 2 are anti-magic. Liang, Wong and Zhu [11] studied trees with many degree 2 vertices, with restriction on the subgraph induced by degree 2 vertices and its complement. They proved that such trees are anti-magic. For an exhaustive survey on anti-magic graphs, we refer the dynamic survey by Gallian [5].

2. Anti-magic labeling of corona product of graphs

In this section, we prove that the corona product of an anti-magic graph G with K_1 and G with K_2 is anti-magic. For that, let us define the corona product of two graphs.

Definition 2.1. *[4][Corona Product of Graphs]*

Let G and H be two graphs and let n be the order of G. The corona product, or simply the corona, of graphs G and H is the graph $G \circ H$

obtained by taking one copy of G and n copies of H and then joining by an edge the ith vertex of G to every vertex in the ith copy of H. The corona product of graphs G and H are denoted as $G \circ H$.

Theorem 2.2. *Let G be an anti-magic regular graph with p vertices and q edges. Then $G \circ K_1$ is anti-magic.*

Proof. Let f be the anti-magic labeling of a regular graph G. For convenience, let us label and arrange the edges of G as $e_1, e_2, \cdots, e_q$ such that $f(e_i) = i$, for $1 \leq i \leq q$. Similarly, let us arrange the vertices of G based on their vertex-sums. That is, arrange the vertices of G as $u_1, u_2, \cdots, u_p$ such that $\varphi_f(u_1) < \varphi_f(u_2) < \cdots < \varphi_f(u_p)$.

To construct a graph $G \circ K_1$ from G, consider the graph G and for $1 \leq i \leq p$, add a vertex u'_i for the vertex u_i and add an edge e'_i between the vertices u_i and u'_i. Now, let us define a bijective function $\psi : E(G \circ K_1) \to \{1, 2, 3, \cdots, p + q\}$ as follows:

$$\psi(e_i) = f(e_i) + p, \text{ for } 1 \leq i \leq q \tag{2.1}$$

$$\psi(e'_i) = i, \text{ for } 1 \leq i \leq p \tag{2.2}$$

From equation (2.1) and equation (2.2), it is clear that the edge labels of $p + q$ edges of $E(G \circ K_1)$ are from the set $\{1, 2, 3, \cdots, p + q\}$. Therefore, ψ is bijective. Since G is a regular graph and observe that the vertex-sum of vertices of $G \circ K_1$ form a monotonically increasing sequence as follows:
$\varphi_\psi(u'_1), \varphi_\psi(u'_2), \varphi_\psi(u'_3), \cdots, \varphi_\psi(u'_p)$
followed by $\varphi_\psi(u_1), \varphi_\psi(u_2), \varphi_\psi(u_3), \cdots, \varphi_\psi(u_p)$ since $\varphi_\psi(u'_1) < \varphi_\psi(u'_2) < \varphi_\psi(u'_3) < \cdots < \varphi_\psi(u'_{m+1}) < \varphi_\psi(u_1) < \varphi_\psi(u_2) < \varphi_\psi(u_3) < \cdots < \varphi_\psi(u_p)$.

Thus, the vertex-sum of vertices of $G \circ K_1$ are distinct. This implies that ψ satisfies the conditions of anti-magic labeling. Therefore, $G \circ K_1$ is an anti-magic graph. Hence the proof. □

Theorem 2.3. *Let G be an anti-magic regular graph with p vertices and q edges. Then $G \circ K_2$ is anti-magic.*

Proof. Let f be the anti-magic labeling of regular graph G. For convenience, let us label and arrange the edges of G as $e_1, e_2, \cdots, e_q$ such that $f(e_i) = i$, for $1 \leq i \leq q$. Similarly, let us arrange the vertices of G based on their vertex-sums. That is, arrange the vertices of G as $u_1, u_2, \cdots, u_p$ such that $\varphi_f(u_1) < \varphi_f(u_2) < \cdots < \varphi_f(u_p)$.

To construct a graph $G \circ K_2$ from G, consider the graph G and for $1 \leq i \leq p$, add an edge $w_i = (u'_i, v'_i)$ for every vertex u_i and correspondingly add edges $x'_i = (u_i, u'_i)$ and $y'_i = (u_i, v'_i)$. Observe that, by the construction, for every vertex in G, we are adding three edges in $G \circ K_2$. Now, let us define a bijective function $\psi : E(G \circ K_2) \to \{1, 2, 3, \cdots, q + 3p\}$ as follows:

$$\psi(e_i) = f(e_i) + 3p, \text{ for } 1 \leq i \leq q \tag{2.3}$$

$$\psi(w_i) = 3i - 2, \text{ for } 1 \leq i \leq p \tag{2.4}$$

$$\psi(x'_i) = 3i - 1, \text{ for } 1 \leq i \leq p \tag{2.5}$$

$$\psi(y'_i) = 3i, \text{ for } 1 \leq i \leq p \tag{2.6}$$

From the above equations, it is clear that the edge labels of $3p+q$ edges of $E(G \circ K_2)$ are from the set $\{1, 2, 3, \cdots, 3p+q\}$. Therefore, ψ is bijective. Since G is a regular graph and we observe that the vertex-sum of vertices of $G \circ K_2$ form a monotonically increasing sequence as follows:

$\varphi_\psi(u'_1), \varphi_\psi(v'_1), \varphi_\psi(u'_2), \varphi_\psi(v'_2), \cdots, \varphi_\psi(u'_i), \varphi_\psi(v'_i), \cdots \varphi_\psi(u'_p), \varphi_\psi(v'_p)$ followed by
$\varphi_\psi(u_1), \varphi_\psi(u_2), \cdots, \varphi_\psi(u_i), \cdots, \varphi_\psi(u_p)$

Thus, the vertex-sum of vertices of $G \circ K_2$ are distinct. This implies that ψ satisfies the conditions of anti-magic labeling. Therefore, $G \circ K_2$ is an anti-magic graph. Hence the proof. $\square$

3. Anti-magicness of edge corona product of graphs

In this section, we prove that the edge corona product of an anti-magic regular graph G with K_1 is anti-magic. To prove one of our main results, let us define the edge corona product of two graphs.

Definition 3.1. *[8][Edge corona of two graphs]*
Let G and H be two graphs and let m be the number of edges in G. The edge corona of G and H, denoted by $G \diamond H$, is the graph obtained by taking one copy of G and m copies of H, and then joining two end vertices of the i-th edge of G to every vertex in the ith copy of H.

Theorem 3.2. *Let G be an anti-magic regular graph with p vertices and q edges such that $\delta(G) \geq 2$. Then $G \diamond K_1$ is anti-magic.*

Proof. Let f be the anti-magic labeling of regular graph G. For convenience, let us label and arrange the edges of G as $e_1, e_2, \cdots, e_q$ such that $f(e_i) = i$, for $1 \leq i \leq q$. Similarly, let us arrange the vertices of G based on their vertex-sums. That is, label and arrange the vertices of G as $u_1, u_2, \cdots, u_p$ such that $\varphi_f(u_1) < \varphi_f(u_2) < \cdots < \varphi_f(u_p)$. By the edge corona product of graph G with K_1, for $1 \leq i \leq q$, a vertex u'_i is added corresponding to the edge $e_i = (u_j, u_k) \in E(G)$. With out loss of generality, let us assume that $\varphi_f(u_j) < \varphi_f(u_k)$. Further, we add the edges $x_i = (u'_i, u_j)$ and $y_i = (u'_i, u_k)$. Note that $G \diamond K_1$ has $p + q$ vertices and $3q$ edges. Now, let us define a function $\psi : E(G \diamond K_1) \to \{1, 2, 3, \cdots, 3q\}$ as follows:

$$\psi(x_i) = 2i - 1, \text{ for } 1 \leq i \leq q \tag{3.1}$$

$$\psi(y_i) = 2i, \text{ for } 1 \leq i \leq q \tag{3.2}$$

$$\psi(e_i) = f(e_i) + 2q, \text{ for } 1 \leq i \leq q \tag{3.3}$$

From the above equations, it is clear that the edge labels of $3q$ edges of $E(G \diamond K_1)$ are from the set $\{1, 2, 3, \cdots, 3q\}$. Therefore, ψ is bijective.

Since $\delta(G) \geq 2$, implies that degree of any vertex u_i for any $1 \leq i \leq p$, is at least 4. Also, $deg(u'_i) = 2$ for $1 \leq i \leq q$. This leads that the vertex-sum of vertices of $G \diamond K_1$ form a monotonically increasing sequence as follows:

$\varphi_\psi(u'_1), \varphi_\psi(u'_2), \cdots, \varphi_\psi(u'_i), \cdots, \varphi_\psi(u'_q)$ followed by
$\varphi_\psi(u_1), \varphi_\psi(u_2), \cdots, \varphi_\psi(u_i), \cdots, \varphi_\psi(u_p)$

Thus, the vertex-sum of vertices of $G \diamond K_1$ are distinct. This implies that ψ satisfies the conditions of anti-magic labeling. Therefore, $G \diamond K_2$ is an anti-magic graph. Hence the proof. □

Theorem 3.3. *$K_{1,n} \diamond K_1$ is anti-magic, for $n \geq 1$.*

Proof. Let f be the anti-magic labeling of the star graph $K_{1,n}$. For convenience, let us label and arrange the edges of $K_{1,n}$ as $e_1, e_2, \cdots, e_n$ such that $f(e_i) = i$, for $1 \leq i \leq n$. Let us denote the central vertex of the star as u and the pendant vertices of star can be labeled and arranged as $u_1, u_2, \cdots, u_n$ such that $\varphi_f(u_1) < \varphi_f(u_2) < \cdots < \varphi_f(u_n)$. By the edge corona product of graph $K_{1,n}$ with K_1, for $1 \leq i \leq n$, a vertex u_i' is added corresponding to the edge $e_i = (u, u_i) \in E(G)$. Further, we add the edges $x_i = (u_i', u_i)$ and $y_i = (u_i', u)$. Note that $K_{1,n} \diamond K_1$ has $2n + 1$ vertices and $3n$ edges. Now, let us define a function $\psi : E(K_{1,n} \diamond K_1) \to \{1, 2, 3, \cdots, 3n\}$ as follows:

$$\psi(e_i) = 3i - 2, \text{ for } 1 \leq i \leq n \tag{3.4}$$

$$\psi(x_i) = 3i - 1, \text{ for } 1 \leq i \leq n \tag{3.5}$$

$$\psi(y_i) = 3i, \text{ for } 1 \leq i \leq n \tag{3.6}$$

From the above equations, it is clear that the edge labels of $3n$ edges of $E(K_{1,n} \diamond K_1)$ are from the set $\{1, 2, 3, \cdots, 3n\}$. Therefore, ψ is bijective.

Further the vertex-sum of vertices of $K_{1,n} \diamond K_1$ form a monotonically increasing sequence as follows:
$\varphi_\psi(u_1), \varphi_\psi(u_1'), \varphi_\psi(u_2), \varphi_\psi(u_2'), \cdots, \varphi_\psi(u_i), \varphi_\psi(u_i'), \cdots, \varphi_\psi(u)$

Thus, the vertex-sum of vertices of $K_{1,n} \diamond K_1$ are distinct. This implies that ψ satisfies the conditions of anti-magic labeling. Therefore, $K_{1,n} \diamond K_2$ is an anti-magic graph. Hence the proof. □

4. Anti-magic labeling of rooted product of two graphs

In this section, we prove that rooted product of two anti-magic graphs is also anti-magic. Now, we introduce some basic definitions to prove our main results.

Definition 4.1. *[6][Rooted product of graphs] Given a graph G of order n and a graph H with root vertex v, the rooted product $G \bullet H$ is defined as the graph obtained from G and H by taking one copy of G and n copies of H and identifying the vertex u_i of G with the vertex v in the ith copy of H for every $1 \leq i \leq n$.*

Remark 4.1. *Suppose T_1 is a tree with m edges and T_2 is a tree with n edges. Then, rooted product of a tree T_1 with T_2 is also a tree with $mn + m + n$ edges.*

Definition 4.2. *Let G be an anti-magic graph whose anti-magic labeling is given by bijective function $f : E(G) \to \{1, 2, \cdots, |E|\}$. Let $k = \max_{u \in V(G)} \{\varphi_f(u)\}$. A vertex $u \in V(G)$ is said to be anti-magic maximum vertex if $\varphi_f(u) = k$ and we denote such vertex as $\hat{u}$.*

Theorem 4.3. *Rooted product of anti-magic trees T_1 and T_2 is anti-magic.*

Proof. Let T_1 be an anti-magic tree with m edges. Let $f_1 : E(T_1) \to \{1, 2, \cdots, m\}$ be the anti-magic labeling. Let us call and arrange the edges of T_1 as $y_1, y_2, \cdots, y_m$ such that $f_1(y_i) = i$, for $1 \leq i \leq m$. Let us label and arrange the vertices of T_1 as $v_1, v_2, \cdots, v_{m+1}$ such that their vertex-sum form a monotonically increasing sequence $\varphi_{f_1}(v_1) < \varphi_{f_1}(v_2) < \cdots < \varphi_{f_1}(v_{m+1})$.

Let T_2 be an anti-magic tree with n edges. Let $f_2 : E(T_2) \to \{1, 2, \cdots, n\}$ be the anti-magic labeling. Let us call and arrange the edges of T_2 as $x_1, x_2, \cdots, x_n$ such that $f_2(x_i) = i$, for $1 \leq i \leq n$. Let us label and arrange the vertices of T_2 as $u_1, u_2, \cdots, u_{n+1}$ such that their vertex-sum form a monotonically increasing sequence $\varphi_{f_2}(u_1) < \varphi_{f_2}(u_2) < \cdots < \varphi_{f_2}(u_{n+1})$.

In the above set up, the anti-magic maximum vertex of T_2 is $\hat{u} = u_{n+1}$ and consider $\hat{u}$ as the root vertex of the tree T_2 in the rooted product of T_1 with T_2. For $1 \leq i \leq m + 1$, denote $T_2^{(i)}$ is the ith copy of the tree T_2 for the ith vertex of the tree T_1. Denote $x_j^{(i)}$ is the jth edge in the arrangement of edges of T_2 in its ith copy. Denote $u_j^{(i)}$ is the jth vertex in the arrangement of vertices of T_2 in its ith copy. Now, let us define a function $\theta : E(T_1 \bullet T_2) \to \{1, 2, 3, \cdots, mn + m + n\}$ as follows:

$$\theta(y_j) = f_1(y_j) + (m + 1)n, \text{ for } 1 \leq j \leq m \tag{4.1}$$

$$\theta\left(x_j^{(i)}\right) = f_2(x_j) + (i - 1) + m(j - 1), \text{ for } 1 \leq i \leq m + 1 \text{ and for } 1 \leq j \leq n \tag{4.2}$$

It is clear that from equation (4.1), θ assigns the edge values from the set $\{mn + n + 1, mn + n + 2, \cdots, mn + n + m\}$. For the ith copy of the tree T_2, the difference between two consecutively arranged edge labels of edges x_j^i and x_{j+1}^i is always $m+1$. Further, the minimum edge label assigned by the function θ in the ith copy of the tree T_2 always i. The edge labels of the edges of $T_2^{(1)}$ assigned by θ form an arithmetic progression $1, 1+(m+1), 1+2(m+1), \cdots, 1+(n-1)(m+1)$. The edge labels of the edges of $T_2^{(2)}$ assigned by θ form an arithmetic progression $2, 2+(m+1), 2+2(m+1), \cdots, 2+(n-1)(m+1)$. Similarly, the edge labels of the edges of $T_2^{(i)}$ assigned by θ form an arithmetic progression $i, i+(m+1), i+2(m+1), \cdots, i+(n-1)(m+1)$. The edge labels of the edges of $T_2^{(m+1)}$ assigned by θ form an arithmetic progression $m+1, m+1+(m+1), m+1+2(m+1), \cdots, m+1+(n-1)(m+1)$. Therefore, the edge labels assigned by the function θ is distinct and are from the set $\{1, 2, \cdots, mn+m+n\}$. Hence θ is bijective.

Now, let us prove that vertex-sum of vertices of $T_1 \bullet T_2$ is distinct. Observe that vertex-sum of vertices of $T_1 \bullet T_2$ form a monotonically increasing sequence as follows:
$\varphi_\theta(u_1^{(1)}), \varphi_\theta(u_1^{(2)}), \varphi_\theta(u_1^{(3)}), \cdots, \varphi_\theta(u_1^{(m+1)}), \varphi_\theta(u_2^{(1)}), \varphi_\theta(u_2^{(2)}), \varphi_\theta(u_2^{(3)}), \cdots,$
$\varphi_\theta(u_2^{(m+1)}), \cdots, \varphi_\theta(u_n^{(1)}), \varphi_\theta(u_n^{(2)}), \varphi_\theta(u_n^{(3)}), \cdots, \varphi_\theta(u_n^{(m+1)})$
followed by $\varphi_\theta(v_1), \varphi_\theta(v_2), \cdots, \varphi_\theta(v_{m+1})$

This implies that the vertex-sum of vertices of $T_1 \bullet T_2$ is distinct. Therefore $T_1 \bullet T_2$ is anti-magic. □

Theorem 4.4. *Let G be an anti-magic graph with p vertices and q edges and let T be an anti-magic tree with m edges. Then, $G \bullet T$ is anti-magic.*

Proof. Let f_1 be the anti-magic labeling of graph G. For convenience, let us label and arrange the edges of G as $e_1, e_2, \cdots, e_q$ such that $f_1(e_i) = i$, for $1 \leq i \leq q$. Similarly, let us arrange the vertices of G based on their vertex-sums. That is, label and arrange the vertices of G as $v_1, v_2, \cdots, v_p$ such that $\varphi_(f_1)(v_1) < \varphi_(f_1)(v_2) < \cdots < \varphi_(f_1)(v_p)$.

Let $f_2 : E(T) \to \{1, 2, \cdots, m\}$ be the anti-magic labeling. Let us call and arrange the edges of T as $x_1, x_2, \cdots, x_m$ such that $f_2(x_i) = i$, for $1 \leq i \leq m$. Let us label and arrange the vertices of T as $u_1, u_2, \cdots, u_{m+1}$ such that their vertex-sum form a monotonically increasing sequence $\varphi_{f_2}(u_1) < \varphi_{f_2}(u_2) < \cdots < \varphi_{f_2}(u_{m+1})$.

In the above arrangement of vertices of T, the anti-magic maximum vertex of T is $\hat{u} = u_{m+1}$. Consider $\hat{u}$ as the root vertex of T in the rooted product of G and T. For $1 \leq i \leq p$, denote $T^{(i)}$ is the ith copy of the tree T for the ith vertex of the graph G. Denote $x_j^{(i)}$ is the jth edge in the arrangement of edges of T in its ith copy. Denote $u_j^{(i)}$ is the jth vertex in the arrangement of vertices of T in its ith copy. Now, let us define a function $\theta : E(G \bullet T) \to \{1, 2, 3, \cdots, mp + q\}$ as follows:

$$\theta(e_j) = f_1(e_j) + mp, \text{ for } 1 \leq j \leq q \tag{4.3}$$

$$\theta\left(x_j^{(i)}\right) = f_2(x_j) + (i-1) + (p-1)(j-1), \text{ for } 1 \leq i \leq p \text{ and for } 1 \leq j \leq m \tag{4.4}$$

It is clear that from equation (4.3), θ assigns the edge values from the set $\{mp+1, mp+2, \cdots, mp+q\}$. For the ith copy of the tree T, the difference between two consecutively arranged edge labels of edges x_j^i and x_{j+1}^i is always p. Further, the minimum edge label assigned by the function θ in the ith copy of the tree T always i. The edge labels of the edges of $T^{(1)}$ assigned by θ form an arithmetic progression $1, 1+p, 1+2p, \cdots, 1+(m-1)p$. The edge labels of the edges of $T^{(2)}$ assigned by θ form an arithmetic progression $2, 2+p, 2+2p, \cdots, 2+(m-1)p$. Similarly, the edge labels of the edges of $T^{(i)}$ assigned by θ form an arithmetic progression $i, i+p, i+2p, \cdots, i+(m-1)p$. The edge labels of the edges of $T^{(p)}$ assigned by θ form an arithmetic progression $p, p+p, p+2p, \cdots, p+(m-1)p$. Therefore, the edge labels assigned by the function θ is distinct and are from the set $\{1, 2, \cdots, mp+q\}$. Hence θ is bijective.

Now, let us prove that vertex-sum of vertices of $G \bullet T$ is distinct. Observe that vertex-sum of vertices of $G \bullet T$ form a monotonically increasing sequence as follows:

$\varphi_\theta(u_1^{(1)}), \varphi_\theta(u_1^{(2)}), \varphi_\theta(u_1^{(3)}), \cdots, \varphi_\theta(u_1^{(m+1)}), \varphi_\theta(u_2^{(1)}), \varphi_\theta(u_2^{(2)}), \varphi_\theta(u_2^{(3)}), \cdots,$ $\varphi_\theta(u_2^{(m+1)}), \cdots, \varphi_\theta(u_m^{(1)}), \varphi_\theta(u_m^{(2)}), \varphi_\theta(u_m^{(3)}), \cdots, \varphi_\theta(u_m^{(m+1)})$ followed by $\varphi_\theta(v_1), \varphi_\theta(v_2), \cdots, \varphi_\theta(v_p)$

This implies that the vertex-sum of vertices of $G \bullet T$ is distinct. Therefore $G \bullet T$ is anti-magic. □

5. Conclusion

Our results in this paper extensively support the conjectures posed by Hartsfield and Ringel. In this paper, we considered various binary graph products such as corona, edge corona and rooted products to generate anti-magic graphs. We proved that the corona product of an anti-magic regular graph with K_1 is anti-magic. Further, we proved that the rooted product of two anti-magic trees are also anti-magic. Also, our results can be used to generate various families of anti-magic graphs. Our results generate anti-magic graphs from regular graphs using various binary products of graphs. In this connection, we raise a related questions that: “Is it possible to generate anti-magic graphs from non-regular anti-magic graphs using binary products?”. In general, is it possible to construct anti-magic graphs from the corona products of two arbitrary anti-magic graphs.

References

[1] N. Alon, G. Kaplan, A. Lev, Y. Roditty and R. Yuster, “Dense graphs are antimagic”, *Journal of Graph Theory,* vol. 47, pp. 297-309, 2004. doi: 10.1002/jgt.20027

[2] T. Cormen, C. E. Leiserson, R. L. Rivest and C. Stein, *Introduction to Algorithms.* 2nd ed. MIT Press, 2001.

[3] D. Cranston, Y. Liang and X. Zhu, “Regular graphs of odd degree are antimagic”, *Journal of Graph Theory,* vol. 80, no. 1, pp. 28-33, 2004. doi: 10.1002/jgt.21836

[4] R. Frucht and F. Harary, “On the corona of two graphs”, *Aequationes mathematicae,* vol. 4, pp. 322-325, 1970. doi: 10.1007/BF01844162

[5] J. A. Gallian, “A Dynamic Survey of Graph Labeling”, *The Electronic Journal of Combinatorics,* 22nd ed., #DS6, 2019.

[6] C. D. Godsil and B. D. McKay, “A new graph product and its spectrum”, *Bulletin of the Australasian Mathematical Society,* vol. 18, no. 1, pp. 21-28, 1978. doi: 10.1017/S0004972700007760

[7] N. Hartsfield and G. Ringel, *Pearls in Graph Theory.* Boston: Academic Press, 1990.

[8] Y. Hou and W.C. Shiu, "The spectrum of the edge corona of two graphs", *Electronic Journal of Linear Algebra,* vol. 20, pp. 586-594, 2010. doi: 10.13001/1081-3810.1395

[9] G. Kaplan, A. Lev and Y. Roditty, "On zero-sum partitions and anti-magic tres", *Discrete Mathematics,* vol. 309, pp. 2010-2014, 2009. doi: 10.1016/j.disc.2008.04.012

[10] Y. Liang and X. Zhu, "Antimagic labeling of cubic graphs", *Journal. Graph Theory,* vol. 75, pp. 31-36, 2013. doi: 10.1002/jgt.21718

[11] Y. Liang, T. Wong and X. Zhu, "Anti-magic labeling of tres", *Discrete Mathematics,* vol. 331, pp. 9-14, 2014. doi: 10.1016/j.disc.2014.04.021

[12] J. Shang, "Spiders are antimagic", *Ars Combinatoria*, vol. 118, pp. 367-372, 2015.

[13] D. B. West, *Introduction to Graph Theory*. 2nd ed. India: Prentice Hall of India, 2001.

P. Ragukumar
Department of Mathematics
School of Advanced Sciences
Vellore Institute of Technology,
Vellore 632 014,
India
e-mail: ragukumar2003@gmail.com