Proyecciones Journal of Mathematics
Vol. 41, N^o 4, pp. 891-902, August 2022.
Universidad Católica del Norte
Antofagasta - Chile

10.22199/issn.0717-6279-5193

Fuzzy S_β-continuous and fuzzy S_β-open mappings

Runu Dhar,
Maharaja Bir Bikram University, India
Received : September 2021. Accepted : February 2022

Abstract

The aim of this paper is to introduce the notion of fuzzy mappings, known as fuzzy S_β-continuous mappings. Some of their basic properties and characterization theorems have been investigated in fuzzy topological spaces. The notion of fuzzy S_β-open mappings have been introduced and some of their characterization theorems and basic properties have also been studied in fuzzy topological spaces.

Keyword: *Fuzzy S_β-open sets, fuzzy S_β-continuous mappings, fuzzy S_β-open mappings, fuzzy topology.*

AMS Subject Classification No.(2020): *54A40, 03E72, 54A05, 54C05, 54C08.*

1. Introduction

The notion of fuzzy sets was introduced by Zadeh [15]. There after realizing its potentiality, researchers investigated on fuzzy sets in different aspects and successfully applied it for further investigations in all the branches of science and technology. Chang [4] introduced the notion of fuzzy topology. Abd El-Monsef et al [1] introduced the concepts of β-open sets and β-continuous functions in general topology and Fath Alla [2] introduced these concepts in fuzzy setting. Khalaf and Ahmed [8] introduced and studied a new class of semiopen sets, called S_β-open sets, then introduced and investigated S_β-continuous functions in general topological spaces. Besides these different researchers [3, 6, 7, 9, 10, 12, 13, 14] contributed lots of work of fuzzy setting. Motivated by these works in this paper we have introduced fuzzy S_β-continuous mappings and have studied their basic properties in fuzzy setting. In section 2, the different concepts of known fuzzy sets and fuzzy mappings have been procured as ready reference. In section 3, the notion of fuzzy S_β-continuous mappings has been introduced and some of their basic properties would be investigated in fuzzy topological spaces. In section 4, the concept of fuzzy S_β-open mappings has been introduced and studied.

2. Preliminaries

In this section, some preliminary results and definitions have been mentioned as ready reference.

Definition 2.1. [15] Let X be an arbitrary set. A fuzzy subset A in X is the collection of ordered pairs $(x, \mu_A(x))$ with $x \in X$ and a membership function

$$\mu_A : X \to [0, 1]$$

The value $\mu_A(x)$ of x denotes the degree to which an element x may be member of A. Thus a fuzzy subset A of X is defined by

$$A = \{(x, \mu_A(x)) : x \in X\}.$$

The null fuzzy set 0_X is the mapping from X into I which assumes only the value 0 and the whole fuzzy set 1_X is the mapping from X into I which takes the value 1 only.

Definition 2.2. [15] Let A and B be two fuzzy sets in a crisp set X and the membership functions of them be μ_A and μ_B respectively. Then

1. A is equal to B, i.e., $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, for all $x \in X$,
2. A is called a subset of B if and only if $\mu_A(x) \leq \mu_B(x)$, for all $x \in X$,
3. the Union of two fuzzy sets A and B is denoted by $A \vee B$ and its membership function is given by $\mu_{A\vee B} = \max(\mu_A, \mu_B)$,
4. the Intersection of two fuzzy sets A and B is denoted by $A \wedge B$ and its membership function is given $\mu_{A\wedge B} = \min(\mu_A, \mu_B)$,
5. the Complement of a fuzzy set A is defined as the negation of the specified membership function. Symbolically it can be written as $\mu_A^c = 1 - \mu_A$.

Definition 2.3. [11] A fuzzy point x_p in X is a fuzzy set in X defined by

$$x_p(y) = \begin{cases} p(0 < p \leq 1) & for y = x \\ 0 & for y \neq x (y \in X), \end{cases}$$

x and p are respectively the support and the value of x_p.
A fuzzy point x_p is said to belong to a fuzzy set A of X if and only if $p \leq A(x)$. A fuzzy set A in X is the union of all fuzzy points which belong to A.

Definition 2.4. [4] Suppose τ is a family of fuzzy subsets in X which satisfies the following axioms :

1. $0_X, 1_X \in \tau$.
2. If $A, B \in \tau$, then $A \wedge B \in \tau$.
3. If $A_j \in \tau$ for all j from the index set $J, \vee_{j\in J} A_j \in \tau$.

Then τ is called a fuzzy topology for X and the pair (X, τ) is called a fuzzy topological space. The elements of τ are called fuzzy open subsets. The complement of each member in τ is defined as a fuzzy closed set in X (with respect to τ) or simply a fuzzy closed set in X.
Throughout the paper, the spaces X and Y always represent fuzzy topological spaces (X, τ) and (Y, σ) respectively.

Definition 2.5. [2] Let X be a fuzzy topological space. A fuzzy set A of X is called fuzzy β-open set if $A \leq clintcl(A)$. The complement of a fuzzy β-open set is fuzzy β-closed set.

Definition 2.6. [2] Let A be a fuzzy set in a fuzzy topological space X. The fuzzy β-closure (βcl) and β-interior (βint) of A are defined as follows :

$$\beta clA = \wedge\{B : A \leq B,\ B \text{ is fuzzy } \beta\text{-closed}\},$$

$$\beta intA = \vee\{B : A \geq B,\ B \text{ is fuzzy } \beta\text{-open}\}.$$

It is obvious that $\beta\mathrm{cl}(A)^c = (\beta intA)^c$ and $\beta\mathrm{int}(A)^c = (\beta clA)^c$.

Definition 2.7. [2] A function $f : X \to Y$ is said to be fuzzy β-continuous (resp. $M\beta$ - continuous) if the inverse image of every fuzzy open (resp. fuzzy β-open) set in Y is fuzzy β-open (resp. fuzzy β-open) set in X.

Definition 2.8. [5] A fuzzy semi open subset A of a fuzzy topological space (X, τ) is said to be fuzzy S_β-open if for each fuzzy point $x_p \in A$ there exists a fuzzy β-closed set F such that $x_p \in F \leq A$. A fuzzy subset B of a fuzzy topological space X is fuzzy S_β-closed if its complement is fuzzy S_β-open.

Theorem 2.9. [5] The union of an arbitrary collection of fuzzy S_β-open sets in a fuzzy topological space (X, τ) is also fuzzy S_β-open set.

Remark 2.10. [5] The intersection of two fuzzy S_β-open sets is not a fuzzy S_β-open set.

3. Fuzzy S_β-continuous mappings

In this section, we introduce the concept of fuzzy S_β-continuity by using fuzzy S_β-open sets. Several relations between these mappings and other types of fuzzy continuous mappings and spaces are also investigated.

Definition 3.1. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy S_β-continuous at a fuzzy point x_p of X if for each fuzzy open set V of Y containing the image of x_p, there exists a fuzzy S_β-open set U in X containing x_p such that $f(U) \leq V$. If f is fuzzy S_β-continuous at every point x_p of X, then it is called fuzzy S_β-continuous.

Proposition 3.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is fuzzy S_β-continuous if and only if the inverse image of every fuzzy open set in Y is fuzzy S_β-open set in X.

Proof. **Necessity**. Let f be a fuzzy S_β-continuous mapping and V be any fuzzy open set in Y. To show that $f^{-1}(V)$ is a fuzzy S_β-open set in X. If $f^{-1}(V) = 0_X$, then $f^{-1}(V)$ is fuzzy S_β-open set in X. If $f^{-1}(V) \neq 0_X$, then there exists $x_p \in f^{-1}(V)$, which implies $f(x_p) \in V$. Since f is fuzzy S_β-continuous, so there exists a fuzzy S_β-open set U in X containing x_p such that $f(U) \leq V$. This implies $x_p \in U \leq f^{-1}(V)$, which shows that $f^{-1}(V)$ is fuzzy S_β-open in X.

Sufficiency. Let V be any fuzzy open set in Y and its inverse be fuzzy S_β-open in X. Since $f(x_p) \in V$, then $x_p \in f^{-1}(V)$ and by hypothesis $f^{-1}(V)$ is fuzzy S_β-open set in X containing x_p, so $f(f^{-1}(V)) \leq V$. Therefore, f is fuzzy S_β-continuous.

Remark 3.3. Every fuzzy S_β-continuous mapping is fuzzy semicontinuous.

Theorem 3.4. Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following statements are equivalent :

1. f is fuzzy S_β-continuous mapping.
2. The inverse image of every fuzzy open set in Y is fuzzy S_β-open set in X.
3. The inverse image of every fuzzy closed set in Y is fuzzy S_β-closed set in X.
4. For each fuzzy set A in X, $f(S_\beta cl(A)) \leq clf(A)$.
5. For each fuzzy set A in X, $intf(A) \leq f(S_\beta int(A))$.
6. For each fuzzy set B in Y, $S_\beta clf^{-1}(B) \leq f^{-1}(clB)$.
7. For each fuzzy set B in Y, $f^{-1}(intB) \leq S_\beta int(f^{-1}(B))$

Proof. $1 \Rightarrow 2$ follows from Proposition 3.2.

$2 \Rightarrow 3$. Let B be any fuzzy closed subset of Y. Then $1_Y - B$ is fuzzy open subset in Y and hence $f^{-1}(1_Y - B) = 1_X - f^{-1}(B)$ is fuzzy S_β-open set in X. Thus $f^{-1}(B)$ is fuzzy S_β-closed set in X.

$3 \Rightarrow 4$. Let A be a fuzzy set in X, then $f(A) \leq Y$. But $f(A) \leq clf(A)$ and by 3, $f^{-1}(clA)$ is fuzzy S_β-closed set in X and $A \leq f^{-1}(clf(A))$, then $S_\beta clA \leq f^{-1}(clf(A))$. This implies $f(S_\beta clA) \leq clf(A)$.

$4 \Rightarrow 5$. Let A be a fuzzy set in X, then $1_X - A \leq X$ and then by 4, $f(S_\beta cl(1_X - A)) \leq clf(1_X - A)$. Therefore, $f(1_X - S_\beta intA) \leq cl(1_Y - f(A))$. This implies $1_Y - f(S_\beta int(A)) \leq 1_Y - intf(A)$. Thus $intf(A) \leq f(S_\beta intA)$.

$5 \Rightarrow 6$. Let B be a fuzzy set in Y, then $f^{-1}(B) \leq X$ and then $1_X - f^{-1}(B) \leq X$. Therefore $intf(1_X - f^{-1}(B)) \leq f(S_\beta int(1_X - f^{-1}(B))$, then $int(1_Y - f(f^{-1}(B)) \leq f(1_X - S_\beta clf^{-1}(B))$, this implies $int(1_Y - B) \leq 1_Y - f(S_\beta clf(f^{-1}(B))$, then $(1_Y - clB) \leq 1_Y - f(S_\beta clf^{-1}(B))$, that is $f(S_\beta clf^{-1}(B)) \leq clB$. Hence $S_\beta clf^{-1}(B) \leq f^{-1}(clB)$.

$6 \Rightarrow 7$. Let B be a fuzzy set in Y, then $1_Y - A \leq Y$. Therefore, by 6, $S_\beta clf^{-1}(1_Y - B) \leq f^{-1}(cl(1_Y - B))$, then $S_\beta cl(1_X - f^{-1}(B)) \leq f^{-1}(1_Y - intB)$, so we get $1_X - S_\beta int(f^{-1}(B)) \leq (1_X - f^{-1}(intB))$, hence $f^{-1}(intB) \leq S_\beta int(f^{-1}(B))$

$7 \Rightarrow 1$. Let x_p be a fuzzy point of X and U be any fuzzy open set of Y containing $f(x_p)$, then by 7,

$f^{-1}(intU) \leq S_\beta int(f^{-1}(U))$, this implies $f^{-1}(U) \leq S_\beta f^{-1}(U)$. Hence $f^{-1}(U)$ is fuzzy S_β-open set in X containing x_p such that $f(f^{-1}(U)) \leq U$. Thus f is fuzzy S_β-continuous mapping.

Theorem 3.5. Let $f : (X, \tau) \to (Y, \sigma)$ be a surjective mapping. Then the following statements are equivalent :

1. f is fuzzy S_β-continuous mapping.
2. For every fuzzy open set B in Y, $intclf^{-1}(B) \leq f^{-1}(clB)$ and $f^{-1}(clB) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$.
3. For every fuzzy set B in Y, $f^{-1}(intB) \leq clintf^{-1}(B)$ and $f^{-1}(intB) = \vee_{i\in\wedge} F_i$ where $F_i \in \beta C(X)$.
4. For each fuzzy set A in X, $f(intclA) \leq clf(A)$ and $f^{-1}(clf(A)) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$.

Proof. $1 \Rightarrow 2$. Let B be a fuzzy set in Y. Then clB is fuzzy closed in Y. Since f is fuzzy S_β-continuous, then by Theorem 3.4., $f^{-1}(clB)$ is fuzzy S_β closed in X. Therefore, by Proposition 3.2., $f^{-1}(clB)$ is fuzzy semiclosed set and $f^{-1}(clB) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$. Thus $intclf^{-1}(clB) \leq f^{-1}(clB)$ and $f^{-1}(clB) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$. Hence $intclf^{-1}(B) \leq f^{-1}(clB)$ and $f^{-1}(clB) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$.

$2 \Rightarrow 1$. Let B be a fuzzy closed set of Y. Then by 2, $intclf^{-1}(B) \leq f^{-1}(clB) = f^{-1}(B)$ and $f^{-1}(B) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$. This implies that $f^{-1}(B) \in SC(X)$ and $f^{-1}(B) = \wedge_{i\in\wedge} V_i$ where $V_i \in \beta O(X)$. Thus by Proposition 3.2., $f^{-1}(B)$ is fuzzy S_β-closed in X. Hence by Theorem 3.4., f is fuzzy S_β-continuous mapping.

$1 \Rightarrow 3$. Let B be a fuzzy set in Y. Then $intB$ is fuzzy open set in Y, since f is fuzzy S_β-continuous. Therefore, $f^{-1}(intB)$ is fuzzy S_β-open in X. This implies $f^{-1}(intB) \in SO(X)$ and $f^{-1}(intB) = \vee_{i\in\wedge} F_i$ where $F_i \in \beta C(X)$. Therefore, $f^{-1}(intB) \leq clintf^{-1}(B)$ and $f^{-1}(intB) = \vee_{i\in\wedge} F_i$ where $F_i \in \beta C(X)$.

$3 \Rightarrow 1$. Let B be any fuzzy open set in Y. Then $intB = B$ and then by 3, $f^{-1}(B) \leq clintf^{-1}(B)$ and $f^{-1}(B) = \vee_{i\in\wedge}F_i$ where $F_i \in \beta C(X)$. This implies $f^{-1}(B) \in S_\beta O(X)$. Hence f is fuzzy S_β continuous.

$2 \Rightarrow 4$. Let A be a fuzzy set of X. Then $f(A)$ is a fuzzy set in Y and then by 2, $intclf^{-1}(f(A)) \leq f^{-1}(clf(A))$ and $f^{-1}(clf(A)) = \wedge_{i\in\wedge}V_i$ where $V_i \in \beta O(X)$. Thus $f(intclA) \leq clf(A)$ and $f^{-1}(clf(A)) = \wedge_{i\in\wedge}V_i$ where $V_i \in \beta O(X)$.

$4 \Rightarrow 2$. Let B be a fuzzy set of Y. Then $f^{-1}(B)$ is a fuzzy set in X. Therefore by 4, $f(intclf^{-1}(B)) \leq clB$ and $f^{-1}(clf(f^{-1}(B))) = \wedge_{i\in\wedge}V_i$ where $V_i \in \beta O(X)$. This implies $intclf^{-1}(B) \leq f^{-1}(clB)$ and $f^{-1}(clB) = \wedge_{i\in\wedge}V_i$ where $V_i \in \beta O(X)$.

4. Fuzzy S_β-open mappings

Definition 4.1 A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy S_β-open if $f(U)$ is fuzzy open in Y for each fuzzy S_β-open set U in X.

Theorem 4.2. A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be fuzzy S_β-open if and only if for every fuzzy subset U of X, $f(S_\beta int(U)) \leq int(f(U))$.

Proof. Let f be a fuzzy S_β-open mapping. Now, we have $S_\beta int(U) \leq U$ and $S_\beta int(U)$ is a fuzzy S_β-open set. Hence we obtain that $f(S_\beta int(U)) \leq f(U)$. Since f is a fuzzy S_β-open mapping, that is $f(U) \in \sigma$ and thus $f(S_\beta int(U)) \leq int(f(U))$.
Conversely, assume that condition holds. Let U be a fuzzy S_β-open set in X.
Then $f(U) = f(S_\beta int(U)) \leq int(f(U))$. But $int(f(U)) \leq f(U)$.
Consequently, $f(U) = int(f(U))$ and hence f is fuzzy S_β-open mapping.

Theorem 4.3. If a mapping $f : (X, \tau) \to (Y, \sigma)$ is fuzzy S_β-open then for every fuzzy subset G of Y, $S_\beta int(f^{-1}(G)) \leq f^{-1}int(G))$.

Proof. Let G be an arbitrary fuzzy subset of Y.

Then $S_\beta int(f^{-1}(G))$ is a fuzzy S_β-open set in X and f is fuzzy S_β-open.

Then $f(S_\beta int(f^{-1}(G))) \leq int(f(f^{-1}(G))) \leq int(G)$.

So that $S_\beta int(f^{-1}(G)) \leq f^{-1}f(S_\beta int(f^{-1}(G))) \leq f^{-1}(int(f(f^{-1}(G)))) \leq f^{-1}int(G)$.

Thus $S_\beta int(f^{-1}(G)) \leq f^{-1}(int(G))$.

Theorem 4.4. Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping. Then the following are equivalent :

1. f is fuzzy S_β-open.
2. For each fuzzy subset U of X, $f(S_\beta int(U)) \leq int(f(U))$.
3. For each fuzzy point x_p in X and each fuzzy S_β-neighborhood U of x_p in X there exists a fuzzy neighborhood V of $f(x_p)$ in Y such that $V \leq f(U)$.

Proof. $1 \Rightarrow 2$. It follows from Theorem 4.2.
$2 \Rightarrow 3$. Assume 2 holds.
Let x_p be a fuzzy point in X and U be an arbitrary fuzzy S_β-neighborhood of x_p in X
Then there exists a fuzzy S_β-open set V in X such that $x_p \in V \leq U$
By 2, we have $f(V) = f(S_\beta int(V)) \leq int(f(V))$ and hence $f(V) = int(f(V))$.
Therefore it follows that $f(V)$ is fuzzy open in Y such that $f(x_p) \in f(V) \leq f(U)$.
Thus 3 holds.
$3 \Rightarrow 1$. Assume 3 holds. Let U be an arbitrary fuzzy S_β-neighborhood in X.
Then for each $y_p \in f(U)$, by 3 there exists a fuzzy neighborhood V_{y_p} of y_p in X such that $V_{y_p} \leq f(U)$.
As V_{y_p} is a fuzzy neighborhood of Y, there exists a fuzzy open set W_{y_p} in Y such that $y_p \in W_{y_p} \leq V_{y_p}$. Thus, $f(U) = \vee\{W_{y_p} : y_p \in f(U)\}$ is a fuzzy open set in Y. This implies that f is fuzzy S_β-open.

Theorem 4.5. A mapping $f : (X, \tau) \to (Y, \sigma)$ is fuzzy S_β-open if and only if for any fuzzy subset B of Y and for any fuzzy S_β-closed set F of X containing $f^{-1}(B)$, there exists a fuzzy closed set A in Y containing B such that $f^{-1}(A) \leq F$.

Proof. Suppose, f is fuzzy S_β-open. Let $B \leq Y$ and F be a fuzzy S_β-closed set in X containing $f^{-1}(B)$. Now put $A = Y - f(X - F)$. It is clear that $f^{-1}(B) \leq F$ implies $B \leq A$.
Since, f is fuzzy S_β-open, we obtain that A is a fuzzy closed set of Y, then we have $f^{-1}(A) \leq F$.
Conversely, let U be a fuzzy S_β-open set in X and $B = Y - f(U)$.
Then $X - U$ is a fuzzy S_β-closed set in X containing $f^{-1}(B)$.
By hypothesis, there exists a fuzzy closed set F of Y such that $B \leq F$ and $f^{-1}(F) \leq X - U$.
Hence, we obtain $f(U) \leq Y - F$.
On the other hand, it follows that $B \leq F$, $Y - F \leq Y - B = f(U)$.
Thus, we obtain $f(U) = Y - F$ which is a fuzzy open set and hence f is fuzzy S_β-open mapping.

Corollary 4.6. A mapping $f : (X, \tau) \to (Y, \sigma)$ is fuzzy S_β-open if and only if $f^{-1}(cl(B)) \leq S_\beta cl(f^{-1}(B))$ for every fuzzy subset B of Y.

Proof. Suppose, f be fuzzy S_β-open. For every fuzzy subset B of Y, $f^{-1}(B) \leq S_\beta cl(f^{-1}(B))$. Therefore by Theorem 4.5., there exists a fuzzy closed F in Y containing B such that $f^{-1}(F) \leq S_\beta cl(f^{-1}(B))$. Because $cl(B) \leq cl(F) = F$, we have $f^{-1}(cl(B)) \leq f^{-1}(F)$. Therefore, we obtain $f^{-1}(cl(B)) \leq S_\beta cl(f^{-1}(B))$.
Conversely, let $B \leq Y$ and F be a fuzzy S_β-closed set of X containing $f^{-1}(B)$. We have $S_\beta cl(f^{-1}(B)) \leq S_\beta cl(F) = F$. Put $W = cl_Y(B)$, then we have $B \leq cl_Y(B) = W$ and W is fuzzy closed in Y and by hypothesis, $f^{-1}(W) = f^{-1}(cl(B)) \leq S_\beta cl(f^{-1}(B))$, so that $f^{-1}(W) \leq F$. Then by Theorem 4.5., the mapping f is fuzzy S_β-open.

Acknowledgement

The author thanks the reviewer for the comments those improved the presentation of the article.

References

[1] M. E. Abd El-Monsef, S. N. El-Deeb, and R. A. Mahmoud, "β-open sets and β-continuous mappings", *Bulletin of the Faculty of Science. Assiut University,* vol. 12, pp. 77-90, 1983.

[2] Fath Alla, "On fuzzy topological spaces", Ph. D. Thesis, Assiut University, Sohag, Egypt, 1984.

[3] K. K. Azad, "Fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity", *Journal of Mathematical Analysis and Applications,* vol. 82, pp. 14-32, 1981.

[4] C. L. Chang, "Fuzzy topological spaces", *Journal of Mathematical Analysis and Applications*, vol. 24, pp. 182-190, 1968.

[5] R. Dhar, "Fuzzy S_β-open sets and fuzzy S_β-operations", *Acta Ciencia Indica*, vol. 47, no. 1-4, pp. 57-66, 2021.

[6] R. Dhar, "Continuity in fuzzy bitopological ordered spaces", *Proyecciones (Antofagasta),* vol. 40, no. 3, pp. 681–696, 2021. doi: 10.22199/issn.0717-6279-4386

[7] R. Dhar, "Some pairwise weakly fuzzy mappings", *Proyecciones (Antofagasta),* vol. 38, no. 4, pp. 707–717, 2019. doi: 10.22199/issn.0717-6279-2019-04-0046

[8] A. B. Khalaf and N. K. Ahmed, "S_β-open sets and S_β-continuity in topological spaces", *Thai Journal of Mathematics,* vol. 11, no. 2, pp. 319-335, 2013. [On line]. Available: https://bit.ly/3zuK6pg

[9] A. Mukherjee and R. Dhar, "Weakly fuzzy δ-semi precontinuous mappings and weakly fuzzy δ-semi preirresolute mappings", *The Journal of Fuzzy Mathematics,* vol. 18, no. 1, pp. 209-216, 2010.

[10] A. Mukherjee and R. Dhar, "Fuzzy δ-semi preopen sets and weakly fuzzy δ-semi preopen functions", *Acta Ciencia Indica, Mathematics,* vol. 35, no. 1, pp. 11-16, 2009.

[11] P. M. Pu and Y. M. Liu, "Fuzzy topology. I. neighborhood structure of a fuzzy point and moore-smith convergence", *Journal of Mathematical Analysis and Applications,* vol. 76, no. 2, pp. 571–599, 1980. doi: 10.1016/0022-247x(80)90048-7

[12] B. C. Tripathy and G. C. Ray, “Fuzzy δ-almost continuous and fuzzy δ-continuous functions in mixed fuzzy ideal topological spaces”, *Proyecciones (Antofagasta)*, vol. 39, no. 2, pp. 435–449, 2020. doi: 10.22199/issn.0717-6279-2020-02-0027

[13] B. C. Tripathy and G. C. Ray, “On δ-continuity in mixed fuzzy topological spaces,” *Boletim da Sociedade Paranaense de Matemática*, vol. 32, no. 2, pp. 175–187, 2014. doi: 10.5269/bspm.v32i2.20254

[14] B. C. Tripathy and G. C. Ray, “Weakly continuous functions on mixed fuzzy topological spaces,” *Acta Scientiarum. Technology*, vol. 36, no. 2, pp. 331–335, 2014. doi: 10.4025/actascitechnol.v36i2.16241

[15] L. A. Zadeh, “Fuzzy sets”, *Information and Control*, vol. 8, pp. 338-353, 1965.

Runu Dhar
Department of Mathematics
Maharaja Bir Bikram University
P. O. Agartala College
College Tilla, Agartala, Tripura,
PIN-799004
India
e-mail : runu.dhar@gmail.com