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Sequence spaces defined via Euler method and matrix transformations

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Abstract

A blend of matrix summability and Euler summability transformation methods is used to define Lacunary sequence spaces defined over n-normed space. Then we present the properties of this space and finally, some inclusion relations are presented.

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1. Introduction

Accelerated convergence techniques are useful in computer science especially in making graphics and to find eigenvalues and eigenvectors of dynamical systems. Euler summability plays a prominent role in improving the convergence of a given series. For a positive real q and a non negative integer n, the Euler transform (see. [1]) E_n^q of sequence $S = (s_n)$ of the partial sum of a series $\sum_{k=0}^{\infty} a_k$ is defined as $E_n^q(S) = \frac{1}{(1+q)^n} \sum_{v=0}^{n} \begin{pmatrix} n \\ v \end{pmatrix} q^{n-v} s_v$. A series $\sum a_n$ is said to be E_n^q−summable to s if $E_n^q(S) \to s$, as $n \to \infty$, further, if $\sum_k |E_k^q(S) - E_{k-1}^q(S)| < \infty$ then the sequence is said to be absolutely E_n^q-summable. Let $x = (x_n)$ be a sequence of scalars, for $n \geq 1$ we will denote by $N_n(x)$ the difference $E_n^q(x) - E_{n-1}^q(x)$, where E_n^q is defined as above.

Another important transform of numerical analysis is "Abel's transform" which is defined as $A_k = \sum_{j=0}^{k} \left[\frac{q}{1+q} \begin{pmatrix} n \\ j \end{pmatrix} - \begin{pmatrix} n-1 \\ j \end{pmatrix} \right] q^{n-(j+1)}$.

Using Abel's transform we have

$$N_n(x) = -\frac{1}{(1+q)^{n-1}} \left(\sum_{j=0}^{n-2} x_{j+1} A_j + s_{n-1} A_{n-1} \right) + \frac{1}{(1+q)^n} \left(s_n - q^{n-1} s_0 \right),$$

and hence, for a scalar λ and sequences $x = (x_n)$, $y = (y_n)$, we have: $N_n(x+y) = N_n(x) + N_n(y)$ and $N_n(\lambda x) = \lambda N_n(x)$.
In this paper, we study certain properties of a class of sequences (defined by using Euler transform) over an n-normed space using Musielak-Orlicz function. We introduce these spaces by using the Euler and some other matrix transformations. Finally, we present some inclusion relations between these spaces. Before proceeding further we present some definitions and results required for the further development of the article.

Definition 1.1. *A continuous, convex, non-decreasing function φ with $\varphi(x) > 0$ for $x > 0$; $\varphi(0) = 0$ and $\varphi(x) \longrightarrow \infty$ whenever $x \longrightarrow \infty$ is called an "Orlicz function".*

Orlicz sequence space [8], denoted ℓ_φ is space of sequences $x = (x_n)$ which satisfy $\displaystyle\sum_{k=1}^{\infty} \varphi\left(\frac{|x_k|}{t}\right) < \infty, t > 0$.

Lemma 1.2. *[6] $(\ell_\varphi, ||.||)$ is a Banach space, where* $||x|| = \inf\Big\{t > 0 : \sum_{j=1}^{\infty} \varphi\Big(\frac{|x_j|}{t}\Big) \leq 1\Big\}$.

Definition 1.3. *A "Musielak-Orlicz function" denoted by Φ is a sequence of Orlicz functions ϕ_i.*

For more information about the complementary function of the Musielak Orlicz function, Musielak Orlicz sequence space, Luxemburg norm we refer to [8].

Remark 1.4. $\mathbf{N}, \mathbf{Z}, \mathbf{R}$ *and* $\mathbf{C}$ *denotes set of natural numbers, integers, real numbers and complex numbers respectively.*

Definition 1.5. *A Lacunary sequence [3] is a sequence $\theta = (k_r)$ of positive integers with $k_0 = 0$, for $0 < k_r < k_{r+1}$ and $h_r = (k_r - k_{r-1}) \to \infty$ as $r \to \infty$.*

Intervals $I_r = (k_{r-1}, k_r]$ are determined by θ and the ratio $\frac{k_r}{k_{r-1}}$ is denoted as q_r.
Another important notation that we require for this article is the notation of an n-norm.

Definition 1.6 (n-norm, see [5, 11]). *A real valued function $||\cdot, \cdots, \cdot||$ defined on V^n, where V is a linear space of dimension d , $d \geq n \geq 2, n \in \mathbf{N}$ over the (real or complex) field $\mathbf{K}$ is called an n-norm ([4, 9]) if the following conditions are satisfied:*

1. $v_1, v_2, \cdots, v_n$ *are linearly dependent iff* $||(v_1, v_2, \cdots, v_n)|| = 0$;
2. $||(v_1, v_2, \cdots, v_n)||$ *is invariant under permutations;*
3. $||(\alpha v_1, v_2, \cdots, v_n)|| = |\alpha| \ \ ||(v_1, v_2, \cdots, v_n)|| \ \forall \ \alpha \in \mathbf{K}$;
4. $||(v + v', v_2, \cdots, v_n)|| \leq ||(v, v_2, \cdots, v_n)|| + ||(v', v_2, \cdots, v_n)||$.

The pair $(V(\mathbf{K}), ||\cdot, \cdots, \cdot||)$ is called an n-normed space.

Sequence spaces defined by Orlicz function have been introduced and their different properties have been investigated by Tripathy and Mahanta [14, 15]. The notion of lacunary sequences have been investigated from different aspects by introducing different classes of lacunary spaces by Tripathy and Baruah [10], Tripathy and Dutta [12], Tripathy et al. [13], Tripathy and Sen [16] and others. For a detailed account of sequence spaces we refer the reader to [1, 2].

2. Main Results

Let

$$\mathbf{A} = [a_{ij}] = \begin{bmatrix} p_1 & w_1^{(1)} & w_1^{(2)} & \dots \\ w_1^{(-1)} & p_2 & w_2^{(1)} & \dots \\ w_1^{(-2)} & w_2^{(-1)} & p_3 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

where $p = (p_i)$ and $w^{(t)} = (w_i)^{(t)}$ are some fixed numerical sequences, $t \in \mathbf{Z}\backslash\{0\}$. For a fixed $k_f \in \mathbf{N}$ we define a finite sequence t_n with k_f terms by, $t_n = \begin{cases} \frac{n+1}{2}, & \text{n is odd;} \\ \frac{-n}{2}, & \text{n is even.} \end{cases}$. We construct a matrix $\mathbf{A}_{(p,w^t,k_f)} = \mathbf{A}$, $w^{t_i} = 0 \; \forall \; i > k_f$ and for $i = 1, 2, \dots, k_f$ we have some fixed sequences w^{t_i} and p.

Example 2.1. *For $k_f = 2$ we have $t_1 = 1, t_2 = -1$, we define $p_i = -1 \; \forall \; i$ and*
$w_i^{(t)} = \begin{cases} 1, & \textit{for } t = 1, -1 \\ 0, & \forall \; t \in \mathbf{Z}\backslash\{0, 1, -1\} \end{cases}$ *then we have,*

$$\mathbf{A} = [a_{ij}] = \begin{bmatrix} -1 & 1 & 0 & \dots \\ 1 & -1 & 1 & \dots \\ 0 & 1 & -1 & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

and hence, $\mathbf{A}_{(p,w^t,2)}x = \left\langle \sum_{j=1}^{\infty} m_{ij}\xi_j \right\rangle_n =< -\xi_1 + \xi_2, \xi_1 - \xi_2 + \xi_3, \xi_2 - \xi_3 + \xi_4, \xi_3 - \xi_4 + \xi_5 \dots >$.

For a Musielak-Orlicz function $\Phi = (\varphi_j)$, we define the following sequence space:

$$E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,j_f)}, ||\cdot, \cdots, \cdot||) = \Big\{x = (x_j) : \lim_r \tfrac{1}{h_r} \textstyle\sum_{j \in I_r} k^{-s} \left[\varphi_j\left(||(\frac{u_j N_j(\mathbf{A}_{(p,w^t,j_f)}x)}{t}, v_1, \cdots, v_{n-1})||\right)\right]^{p_j} < \infty, \;\; s \geq 0, \;\; \text{for some} \;\; t > 0\Big\}.$$

Here, $p = (p_j)$ and $u = (u_j)$ are the bounded sequence of non-negative reals and sequence of positive reals; h_r is as defined in definition 1.5; and $v_i \in V$.

Lemma 2.2 (Maddox, [7]). *If $K = \max(1, 2^{H-1})$ and $0 \leq p_j \leq \sup p_j = H$ then we have, $|a_j + b_j|^{p_j} \leq K\{|a_j|^{p_j} + |b_j|^{p_j}\}$ for all j and $a_j, b_j \in \mathbf{C}$. Further we have, $|a|^{p_j} \leq \max(1, |a|^H)$ for all $a \in \mathbf{C}$.*

Theorem 2.3. *$E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||)$ is a vector space over the field of complex numbers and is paranormed by the paranorm $g(x)$ defined by:*

$$(x) = \inf\Big\{t^{p_n/H} : \Big(\lim_r \tfrac{1}{h_r}\textstyle\sum_{k\in I_r}\tfrac{1}{k^s}\Big[\phi_k ||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1, \cdots, v_{n-1})||\Big]^{p_k}\Big)^{\frac{1}{H}} \leq 1,\ n = 1, 2, 3, \cdots\Big\},$$

here $H = \max(1, \sup_k p_k)$.

Proof. Let, $x = (x_k)$ and $y = (y_k) \in E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||)$,$\alpha, \beta \in \mathbf{C}$. There exist $t_1, t_2 \in \mathbf{N}$ such that,

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t_1}, v_1, \cdots, v_{n-1})||\Big]^{p_k} < \infty \text{ and}$$

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}y)}{t_2}, v_1, \cdots, v_{n-1})||\Big]^{p_k} < \infty.$$

Since (ϕ_k) is a sequence of convex and non-decreasing functions. Let $t_3 = \max(2|\alpha|t_1, 2|\beta|t_2)$ then we have,

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}(\alpha x + \beta y))}{t_3}, v_1, \cdots, v_{n-1})||\Big]^{p_k}$$

$$\lim_r \tfrac{1}{h_r}\textstyle\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}\alpha x)}{t_3}, v_1, \cdots, v_{n-1})|| + ||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}\beta y)}{t_3}, v_1, \cdots, v_{n-1})||\Big]^{p_k}$$

$$K \lim_r \tfrac{1}{h_r}\textstyle\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t_1}, v_1, \cdots, v_{n-1})||\Big]^{p_k} + K \lim_r \tfrac{1}{h_r}\textstyle\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}y)}{t_2}, v_1, \cdots, v_{n-1})||\Big]^{p_k} < \infty.$$

Therefore, $\alpha x + \beta y \in E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||)$. Hence, the space $E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||)$ is linear.
Further, we have, $g(x + y) \leq g(x) + g(y)$ and $g(x) = g(-x)$. Now, as $M_k(0) = 0$, we have $\inf\{t^{p_n/H}\} = 0$ if $x = 0$.
For any number λ,

$$(\lambda x) = \inf\Big\{t^{p_n/H} : \lim_r \tfrac{1}{h_r}\textstyle\sum_{k\in I_r} k^{-s}\Big[\phi_k ||(\tfrac{\lambda u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1, \cdots, v_{n-1})||\Big]^{p_k} \leq 1, n = 1, 2, 3, \cdots\Big\}$$

implies

$$(\lambda x) = \inf\Big\{(\lambda s)^{p_n/H} : \lim_r \tfrac{1}{h_r}\sum_{k\in I_r} k^{-s}$$
$$\Big[\phi_k||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{s}, v_1,\cdots,v_{n-1})||\Big]^{p_k} \le 1, n=1,2,3,\cdots\Big\},$$

where, $s = \frac{t}{|\lambda|}$.

By Theorem 2.2 we get, $|\lambda|^{p_k/H} \le \Big(\max\Big(|\lambda|^H, 1\Big)\Big)^{\frac{1}{H}}$ and this gives us,

$$(\lambda x) \le \Big(\max\Big(|\lambda|^H,1\Big)\Big)^{\frac{1}{H}} \inf\Big\{(s)^{p_n/H} : \Big(\lim_r \tfrac{1}{h_r}\sum_{k\in I_r} k^{-s}$$
$$\Big[\phi_k||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}\Big)^{1/H} \le 1; n=1,2,\cdots\Big\}.$$

Clearly, $g(x)\to 0$ when $x\to 0$ in $E_n^q(\Phi,u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$. Now let, $\lambda_n \to 0$ and $x\in E_n^q(\Phi,u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$. For any $\epsilon>0$, let $n_0\in\mathbf{N}$ such that

$$\lim_r \frac{1}{h_r}\sum_{k=n_0+1\in I_r} k^{-s}\Big[\phi_k||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k} < \frac{\varepsilon}{2}$$

for some $t>0$. This gives us

$$\Big(\lim_r \frac{1}{h_r}\sum_{k=n_0+1\in I_r} k^{-s}\Big[\phi_k||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}\Big)^{\frac{1}{H}} \le \frac{\varepsilon}{2}.$$

Let $0<|\lambda|<1$, then by using the convexity of (ϕ_k), we have

$$\lim_r \tfrac{1}{h_r}\sum_{k=n_0+1\in I_r} k^{-s}\Big[\phi_k\Big(||(\tfrac{\lambda u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}$$
$$< |\lambda| \lim_r \tfrac{1}{h_r}\sum_{k=n_0+1\in I_r} k^{-s}\Big[\phi_k||(\tfrac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k} < \Big(\tfrac{\varepsilon}{2}\Big)^H.$$

Since (ϕ_k) is continuous everywhere on $[0,\infty)$, so

$$h(t) = \lim_r \frac{1}{h_r}\sum_{k=1}^{n_0} k^{-s}\Big[\phi_k||(\frac{t u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}$$

is also continuous at zero. Hence, there exists $0<\delta<1$ such that for some $0<t<\delta$ we have $|h(t)|<\varepsilon/2$.
Let, $K(<n)$ be such that $|\lambda_n|<\delta$ then, we get

$$\Big(\lim_r \frac{1}{h_r}\sum_{k=1}^{n_0} k^{-s}\Big[\phi_k||(\frac{\lambda_n u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}\Big)^{\frac{1}{H}} < \frac{\varepsilon}{2}.$$

Thus for $n > K$

$$\left(\lim_r \frac{1}{h_r}\sum_{k\in I_r} k^{-s}\Big[\phi_k||(\frac{\lambda_n u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}\right)^{\frac{1}{H}} < \varepsilon.$$

Hence, $g(\lambda x)$ tends to 0 as λ tends to 0 and hence the result. □

Theorem 2.4. *The space $E_n^q(\Phi,u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$ is complete with paranorm $g(x)$ as defined in 2.3.*

Proof. The proof is routine verification, hence omitted. □

Theorem 2.5. *For "Musielak-Orlicz functions" $\Phi' = (\phi'_k)$ and $\Phi'' = (\phi''_k)$ and for non-negative real numbers s, s_1, s_2, we have*
(i) $E_n^q(\Phi',u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)\cap E_n^q(\Phi'',u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||) \subseteq E_n^q(\Phi'+\Phi'',u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$,
(ii) If $s_2 \geq s_1$, then

$$E_n^q(\Phi',u,p,s_1,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||) \subseteq E_n^q(\Phi',u,p,s_2,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||),$$

(iii) If Φ' and Φ'' are equivalent , then
$E_n^q(\Phi',u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||) = E_n^q(\Phi'',u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$.

Proof. The proof of the theorem can be established using standard techniques, so omitted. □

Theorem 2.6. $\; {}_n^q(\Phi,u,r,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||) \subseteq E_n^q(\Phi,u,p,s,\mathbf{A}_{(p,w^t,k_f)}, ||\cdot,\cdots,\cdot||), 0 < r_k \leq p_k < \infty.$

Proof. Let, $x \in E_n^q(\Phi,u,r,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$. Then, there exist some $t > 0$ such that

$$\lim_r \frac{1}{h_r}\sum_{k\in I_r} k^{-s}\Big[\phi_k||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{r_k} < \infty.$$

Hence, $\phi_k||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})|| \leq 1$ for sufficiently large k. For some fixed $k_0 \in \mathbf{N}$ let, $k \geq k_0$. As (M_k) is non-decreasing so
$\lim_r \frac{1}{h_r}\sum_{k\geq k_0\in I_r} k^{-s}\Big[\phi_k||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{p_k}$
$\leq \lim_r \frac{1}{h_r}\sum_{k\geq k_0\in I_r} k^{-s}\Big[\phi_k||(\frac{u_k N_k(\mathbf{A}_{(p,w^t,k_f)}x)}{t}, v_1,\cdots,v_{n-1})||\Big]^{r_k} < \infty.$
Hence, $x \in E_n^q(\Phi,u,p,s,\mathbf{A}_{(p,w^t,k_f)},||\cdot,\cdots,\cdot||)$. □

We state the following result without proof in view of Theorem 2.6.

Theorem 2.7. *(i) If $0 < p_k \leq 1$, then we have*
$E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||) \subseteq E_n^q(\Phi, u, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||)$.
(ii) If $p_k \geq 1$ for every natural number k, then
$E_n^q(\Phi, u, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||) \subseteq E_n^q(\Phi, u, p, s, \mathbf{A}_{(p,w^t,k_f)}, ||\cdot, \cdots, \cdot||)$.

3. Conclusion

We have defined and studied the properties of certain new Lacunary sequence spaces defined over an n-normed space. The new space is defined by using a combination of matrix and Euler summability transformation methods. Some inclusion relations between these spaces are proved.

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