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Grundy number of corona product of some graphs

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Abstract

The Grundy number of a graph G is the maximum number k of colors used to color the vertices of G such that the coloring is proper and every vertex u colored with color i, $1 \leq i \leq k$, is adjacent to $i-1$ vertices colored with each color j, $1 \leq j \leq i-1$. In this paper we obtain the Grundy number of corona product of some graphs, denoted by $G \circ H$. First, we consider the graph G be 2-regular graph and H be a cycle, complete bipartite, ladder graph and fan graph. Also we consider the graph G and H be a complete bipartite graphs, fan graphs.

Keywords: *Grundy number, corona product, 2-regular graph, cycle, complete bipartite, ladder graph, fan graph.*

AMS Subject Classification: *05C15, 05C75*

1. Introduction

We consider graphs without loops or multiple edges. Let G be a graph on vertices $u_1, u_2, \ldots, u_m$ with vertex set $V(G)$ and edge set $E(G)$. Let $d(u)$ be the degree of the vertex u of G and let $\Delta(G)$ be the maximum degree of G.

A $k-$coloring of a graph G is a surjective mapping $g : V(G) \rightarrow \{1, 2, \ldots, k\}$ (we say that vertex u is colored with $g(u)$). A $k-$coloring g is proper if any two adjacent vertices receive different colors in g. The chromatic number $\chi(G)$ of G is the smallest k such that G has a proper $k-$coloring. Determining the chromatic number of a graph is one of the most fundamental problems in graph theory. Given a graph G and an ordering $g = u_1, u_2, \ldots u_m$ of $V(G)$, the first$-$fit coloring algorithm colors vertex u_i with the smallest color of $u_1, u_2, \ldots u_{i-1}$ that is not present among the set of neighbors of u_i. The Grundy number $\Gamma(G)$ [5, 7, 16, 10, 12] is the largest k such that G admits a vertex ordering on which the first$-$fit algorithm yields a proper $k-$coloring. First$-$fit is presumably the simplest heuristic to compute a proper coloring of a graph. In this sense, the Grundy number gives an algorithmic upper bound on the performance of any heuristic for the chromatic number.
Greedy Coloring Algorithm: [2, 13] Assume that a graph G's vertices are given in the following order $u_1, u_2, \ldots, u_m$.

- The vertex u_1 is assigned the color 1.
- Once the vertices $u_1, u_2, \ldots u_j$ have been assigned colors, where $1 \leq j \leq n$, the vertex u_{j+1} is assigned the smallest color that is not assigned to any neighbor of u_{j+1} belonging to the set $\{u_1, u_2, \ldots, u_j\}$.

While the Greedy coloring algorithm is efficient in the sense that the vertex coloring that it produces, regardless of the order in which its vertices are listed, is done in polynomial time (a polynomial in the order n of the graph), the number of colors in the coloring obtained need not equal or even be close to the chromatic number of the graph. Indeed, there is reason not to be optimistic about finding any efficient algorithm that produces a coloring of each graph where the number of colors is close to the chromatic number of the graph since Michael R. Garey and David S. Johnson,[8] have shown that if there should be an efficient algorithm that produces a coloring of every graph G using at most $2\chi(G)$ colors, then there is an efficient algorithm that determines $\chi(G)$ exactly for every graph G.

This notion was first studied by Grundy in 1939 in the context of digraphs and games [1, 11], and formally introduced by Christen and Selkow [3] in 1979. In [16], Hedetniemi et al. gave a linear algorithm for the Grundy number of a tree and established a relation between the chromatic number, the Grundy number and the achromatic number: $\chi(G) \leq \Gamma(G) \leq \psi(G)$, where the achromatic number $\psi(G)$ is the maximum number of colors used for a proper coloring of G such that each pair of colors appears on at least one edge of G. In 1997, Telle and Proskurowski [19] gave an algorithm for the Grundy number of partial $k-$trees in $O(n_3k_2)$ and bounded this parameter for these graphs by the value $1 + k\log_2 n$, where n is the graph order. In 2000, Dunbar et al. used the Grundy number to bound new parameters that they introduced in [4], the chromatic and the achromatic numbers of a fall coloring. Recently, Germain and Kheddouci studied in [9] the Grundy coloring of power graphs. They gave bounds for the Grundy number of the power graphs of a path, a cycle, a caterpillar and a complete binary tree. Such colorings are also explored for other graphs like chessboard graphs [17].

2. Preliminaries

All graphs we consider are simple and fnite. A closed trail whose origin and internal vertices are distinct is called a cycle. A regular graph [13] of degree 0 has no lines at all. If G is regular of degree 1, then every component contains exactly one line; if it is regular of degree 2, every component is a cycle, and conversely of course. A bigraph [13] (or bipartite graph) G is a graph whose point set V can be partitioned into two subsets V_1 and V_2 such that every line of G joins V_1 with V_2. If G contains every line joining V_1 and V_2, then G is a complete bigraph. If V_1 and V_2 have m and n points, we write $G = K_{m,n}$. A fan graph [21] $F_{m,n}$, is defined as the graph join $\overline{K_m} + P_n$, where $\overline{K_m}$ is the empty graph on m vertices and P_n is the path graph on n vertices, where $V(F_{1,n-1}) = \{u_n\} \cup \{u_1, u_2, \dots u_{n-1}\}$. The Ladder graph[18, 22] L_n is defined by $L_n = P_nK_2$, where P_n is a path with n vertices, denotes the Cartesian product and K_2 is a complete graph with two vertices.

The corona [6, 14, 15, 20] of two graphs G and H is the graph $G \circ H$ formed from one copy of G and $|V(G)|$ copies of H, where the i^{th} vertex of G is adjacent to every vertex in the i^{th} copy of H. Such type of graph products was introduced Frucht and Harary in 1970 [6]. For example the corona $P_n \circ K_1$ is a comb graph.

In the following section, we obtain the Grundy number of corona product of some graphs, denoted by $G \circ H$. First, we consider the graph G be $2-$ regular graph and H be a cycle, complete bipartite, ladder graph and fan graph. Also we consider the graph G and H be a complete bipartite graphs, fan graphs.

3. Main Results

Now we consider G and H be a 2-regular graphs of order $m > 4$ and $n > 4$. Let $V(G) = \{u_i : 1 \leq j \leq m\}$ and let $V(H) = \{v_j : 1 \leq j \leq n\}$. Let $V(G \circ H) = V(G) \cup \bigcup_{i=1}^{m} \{v_{i,j} : 1 \leq j \leq n\}$.

Theorem 1. *Let G and H be a 2-regular graphs of order $m > 4$ and $n > 4$. then $\Gamma(G \circ H) = 6$.*

Proof: Define a mapping, $g : V(G \circ H) \to \mathbf{N}$ as follows:

Case (i): For $m \equiv 0, 23$.
For $1 \leq i \leq m$,

$$g(u_i) = \begin{cases} 4, & \text{if } i \equiv 13 \\ 5, & \text{if } i \equiv 23 \\ 6, & \text{if } i \equiv 03 \end{cases} \tag{3.1}$$

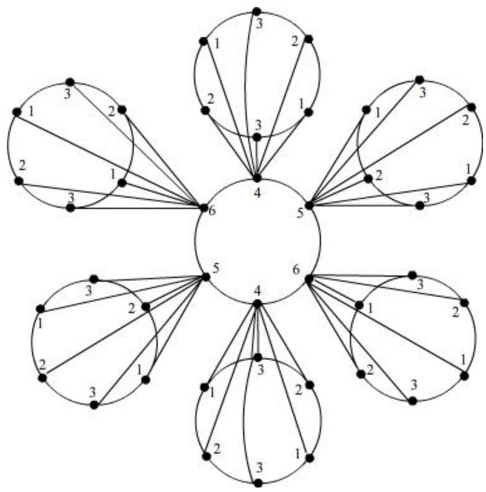

Figure 1: Grundy number of $C_6 \circ C_6$ is 6.

For $n \equiv 0, 23$.

For $1 \leq i \leq m$ and $1 \leq j \leq n$,

$$g\left(v_{i,j}\right) = \begin{cases} 1, & \text{if } j \equiv 13 \\ 2, & \text{if } j \equiv 23 \\ 3, & \text{if } j \equiv 03 \end{cases} \tag{3.2}$$

For $n \equiv 13$.

Using the color pattern as followed from equation (3.2) of order $n-2$. Also $g\left(v_{i,n-1}\right) = 1$; $g\left(v_{i,n}\right) = 2$.

Case (ii): For $m \equiv 13$

Using the color pattern as followed from Case(i) of order $m-2$. Also $g\left(u_{m-1}\right) = 4$; $g\left(u_m\right) = 5$.

Assume $\Gamma\left(G \circ H\right) \leq 7$. Consider a Greedy algorithm that assigns a colors 1, 2 and 3 to the vertices $\{v_{i,j} \forall\ i = 1 \text{ to } m \text{ and } \ j = 1 \text{ to } n\}$ in order to satisfy the Grundy number. Next we assigns a color to the vertices u_i, $(1 \leq i \leq m)$ in such a way that they receive the color 4 or 5 or 6 which is not assigned to the vertices in $N(u_i) = \{v_{i,j}, u_{i+1}, u_{i-1} \forall\ i = 1 \text{ to } m \text{ and } \ j = 1 \text{ to } n\}$. Then any vertex of $u_i, (1 \leq i \leq m)$ is given the color 7, the in-order to satisfy the Grundy number, the vertex which is given the color 7 must be adjacent to all neighbourhood vertices which is given the colors 1, 2, 3, 4, 5 and 6. Which is contradiction. Hence $\Gamma\left(G \circ H\right) \leq 6$. □

Now we consider G and H be a complete bipartite graphs. Let $V(G) = \{u_i : 1 \leq i \leq l\} \cup \{v_j : 1 \leq j \leq p\}$ and let $V(H) = \{x_r : 1 \leq r \leq m\} \cup \{y_s : 1 \leq s \leq n\}$. Let $V\left(G \circ H\right) = V(G) \cup \bigcup\limits_{i=1}^{l} \{x_{i,r}, y_{i,s} : 1 \leq r \leq m,\ 1 \leq s \leq n\} \cup \bigcup\limits_{j=1}^{p} \{x_{j,r}, y_{j,s} : 1 \leq r \leq m,\ 1 \leq s \leq n\}$.

Theorem 2. *Let G and H be a complete bipartite graphs of order l, p, m,and n, then $\Gamma\left(G \circ H\right) = 4$.*

Proof: Define a mapping, $g : V\left(G \circ H\right) \to \mathbf{N}$ as follows:
$g\left(u_i\right) = 4$; $g\left(v_j\right) = 3$; $g\left(x_{i,r}\right) = g\left(x_{j,r}\right) = 1$; $g\left(y_{i,s}\right) = g\left(y_{j,s}\right) = 2$.

Assume $\Gamma\left(G \circ H\right) > 4$. Let $1, 2, 3, 4, 5$ be the distinct colors. Suppose we assign the color 5 to anyone vertex of $u_i(1 \leq i \leq m)$. Since the neighbourhood vertices are assigned the colors $4, 3, 2, 1$ to the vertices $v_j, x_{i,r}, y_{i,s}, x_{j,r}, y_{i,s}(1 \leq i \leq l, 1 \leq j \leq p, 1 \leq r \leq m, 1 \leq s \leq n)$. Here either every two colors a and b with ab or every vertex colored b has not a neighbor colored a. it contradicts by the Grundy chromatic number. Since $\Gamma(G \vee H) \geq 4$. Therefore $\Gamma(G \vee H) = 4$. □

Now we consider G be a 2-regular graph and H be a complete bipartite graphs. Let $V(G) = \{u_i : 1 \leq i \leq l\}$ and let $V(H) = \{x_r : 1 \leq r \leq m\} \cup \{y_s : 1 \leq s \leq n\}$. Let $V(G \circ H) = V(G) \cup \bigcup_{i=1}^{l} \{x_{i,r} : 1 \leq r \leq m\} \cup \bigcup_{i=1}^{l} \{y_{i,s} : 1 \leq s \leq n\}$.

Theorem 3. *Let G be a 2-regular graph of order $l > 4$ and H be a complete bipartite graph of order m and n, then $\Gamma(G \circ H) = 5$.*

Proof: Define a mapping, $g : V(G \circ H) \to \mathbf{N}$ as follows:

Case (i): For $l \equiv 0, 23$.
For $1 \leq i \leq l$,

$$g(u_i) = \begin{cases} 3, & \text{if } i \equiv 13 \\ 4, & \text{if } i \equiv 23 \\ 5, & \text{if } i \equiv 03 \end{cases}$$

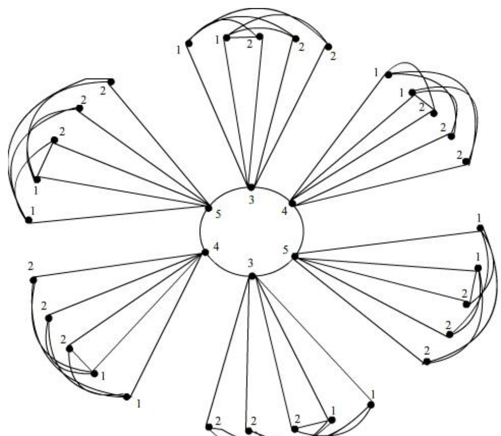

Figure 2: Grundy number of $C_6 \circ K_{2,3}$ is 5.

$g(x_{i,r}) = 1, 1 \leq r \leq m; \quad g(y_{i,s}) = 2, 1 \leq s \leq n.$

Case (ii): For $l \equiv 13$
Using the color pattern as followed from Case(i) of order $l-2$. Also $g(u_{l-1}) = 3$; $g(u_l) = 4$.

Assume $\Gamma(G \circ H) \leq 8$. Consider a Greedy algorithm that assign the colors 1 and 2 to the vertices $\{x_{i,r}, y_{i,s}, \forall\, i = 1$ to $l,\ r = 1$ to m and $s = 1$ to $n\}$ in order to satisfy the Grundy number. Next we assigns a color to the vertices u_i, $(1 \leq i \leq m)$ in such a way that they receive the color 3 or 4 or 5 which is not assigned to the vertices in $N(u_i) = \{u_{i+1}, u_{i-1}, x_{i,r}, y_{i,s}, \forall\, i = 1$ to $l,\ r = 1$ to m and $s = 1$ to $n\}$. Then any vertex of $u_i, (1 \leq i \leq m)$ is given the color 6, the in-order to satisfy the Grundy number, the vertex which is given the color 6 must be adjacent to all neighbourhood vertices which is given the colors 1, 2, 3, 4 and 5. Which is contradiction. Hence $\Gamma(G \circ H) \leq 5$. □

Now we consider G be a 2-regular graph and H be a ladder graph. Let $V(G) = \{u_i : 1 \leq i \leq m\}$ and let $V(H) = \{v_j, v'_j : 1 \leq j \leq n\}$. Let $V(G \circ H) = V(G) \cup \bigcup_{i=1}^{m} \{v_{i,j}, v'_{i,j} : 1 \leq j \leq n\}$.

Theorem 4. *Let G be a 2-regular graph of order $m > 4$ and H be a ladder graph of order $n > 4$, then $\Gamma(G \circ H) = 7$.*

Proof: Define a mapping, $g : V(G \circ H) \to \mathbf{N}$ as follows:

Case (i): For $m \equiv 0, 23$.

For $1 \leq i \leq m$,

$$g(u_i) = \begin{cases} 5, & \text{if } i \equiv 13 \\ 6, & \text{if } i \equiv 23 \\ 7, & \text{if } i \equiv 03 \end{cases} \tag{3.3}$$

Subcase (i): For $n \equiv 1, 34$.

For $1 \leq i \leq m$ and $1 \leq j \leq n$,

$$g(v_{i,j}) = \begin{cases} 1, & \text{if } j \equiv 14 \\ 2, & \text{if } j \equiv 24 \\ 3, & \text{if } j \equiv 34 \\ 4, & \text{if } j \equiv 04 \end{cases} \tag{3.4}$$

For $1 \leq i \leq m$ and $3 \leq j \leq n$,

$$g\left(v'_{i,j}\right) = \begin{cases} 1, & \text{if } j \equiv 34 \\ 2, & \text{if } j \equiv 04 \\ 3, & \text{if } j \equiv 14 \\ 4, & \text{if } j \equiv 24 \end{cases} \tag{3.5}$$

$g\left(v'_{i,1}\right) = 2;\ g\left(v'_{i,2}\right) = 3;$

Subcase (ii): For $n \equiv 24$.

Using the color pattern as followed from the subcase (i) except the vertices $v'_{i,n}$, $(1 \leq i \leq m)$.

So $g\left(v'_{i,n}\right) = 1$.

Subcase (iii): For $n \equiv 04$.

Using the color pattern as followed from the subcase (i) except the vertices $v_{i,n}$, $(1 \leq i \leq m)$.

So $g\left(v_{i,n}\right) = 1$.

Case (ii): For $m \equiv 13$

Using the color pattern as followed from Case(i) of order $m-2$. Also $g\left(u_{m-1}\right) = 5$; $g\left(u_m\right) = 6$.

Assume $\Gamma\left(G \circ H\right) \leq 8$. Consider a Greedy algorithm that assign the colors 1, 2, 3 and 4 to the vertices $\{v_{i,j}, v'_{i,j} \forall\ i = 1 \text{ to } m \text{ and } \ j = 1 \text{ to } n\}$ in order to satisfy the Grundy number. Next we assigns a color to the vertices u_i, $(1 \leq i \leq m)$ in such a way that they receive the color 5 or 6 or 7 which is not assigned to the vertices in $N(u_i) = \{v_{i,j}, v'_{i,j}, u_{i+1}, u_{i-1} \forall\ i = 1 \text{ to } m \text{ and } \ j = 1 \text{ to } n\}$. Then any vertex of $u_i, (1 \leq i \leq m)$ is given the color 8, the in-order to satisfy the Grundy number, the vertex which is given the color 8 must be adjacent to all neighbourhood vertices which is given the colors 1, 2, 3, 4, 5, 6 and 7. Which is contradiction. Hence $\Gamma\left(G \circ H\right) \leq 7$. □

Now we consider G and H be a fan graph. Let $V(G) = \{u_i : 1 \leq i \leq m-1\} \cup \{u_m\}$, wher u_m be a centre vertex of $u_i, (1 \leq i \leq m-1)$ and let $V(H) = \{v_j : 1 \leq j \leq n-1\} \cup \{v_n\}$, wher v_n be a centre vertex of $v_j, (1 \leq j \leq n-1)$. Let $V\left(G \circ H\right) = V(G) \cup \bigcup_{i=1}^{m} \{v_{i,j} : 1 \leq j \leq n\}$.

Theorem 5. *Let G and H be a fan graph of order $m > 4$ and $n > 4$, then* $\Gamma\left(G \circ H\right) = 8$.

Proof: Define a mapping, $g : V\left(G \circ H\right) \to \mathbf{N}$ as follows:

Case (i): For $m-1 \equiv 1, 23$.

For $1 \leq i \leq m-1$,

$$g\left(u_i\right) = \begin{cases} 6, & \text{if } \ i \equiv 13 \\ 7, & \text{if } \ i \equiv 23 \\ 8, & \text{if } \ i \equiv 03 \end{cases} \tag{3.6}$$

$g\left(u_m\right) = 5$.

Subcase (i): For $n-1 \equiv 1, 23$.

For $1 \leq i \leq m$ and $1 \leq j \leq n-1$,

$$g\left(v_{i,j}\right)=\begin{cases} 1, & \text{if } j \equiv 13 \\ 2, & \text{if } j \equiv 23 \\ 3, & \text{if } j \equiv 03 \end{cases} \tag{3.7}$$

$g\left(v_{i,n}\right)=4$.

Subcase (ii): For $n-1 \equiv 03$.

Using the color pattern as followed from the subcase (i) except the vertices $v_{i,n-1}$, $(1 \leq i \leq m)$.

So $g\left(v_{i,n-1}\right)=1$.

Case (ii): For $m-1 \equiv 03$

Using the color pattern as followed from Case(i) except the vertex u_{m-1}. So $g\left(u_{m-1}\right)=6$.

Assume $\Gamma(G \circ H) \leq 9$. Consider a Greedy algorithm that assign the colors 1, 2, 3 and 4 to the vertices $\{v_{i,j} \forall\ i=1 \text{ to } m \text{ and } \ j=1 \text{ to } n\}$ in order to satisfy the Grundy number. Next we assigns a color to the vertices u_i, $(1 \leq i \leq m)$ in such a way that they receive the color 5 or 6 or 7 or 8 which is not assigned to the vertices in $N(u_i) \forall\, i=1$ to m and $\ j=1$ to $n\}$. Then any vertex of $u_i, (1 \leq i \leq m)$ is given the color 9, the in-order to satisfy the Grundy number, the vertex which is given the color 9 must be adjacent to all neighbourhood vertices which is given the colors 1, 2, 3, 4, 5, 6, 7 and 8. Which is contradiction. Hence $\Gamma(G \circ H) \leq 8$. □

Now we consider G be a 2-regular graph and H be a fan graph. Let $V(G)=\{u_i : 1 \leq i \leq m\}$ and let $V(H)=\{v_j : 1 \leq j \leq n-1\} \cup \{v_n\}$, wher v_n be a centre vertex of $v_j, (1 \leq j \leq n-1)$. Let $V(G \circ H)=V(G) \cup \bigcup_{i=1}^{m}\{v_{i,j} : 1 \leq j \leq n\}$.

Theorem 6. *Let G be a 2-regular graph of order $m>4$ and H be a fan graph of order $n>4$, then $\Gamma(G \circ H)=7$.*

Proof: Define a mapping, $g : V(G \circ H) \rightarrow \mathbf{N}$ as follows:

Case (i): For $m \equiv 0, 23$.

For $1 \leq i \leq m$,

$$g\left(u_i\right)=\begin{cases} 5, & \text{if } i \equiv 13 \\ 6, & \text{if } i \equiv 23 \\ 7, & \text{if } i \equiv 03 \end{cases} \tag{3.8}$$

Subcase (i): For $n-1 \equiv 1,2 3$.

For $1 \leq i \leq m$ and $1 \leq j \leq n-1$,

$$g(v_{i,j}) = \begin{cases} 1, & \text{if } j \equiv 1 3 \\ 2, & \text{if } j \equiv 2 3 \\ 3, & \text{if } j \equiv 0 3 \end{cases} \tag{3.9}$$

$g(v_{i,n}) = 4$.

Subcase (ii): For $n-1 \equiv 0 3$.

Using the color pattern as followed from the subcase (i) except the vertices $v_{i,n-1}$, $(1 \leq i \leq m)$.

So $g(v_{i,n-1}) = 1$.

Case (ii): For $m \equiv 1 3$

Using the color pattern as followed from Case(i) of order $m-2$. Also $g(u_{m-1}) = 5$; $g(u_m) = 6$.

Assume $\Gamma(G \circ H) \leq 8$. Consider a Greedy algorithm that assign the colors 1, 2, 3 and 4 to the vertices $\{v_{i,j} \forall\ i = 1$ to m and $j = 1$ to $n\}$ in order to satisfy the Grundy number. Next we assigns a color to the vertices u_i, $(1 \leq i \leq m)$ in such a way that they receive the color 5 or 6 or 7 which is not assigned to the vertices in $N(u_i) = \{v_{i,j}, u_{i+1}, u_{i-1} \forall\ i = 1$ to m and $j = 1$ to $n\}$. Then any vertex of $u_i, (1 \leq i \leq m)$ is given the color 8, the in-order to satisfy the Grundy number, the vertex which is given the color 8 must be adjacent to all neighbourhood vertices which is given the colors 1, 2, 3, 4, 5, 6 and 7. Which is contradiction. Hence $\Gamma(G \circ H) \leq 7$. □

Conclusion

In this paper, we have demonstrated the new results on Grundy number of corona product of 2-regular graph with cycle, complete bipartite, ladder graph and fan graph. Also we found the results on Grundy number of corona product of complete bipartite with complete bipartite and fan graph with fan graph. Furthermore, we extend our work to generalized Grundy number of some other product of graphs.

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