Power domination in splitting and degree splitting graph

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Abstract

A vertex set S is called a power dominating set of a graph G if every vertex within the system is monitored by the set S following a collection of rules for power grid monitoring. The power domination number of G is the order of a minimal power dominating set of G . In this paper, we solve the power domination number for splitting and degree splitting graph.

Keywords: *Power dominating set; Electrical power network; splitting graph; degree splitting graph.*

1. Introduction

A graph $G = (V, E)$ is an ordered pair of a finite nonempty set of vertices $V = V(G)$ and a set of edges $E = E(G)$ consists of unordered pairs of different vertices. The order of G is denoted by $|G| = |V(G)|$. We are saying that the vertices u and v are adjacent, if $uv \in E$. For $v \in V(G)$. The open neighbourhood of v , denoted by $N(v)$, is the set of vertices adjacent to v ; the closed neighbourhood of v , denoted by $N[v]$, is $N_G(v) \cup \{v\}$. Let $S \subseteq V(G)$. Then open neighbourhood of S is defined as $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighbourhood of S is defined as $N[S] = N(S) \cup S$ [3].

For a graph $G(V, E)$, $S \subseteq V$ is a dominating set of G if every vertex in $V \setminus S$ has at least one neighbour in S . The domination number of G , denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G . [3].

For a graph G , let $S \subseteq V(G)$. We define the sets $M^j(S)$ of vertices monitored by S at level j , $j \geq 0$, inductively as follows:

1. $M^0(S) = N[S]$.
2. $M^{j+1}(S) = M^j(S) \cup \{w : \exists v \in M^j(S), N(v) \cap (V(G) \setminus M^j(S)) = w\}$.

If $M^\infty(S) = V(G)$, then the set S is said to be a power dominating set of G . The minimum number of a power dominating set in G is called the power domination number of G and is denoted by $\gamma_p(G)$ [3].

The power domination problem is *NP*-complete [1]. The problem has been exhibited to be *NP*-complete even when restricted to bipartite graphs and chordal graphs [1]. The bounds on the power domination number for any graph G , $1 \leq \gamma_p(G) \leq \gamma(G)$, were realized in [1] and also power domination problem is well investigated for trees [1]. In 2005 Liao et al. have obtained that the power domination number for split graphs, a subclass of chordal graphs, an *NP*-complete. The power domination problem was solved for block graphs [4], product graphs [5], cylinder, torus and generalized Petersen graphs [6], certain chemical structures [7], honeycomb network [3], hexagonal grid [8] and so on.

- The *central graph* of G , written as $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in $C(G)$ [9].
- Let $v \in G$ and let v' be a new vertex. Join v' to all vertices of G adjacent to v . The graph $S(G)$ thus obtained is named the *splitting*

of a graph G . Let G be a graph with $V = D_1 \cup D_2 \dots \cup D_r \cup L$ where each D_i may be a set of vertices having a minimum of two vertices and having the identical degree and $L = V - \cup D_i$, $1 \leq i \leq r$. The *degree splitting graph* of G , denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, \dots, w_r$ and joining w_i to every vertex of D_i [11].

- Let T be the tree constructed from a $K_{1,n}$ by subdividing any number of its edges any number of times; that is, T has at the most one vertex of degree 3 or more. We call such a tree T a *spider*, denoted by $S(n)$ [1]. A tree is a connected graph that contains no cycles. The most common type of tree is the binary tree. It is so called because each node can have at most two descendants. A k -ary tree is said to be a complete k -ary tree if each internal node has exactly k - descendants [1].
- The *friendship graph* F_m may be formulated by joining m copies of the cycle graph C_m with a typical vertex. A *wheel graph* W_m of order m , called an m -wheel is, a graph that contains a cycle of order $m - 1$, and for which every graph vertex within the cycle is connected to at least one other graph vertex (which is understood because hub). The edges of a wheel which include the hub are called spokes. The *helm graph* H_m is the graph obtained from an m -wheel graph by adjoining a pendant edge at every vertex of the cycle [10].
- The *spiked cycle* with $2n$ vertices, denoted as SC_n , is that the graph with vertex set $\{v_0, \dots, v_{n-1}\} \cup \{u_0, \dots, u_{n-1}\}$ and edges $v_i v_{i+1}$ and $v_i u_i$, for all $i = 0, \dots, n - 1$ with the subscript addition modulo n . It is observed that the vertices $\{v_0, \dots, v_{n-1}\}$ are cyclically connected by the edges $v_i v_{i+1}$ while the edges $v_i u_i$ add a spoke on every vertex of the cycle [6].
- The Cartesian product GH of two graphs G and H is the graph with vertex set $V(G) \times V(H)$, two vertices (r, t) and (s, p) are adjacent if and only if either $r = s$ and $tp \in E(H)$ or $t = p$ and $rs \in E(G)$. The n -ladder graph L of length n is defined as $P_2 P_{n+1}$, where P_{n+1} may be a path on $n + 1$ vertices, $n \geq 1$. The *circular ladder* CL_n of length $n \geq 3$ is that the Cartesian product $CL_n = C_n K_2$. Möbius Ladder graphs are formulated by introducing a twist in a circular ladder and is denoted by M_n [14].

- The graph $P_m P_n$ is termed $s \times r$ grid graph, and is denoted by $M(m, n)$, $m, n \geq 2$. The cylinder $C_m P_n$, where $m, n \geq 3$ is a $P_m P_n$ grid with wraparound edge in each row. It is observed that the vertex set of $P_m \times P_n$ is $V = \{u_1 u_2 : 0 \leq u_i \leq d_{i-1}, i = 1, 2\}$ and two vertices $u = u_1 u_2$ and $v = v_1 v_2$ are linked by an edge, if $|u_1 - v_1| + |u_2 - v_2| = 1$ [13].

2. Main Results

In this section, we solve the power domination number for central graph, splitting graph and degree splitting graph for extended cycle of ladder, circular ladder and corona graph.

Observations I:

The following results hold good:

- For any graph G of order $r \geq 2$, $\gamma_p(C(G)) = 1$.
- For any graph G , $1 \leq \gamma_p(S(G)) \leq \gamma(G)$.
- Let G be a graph. Then $\gamma_p(S(G)) = 1$, where $G \in \{P_n, K_n, F_m, S(n), W_m\}$. See Figure 1(a).

Let G be a cycle $C_n, n \geq 3$. Then $\gamma_p(S(G)) = 2$.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices of C_n which are replicated by the vertices $v'_1, v'_2, \dots, v'_n$ and let $e_1, e_2, \dots, e_n$ be the edges of C_n . Then the graph $S(C_n)$ will have $2n$ vertices and $3n$ edges.

Let S be a power dominating set of $S(C_n)$. We claim that $|S| = 2$.

($\geq$): Suppose $|S| = 1$. Without loss of generality, let $S = \{v_1\}$ be the power dominating set of $S(C_n)$. Then $M^0(S)$ monitors the vertices say, $\{v_1, v_2, v_n, v'_2, v'_n\}$. See Figure 1(c). Now the vertices in $M^0(S)$ say, v'_2 and v'_n are adjacent to exactly one unmonitored vertices say, v_3 and v_{n-1} respectively. Then these vertices are monitored in $M^1(S)$. Now for every vertex $u \in M^1(S)$, $|N[u] \setminus M^1(S)| \geq 2$, a contradiction. See Figure 1(c).

($\leq$): Select the vertices $\{v_1, v_5\}$ in S . Then $M^0(S) = \{v_1, v_n, v_2, v'_2, v'_n, v_5, v_4, v_6, v'_4, v'_6\}$. In the next step vertices labelled as v'_2, v'_6 and v'_n in $M^0(S)$ is adjacent to exactly one unmonitored vertices say, v_3, v_7 and v_{n-1} respectively, these vertices can monitor in $M^1(S)$. Proceeding inductively, for every vertex $u \in M^i(S)$, $|N[u] \setminus M^i(S)| = 1$, at every inductive step $i \geq 2$. Thus $M^{n-1}(S) = V(G)$. Hence $|S| = 2$. Therefore, $\gamma_p(G) = 2$. $\square$

Figure 2.1: Red colored squared vertices indicates a minimum power dominating set of (a) Path $S(P_7)$ (b) Ladder $S(L_7)$ (c) Cycle $S(C_7)$ (d) Spiked Cycle $S(SC_7)$.

2.1. Graphs with $\gamma_p(S(G)) = \gamma_p(G)$

In this section, we realize graphs for which $\gamma_p(S(G)) = \gamma_p(G)$, where G is isomorphic to complete bipartite graph, spiked cycle and circular ladder .

Let G be a spiked cycle $S(SC_n)$, $n \geq 3$. Then $\gamma_p(G) = \lceil \frac{n}{3} \rceil$.

Proof. Label the cyclically connected vertices of SC_n as $v_0, v_2, \dots, v_n$ which are replicated by the vertices $v'_0, v'_2, \dots, v'_n$ and let $u_0, u_2, \dots, u_n$ be the spoke vertices of SC_n which are replicated by the vertices $u'_0, u'_2, \dots, u'_n$. Then the graph $S(SC_n)$ will have $4n$ vertices.

($\leq$): Let S be a power dominating set of $S(SC_n)$. Let $S = \{v_i : i \equiv 0 \pmod 3\}$ where $1 \leq i \leq n$ with $|S| = \lceil \frac{n}{3} \rceil$. Then $M^0(S) = \{v_i, u_i, u'_i, v_{i+1}, v_{i-1}, v'_{i+1}, v'_{i-1} : i \equiv 0 \pmod 3\}$. Now the vertices labelled as $\{v'_{i+1}, v'_{i-1}\}$ in $M^0(S)$ are adjacent to exactly one unmonitored vertices say, $\{u_{i+1}, u_{i-1}\}$. These vertices monitored in $M^1(S)$. In the next step, vertices labelled as $\{v_{i+1}, v_{i-1}\}$ are adjacent to exactly one unmonitored vertices say, $\{u'_{i+1}, u'_{i-1}\}$ and these vertices monitored in $M^2(S)$. Hence $M^2(S) = V(S(SC_n))$. This implies that, $\gamma_p(S(SC_n)) \leq \lceil \frac{n}{3} \rceil$.

($\geq$): As $\Delta(S(SC_n)) = 6$, we notice that $\deg(v) = 6$ for every $v \in S$. In particular, every vertex in S lies on the cycle $(v_1, v_2, \dots, v_n)$ of $S(SC_n)$. Suppose that $\gamma_p(S(SC_n)) < \lceil \frac{n}{3} \rceil$. Then there is a vertex say, $v_i \in S(SC_n)$ such that $v_i \notin S$ and $d(v_i, N(s)) = 1$. So $v_i \notin N[S]$. This vertex can be monitor only if at least one edge incident to it is monitored. The edge $u_i v_i$ and $u'_i v_i$ cannot be monitor unless if $v_{i-1} v_i$, $v_{i+1} v_i$, $v_{i-1} v'_i$ and $v_{i+1} v'_i$ are monitored. So $v_{i-1} v_i$ is also not monitored. See Figure 1(d). Hence S is not a power dominating set, a contradiction. Therefore $\gamma_p(G) \geq \lceil \frac{n}{3} \rceil$. Hence the proof. $\square$

Let G be a ladder L_n or circular ladder CL_n , $n \geq 4$, we have $\gamma_p(S(G)) \geq 2$.

Proof. Let S be a power dominating set of $S(CL_r)$. We claim that $|S| \geq 2$. Suppose not, consider $|S| = 1$. Without loss of generality, let $S = \{w\}$. Then for every vertex in $u \in N(w)$ is adjacent to two neighbouring vertices, a contradiction. See Figure 1(b). $\square$

Let G be a circular ladder CL_n , $n \geq 4$ or a Möbius ladder M_n , $n \geq 4$. Then $\gamma_p(S(G)) = 2$.

Proof. Let $v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ be the vertices of $CL(n)$ which are replicated by the vertices $v'_1, v'_2, \dots, v'_n$ and $u'_1, u'_2, \dots, u'_n$. Then the graph $S(CL(n))$ will have $4n$ vertices. Select the vertices $\{v_1, v_4\}$ in S as shown in Figure 2(a). Then $M^0(S) = \{v_1, v_n, v_2, v'_2, v'_n, u'_1, u_1, v_4, v_3, v_5, v'_3, v'_5, u'_4, u_4\}$. In the next step vertices say, v'_2 and v'_3 in $M^0(S)$ are adjacent to exactly one unmonitored vertices say, u_3 and u_2 respectively, these vertices can be monitored in $M^1(S)$. Then vertices labelled as v_3 and v_2 in $M^0(S)$ monitor u'_3 and u'_2 in $M^2(S)$. Proceeding inductively, for every vertex $u \in M^i(S)$, $|N[u] \setminus M^i(S)| \leq 1$, $i \geq 1$. Thus $M^{n-1}(S) = V(G)$. Hence $|S| = 2$. Therefore, $\gamma_p(G) = 2$. $\square$

Observation 1. Let G be a Hexagonal chain H_n , $n \geq 3$.

Then $\gamma_p(S(G)) = 2$.

Observation 2. For the complete bipartite graph $K_{m,n}$, $m, n \geq 3$, we have $\gamma_p(S(K_{m,n})) = 2$.

2.2. Extended Cycle-of-Ladder

Jywe-Fei Fang proposed a network called cycle-of-ladder in 2008. It is a spanning subgraph of the hypercube network, thereby showing that hypercube network is bipancyclic [14]. The graph obtained appears like a ladder having two rails and $n + 1$ rungs between them. The length of the ladder is defined as n .

[14] A *cycle-of-ladder* is a graph, composed of a cycle C_s of length $2l$ termed the spine cycle such that removal of alternate edges on C_s leaves l distinct components $L_1, L_2, \dots, L_l$, each of which is isomorphic to a ladder. If $r_1, r_2, \dots, r_l$ represents the number of rungs in the ladders $L_1, L_2, \dots, L_l$ respectively, then cycle of ladders is written as $CL(2l, r_1, r_2, \dots, r_k)$. Let R_j^i , $1 \leq j \leq r_i$ represent the rungs of L_i such that the bottom rung R_1^i is the edge of C_s in L_i , $1 \leq i \leq k$. For brevity, we represent $(r_1, r_2, \dots, r_k)$ as s and we represent the cycle-of-ladder as $CL(2l, s)$, where l and s denote the number of ladders and the length of each ladder respectively. For convenience, we label the vertices of L_i as $l_{j,1}^i$ and $l_{j,2}^i$ where $0 \leq j \leq s$ and $1 \leq i \leq l$ in $CL(2l, s)$.

We add l number of edges to $CL(2l, s)$ to obtain a 3-regular graph and call it the extended cycle-of-ladder $ECL(2l, s)$.

[14] The extended cycle-of-ladder $ECL(2l, s)$ is obtained from $CL(2l, s)$ by adding edges between $(l_{s,2}^i, l_{s,1}^{i+1})$, $1 \leq i \leq l - 1$ and $l_{s,2}^l, l_{s,1}^1$.

Figure 2.2: Red colored squared vertices indicates a power dominating set of (a) $CL(8, 5)$ and (b) $CL(8, 5)$.

Let G be an extended cycle-of-ladder $ECL(2l, s)$, $l, s \geq 4$. Then $\gamma_p(S(G)) \geq l$.

Proof. $CL(2l, s)$ contains l vertex disjoint copies of $L(i)$, say $L(1), L(2), \dots, L(l)$. Let S be a power dominating set of $ECL(2l, s)$. We claim that $|S| \geq l$. Suppose $|S| = l - 1$. Without loss of generality, consider $L(1)$. The worst case arises when all bone cycle vertices and their duplicated vertices $l_{0,1}^1, l_{0,2}^1, l_{0,1}^{1'}, l_{0,2}^{1'}$ of $L(1)$ are monitored by vertices of S not in $L(1)$. Then the vertices $l_{0,1}^1, l_{0,2}^1$ are adjacent to two unmonitored vertices each say, $l_{1,1}^{1'}, l_{0,2}^{1'}$ and $l_{1,2}^{1'}, l_{0,1}^{2'}$ respectively and $l_{0,1}^{1'}, l_{0,2}^{1'}$ are adjacent to exactly one unmonitored vertices say, $l_{1,1}^1, l_{1,2}^1$ each of which in turn is adjacent to at least two unmonitored vertices, a contradiction. Therefore, $|S| \geq l$. See Figure 2(a). $\square$

Algorithm for Power Domination in Splitting graph of Cycle-of-Ladder $S(CL(2l, s))$

Input: Splitting graph of Cycle-of-ladder $S(CL(2l, s))$, $l, s \geq 4$.

Algorithm: Label the vertices of splitting graph of cycle-of-ladder as $\{l_{j,1}^i \cup l_{j,2}^i \cup (l_{j,1}^i)' \cup (l_{j,2}^i)'\} : 0 \leq j \leq s, 1 \leq i \leq l\}$ and select the vertices $\{l_{1,1}^i, l_{0,1}^j : i = 1, 3, 5, \dots, l-1, j = 2, 4, 6, \dots, l\}$ in S . See Figure 2(b).

Output: $\gamma_p(S(CL(2l, s))) = l$.

Proof of Correctness: Let S be a power dominating set of $S(ECL(2l, s))$. Now vertices in S monitor all the vertices of bone cycle and the replicated vertices of bone cycle say, $l_{0,1}^1, l_{0,2}^1, l_{0,1}^{1'}, l_{0,2}^{1'}, \dots, l_{0,1}^l, l_{0,2}^l, l_{0,1}^{l'}, l_{0,2}^{l'}$. Then the vertices labelled as $l_{0,1}^{1'}, l_{0,2}^{1'}, \dots, l_{0,1}^{l'}, l_{0,2}^{l'}$ are adjacent to exactly one unmonitored vertices say, $l_{1,1}^1, l_{1,2}^1, \dots, l_{1,1}^l, l_{1,2}^l$ these vertices monitored in $M^1(S)$. In the next step, the vertices labelled as $l_{0,1}^1, l_{0,2}^1, \dots, l_{0,1}^l, l_{0,2}^l$ are adjacent to exactly one unmonitored vertices say, $l_{1,1}^{1'}, l_{1,2}^{1'}, \dots, l_{1,1}^{l'}, l_{1,2}^{l'}$ these vertices monitored in $M^2(S)$. Now the vertices labelled as $l_{1,1}^{1'}, l_{1,2}^{1'}, \dots, l_{1,1}^{l'}, l_{1,2}^{l'}$ are adjacent to exactly one unmonitored vertices say, $l_{2,1}^1, l_{2,2}^1, \dots, l_{2,1}^l, l_{2,2}^l$ these vertices monitored in $M^3(S)$. Proceeding inductively, for every vertex $u \in M^i(S)$, $|N[u] \setminus M^i(S)| \leq 1$, $i \geq 4$. Hence S is a power dominating set of $S(CL(2l, s))$ with $|S| = l$. Therefore, $\gamma_p(S(CL(2l, s))) = |S| = l$, which we present in the following:

Let G be the graph isomorphic to cycle-of-ladder as $CL(2l, s)$, $l, s \geq 4$ or an extended cycle-of-ladder $ECL(2l, s)$, $l, s \geq 4$. Then $\gamma_p(G) = l$

Let T be a complete k -array tree of level r , $r \geq 2$, $k \geq 2$. Then $\gamma_p(S(T)) = k^{r-1}$.

Proof. Name the vertices of a complete k -array tree of level r , $r \geq 2$ as $v_1, v_2, \dots, v_n$ which are replicated by the vertices $v'_1, v'_2, \dots, v'_n$. Then the splitting graph of k -array tree of level r , $r \geq 2$ will have $k^{r+2} - 2$ vertices. In T , the vertices in level $r-1$ as $2k$ children adjacent to it. Any minimum power dominating set for a complete k -array tree contains

vertices of level $r - 1$. For, if not, even if all vertices in level $(r - 1)$ are monitored, their children will be left unmonitored. Select k^{r-1} vertices in level $r - 1$ of splitting complete k -array tree $S(T)$ in set S . The vertices in level $r - 1$ monitor k^{r+1} vertices in level r and k^{r-2} vertices in level $r - 2$. Now k^r vertices in level r monitor k^{r-1} replicated vertices in level $r - 1$. Proceeding inductively, then the vertices in level $r - t$ monitor vertices in level $r - t - 1$, $2 \leq t \leq r - 1$. Hence S is a power dominating set of $S(T)$ with $|S| = k^{r-1}$. Therefore, $\gamma_p(S(T)) = |S| = k^{r-1}$. $\square$

For the Helm graph H_m , $m \geq 3$, we have $\gamma_p(S(H_m)) = m$.

Proof. Label the vertices of helm graph H_m as $v_1, v_2, \dots, v_m$ which are replicated by the vertices $v'_1, v'_2, \dots, v'_m$. Then the graph $S(G)$ will have $4m + 2$ vertices. Let S be a power dominating set of $S(G)$. Suppose not, by definition there are m vertices of H_m in spoke cycle, each of which is adjacent to two pendent vertices in it. Deleting at least one vertex from S we see that at least one vertex of $S(H_m)$ is not monitored by S , a contradiction. Thus $|S| \geq m$. Now $S = \{v_1, v_2, \dots, v_m\} = V(S(H_m))$. Hence $M^0(S) = V(H_m)$ is monitored by S . Therefore, $\gamma_p(S(H_m)) = m$. Hence the proof.

2.3. Graphs with $\gamma_p(S(G)) = \gamma_p(G) = \gamma(G)$

[1] Let G and H be two graphs of order r and s respectively. The *corona* of G and H , denoted by $G \circ H$ is a graph composed from one copy of G and r copies of H where k^{th} vertex of G is adjacent to every vertex in the k^{th} copy of H , $1 \leq k \leq |V(G)|$.

Rajasingh et.al. [12] have proved that domination number and power domination number are equal for corona graphs $(G \circ H)$ and we solve the power domination number for splitting graph of corona graph. Let G be a graph of order $r \geq 2$ and H be a graph of order $s \geq 2$. Then $\gamma_p(S(G \circ H)) = r$.

Proof. Label the vertices of G as $v_1, v_2, \dots, v_r$ and let $u_1, u_2, \dots, u_s$ be the vertices of H which are replicated by the vertices $v'_1, v'_2, \dots, v'_r$ and $u'_1, u'_2, \dots, u'_s$. Then the graph $S(G \circ H)$ will have $2(r + rs)$ vertices. Let S be a power dominating set of $S(G \circ H)$. Suppose not, by definition, there are r vertex disjoint copies of H_k , $1 \leq k \leq r$ adjacent to each vertex of G in $S(G \circ H)$. With the deletion of a vertex say, k^{th} vertex from S we get that k^{th} copy of H and replicated vertices of k^{th} copy of H not monitored by S ,

a contradiction. Thus $|S| \geq r$. Now let $S = \{v_1, v_2, \dots, v_r\} = V(S(G \circ H))$. Hence $M^0(S) = V(G)$ is monitored by S . Therefore, $\gamma_p(S(G \circ H)) = r$. Hence the proof. $\square$

3. Degree Splitting Graph

In this section, we compute the power domination number for degree splitting graph.

Basavanagoudi et al. [11] have given a lower bound and upper bound for the domination number of degree splitting graph for any graph G and is quoted below.

[11] For any graph G , $1 \leq \gamma(DS(G)) \leq |w_i \cup L|$, for every w_i , $1 \leq i \leq r$.

Observations:

The following results hold good:

- For any graph G , $1 \leq \gamma_p(DS(G)) \leq |w_i \cup L|$, for every w_i , $1 \leq i \leq r$.
- For $n \geq 3$, $\gamma_p(DS(G)) = 1$, $G \in \{P_n, F_m, W_m, H_m, K_{1,n}, K_{m,n}, P_m P_n, P_m C_n, P_n K_m, P_n K_{1,m}, P_n W_m, C_n K_m, C_n K_{1,n}, C_n W_m, W_n W_m, W_n K_{1,m}, K_n K_m, P_m \times P_n, P_m \times C_n\}$. See Figure 3(a).
- Let G be a regular graph of order $n \geq 3$. Then $\gamma_p(DS(G)) = 1$.
- Let G be a spider tree $S(n)$. Then $\gamma_p(S(G)) = 1$.
- For any graph G , $\gamma_p(DS(G)) = \gamma_p(G)$, $G \in \{P_n, C_n, F_n, W_n, K_n, K_{1,n}, K_{m,n}, S(n)\}$, $m \leq n, m, n \geq 3$.

[8] Let T_r be the triangular grid graph. The vertices of T_r are ordered triples (i, j, k) of non-negative integers summing to r , and two vertices are connected by an edge if they admit in one co-ordinate and vary by one in the other two co-ordinates. Vertices in a r -dimensional hexagonal graph are placed at the vertices of a r -triangular tessellation, so that each vertex has up to $2k+2$ adjacent neighbours, denoted by $HX(r)$. See Figure 3(b).

Figure 3.1: Red colored squared vertices indicates a minimum power dominating set of (a) P_6P_7 (b) Hexagonal grid $HX(3)$.

Let G be a Hexagonal grid $HX(r)$, $n \geq 2$ or triangular grid $T(r)$, $n \geq 2$. Then $\gamma_p(DS(G)) = 2$.

Proof. Let S be a power dominating set of $DS(HX(r))$ with $|S| = 2$. Suppose not, let $|S| < 2$.

Case (i): Let $S = \{w : \deg(w) = 3\}$. Then for every vertex in $v \in N(w)$ is adjacent to two unmonitored vertices of degree four, a contradiction. Therefore, $|S| \neq 1$. Thus $|S| \geq 2$.

Case (ii): Let $S = \{w : \deg(w) = 4\}$. Then at least one vertex in $u \in N(w)$ is adjacent to two unmonitored vertices, a contradiction.

Case (iii): Let $S = \{w : \deg(w) = 6\}$. Then at least one vertex in $u \in N(w)$ is adjacent to two unmonitored vertices, a contradiction.

($\leq$:) Let $S = \{w_1, w_2\}$. Then S monitor all vertices of degree 4 and 6. Now every vertex in $M^0(S)$ is adjacent to exactly one unmonitored vertices of degree 3. These vertex monitor in $M^1(S)$. Now vertex w_3 monitor in $M^2(S)$. Hence $M^2(S) = V(DS(G))$ is monitored by S . Therefore, $\gamma_p(DS(G)) = 2$. Hence the proof. $\square$

Observation 3. For the complete k - array tree T of level r , $r \geq 2$, $k \geq 2$, $\gamma_p(DS(T)) = 2$.

4. Conclusion and Open Problem

In this paper, we have obtained the power domination numbers for splitting graph and degree splitting graph. Further, we realized partial solutions of graphs for which $\gamma_p(S(G)) = \gamma_p(G)$, $\gamma_p(S(G)) = \gamma_p(G) = \gamma(G)$ and $\gamma_p(DS(G)) = \gamma_p(G)$. Further, identifying classes of graphs for which $\gamma_p(S(G)) = \gamma_p(G) = \gamma(G)$ is open.

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