Corrigendum to “Orlicz-Lorentz Spaces and their Composition Operators” [Proyecciones (Antofagasta). 34, (2015) No 1, pp. 85-105.]

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Abstract

After publication of the above article in Proyecciones (Antofagasta). Volume 34, (2015) No 1, pp. 85-105, it has come to the attention of the authors that an extra condition was missing in Theorem 3.2.

1 Introduction

Recently it was called to our attention by professor Lech Maligranda that Theorem 3.2 in Orlicz-Lorentz Spaces and their Composition Operators (see [1]) fails to be true when the weight $w \equiv 1$, it is necessary to additionally suppose the Δ_2-condition.

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Now, in order that everything will be in a good shape, we rewrite Theorem 3.2 as follows

Theorem 1.1. *Let $T : X \to X$ be a non-singular measurable transformation. If C_T induced by T is bounded, then there exists $M \geq 1$ such that $\nu\left(T^{-1}(A) \leq M\nu(A)\right)$.*

In addition, if φ satisfies the Δ_2-condition for all $t > 0$ and $\nu\left(T^{-1}(A)\right) \leq M\nu(A)$ for $A \in \mathcal{A}$, then C_T is bounded. Moreover,

$$\|C_T f\| = \sup_{0<\nu(A)<\infty} \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right). \tag{1}$$

Proof. We empathize that the fist part of the proof of this Theorem goes the same as in the proof of Theorem 3.2 in [1] and it holds without the Δ_2-condition. Conversely, if $\nu(T^{-1}(A)) \leq M\nu(A)$ holds for all $A \in \mathcal{A}$, then

$$\begin{aligned}
&\int_0^{\mu\left(T^{-1}(A)\right)} w(t)\,dt \leq M \int_0^{\mu(A)} w(t)\,dt \\
\Rightarrow &\int_0^{\infty} \chi_{(0,\mu(T^{-1}(A)))}(t) w(t)\,dt \leq M \int_0^{\infty} \chi_{(0,\mu(A))}(t) w(t)\,dt \\
\Rightarrow &\int_0^{\infty} \chi^*_{T^{-1}(A)}(t) w(t)\,dt \leq M \int_0^{\infty} \chi^*_A(t) w(t)\,dt \\
\Rightarrow &\int_0^{\infty} \left(\chi_E \circ T\right)^*(t) w(t)\,dt \leq M \int_0^{\infty} \chi^*_A(t) w(t)\,dt \\
\Rightarrow &\int_0^{\infty} \left(f \circ T\right)^*(t) w(t)\,dt \leq M \int_0^{\infty} f^*(t) w(t)\,dt.
\end{aligned}$$

Therefore

$$(f \circ T)^*(t) \leq M f^*(t) \quad \text{a.e.}$$

Since φ satisfies the Δ_2-condition for all $t > 0$, it follows that

$$k = \sup_{t>0} \frac{\varphi(\alpha t)}{\varphi(t)} < \infty,$$

and thus

$$\varphi(\alpha t) \leq k\varphi(t),$$

then, with $k = 1/M$, we have

$$\begin{aligned}
\int_0^{\infty} \varphi\left(\frac{(f \circ T)^*(t)}{M\|f\|_{\varphi,w}} \right) w(t)\,dt &\leq \frac{1}{M} \int_0^{\infty} \varphi\left(\frac{f^*(t)}{\|f\|_{\varphi,w}} \right) w(t)\,dt \\
&\leq \int_0^{\infty} \varphi\left(\frac{f^*(t)}{\|f\|_{\varphi,w}} \right) w(t)\,dt \leq 1.
\end{aligned}$$

Finally

$$\|f \circ T\|_{\varphi,w} \leq M\|f\|_{\varphi,w},$$

that is

$$\|C_T f\|_{\varphi,w} \leq M\|f\|_{\varphi,w}.$$

On the one hand, let us prove (1). Indeed, let

$$N = \sup_{0<\nu(A)<\infty} \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right),$$

then

$$\nu\left(T^{-1}(A)\right) \leq N\nu(A)$$

and thus

$$\|C_T f\|_{\varphi,w} \leq N\|f\|_{\varphi,w}, \quad \forall\ f \in L_{\varphi,w}$$

hence

$$\frac{\|C_T f\|_{\varphi,w}}{\|f\|_{\varphi,w}} \leq N, \quad \text{for all } 0 \neq f \in L_{\varphi,w}.$$

Therefore

$$\begin{aligned} \|C_T\| &= \sup_{f\neq 0} \frac{\|C_T(f)\|_{\varphi,w}}{\|f\|_{\varphi,w}} \\ &< N = \sup_{0<\nu(A)<\infty} \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right). \end{aligned}$$

That is

$$\|C_T\| \leq \sup_{0<\nu(A)<\infty} \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right). \tag{2}$$

On the other hand, let us consider

$$M = \|C_T\| = \sup_{f\neq 0} \frac{\|C_T(f)\|_{\varphi,w}}{\|f\|_{\varphi,w}},$$

then

$$\frac{\|C_T(f)\|_{\varphi,w}}{\|f\|_{\varphi,w}} \leq M \quad \forall\ 0 \neq f \in L_{\varphi,w}.$$

In particular, if $f = \chi_A$ such that $0 < \mu(A) < \infty$, $A \in \mathcal{A}$, then

$$\frac{\|C_T(\chi_A)\|_{\varphi,w}}{\|\chi_A\|_{\varphi,w}} = \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right) \leq M,$$

therefore

$$\sup_{0<\nu(A)<\infty} \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right) \leq M = \|C_T\|. \tag{3}$$

Combining (2) and (3) we have

$$\|C_T\| = \sup_{0<\nu(A)<\infty} \left(\frac{\nu\left(T^{-1}(A)\right)}{\nu(A)} \right). \qquad \square$$

Remark 1.2. We like to point it out that when the weight $w(t) = 1$ we obtain the characterization for boundedness of the composition operator in Orlicz space (see Theorem 2.2. in [2]).

2 Acknowledgement

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References

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[2] Y. Cui, H. Hudzik, Romesh Kumar and L. Maligranda, Composition operators in Orlicz spaces, J. Austral. Math. Soc., Vol. **76**(2) (2004), 189-206.