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Energy of commuting graph of finite AC-groups

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Abstract

Let Γ *be a graph with the adjacency matrix* A*. The energy of* Γ *is the sum of the absolute values of the eigenvalues of* A*. In this article we compute the energies of the commuting graphs of some finite groups and discuss some consequences regarding hyperenergetic and borderenergetic graphs.*

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1. Introduction

The commuting graph of a non-Abelian group G with center $Z(G)$ is an undirected graph with vertex set $G\setminus Z(G)$, and two distinct vertices a and b are adjacent whenever $ab = ba$. We write Γ_G to denote this graph. Various properties of Γ_G have been studied in [3],[4],[15],[16],[17] and [20]. It is noted in [19] that the complement of Γ_G, known as non-commuting graph of G (see [1]), was first considered by Erdös in 1975. Let g be any element of a group G. Then the subgroup $C_G(g) = \{a \in G \mid ag = ga\}$ is called centralizer of g. If $C_G(g)$ is Abelian for every $g \in G \setminus Z(G)$ then the group G is called an AC-group. If $\mathrm{Cent}(G) = \{C_G(g) \mid g \in G\}$ and $|\mathrm{Cent}(G)| = n$ then G is called an n-centralizer group (see [5] and [2]).

Let Γ be a graph with the adjacency matrix A. Let $\lambda_1, \lambda_2, \ldots, \lambda_t$ be the distinct eigenvalues of A with corresponding mutiplicites $m_1, m_2, \ldots, m_t$. The spectrum of Γ is defined as

$$\mathrm{Spec}(\Gamma) = \left\{\lambda_1^{m_1}, \lambda_2^{m_2}, \ldots, \lambda_t^{m_t}\right\}.$$

The energy of Γ (see [12] and [14]) is given by

$$\mathcal{E}(\Gamma) = \sum_{i=1}^{t} m_i|\lambda_i|. \tag{1.1}$$

It may be mentioned here that $\mathcal{E}(K_n) = 2n - 2$, where K_n is the complete graph on n vertices. Spectral aspects of Γ_G are considered in [6], [7], [8], [9], [10] and [18] recently for various families of finite groups. In this paper we compute energy of Γ_G for several classes of finite AC-groups and discuss some consequences regarding hyperenergetic and borderenergetic graphs. Throughout the paper same notations are used to denote various families of groups as in [6] and [7].

2. Results and consequences

In this section, we compute the energy of Γ_G for some particular families of finite non-Abelian groups and derive some consequences. Using (1.1) and the spectrum related results available in [6] and [7] we get the following results.

Theorem 1. *We have*

(i) $\mathcal{E}(\Gamma_{M_{2mn}}) = \begin{cases} 4mn - 2m - 2n - 2, & \textit{if } m \textit{ is odd} \\ 4mn - 4n - m - 2, & \textit{if } m \textit{ is even.} \end{cases}$

(ii) $\mathcal{E}(\Gamma_{D_{2m}}) = \begin{cases} 2m - 4, & \textit{if } m \textit{ is odd} \\ 3m - 6, & \textit{if } m \textit{ is even.} \end{cases}$

(iii) $\mathcal{E}(\Gamma_{Q_{4m}}) = 6m - 3$.

(iv) $\mathcal{E}(\Gamma_{U_{6n}}) = 10n - 8$.

Theorem 2. *Let G be a finite group. Then we have the following.*

(i) *If $G = QD_{2^n}$ then $\mathcal{E}(\Gamma_G) = 2^n + 2^{n-1} - 6$.*

(ii) *If $G = PSL(2, 2^k)$ then $\mathcal{E}(\Gamma_G) = 2^{3k+1} - 2^{2k+1} - 2.2^{k+1} - 4$.*

(iii) *If $G = GL(2, q)$ then $\mathcal{E}(\Gamma_G) = 2q^4 - 2q^3 - 4q^2 - 2q$.*

(iv) *If $\frac{G}{Z(G)} \cong Sz(2)$ then $\mathcal{E}(\Gamma_G) = 38|Z(G)| - 12$.*

(v) *If $G = A(n, \vartheta)$ then $\mathcal{E}(\Gamma_G) = 2(2^n - 1)^2$.*

(vi) *If $G = A(n, p)$ then*

$$\mathcal{E}(\Gamma_G) = (p^{3n} - 2p^n - 1) + (p^n + 1)(p^{2n} - p^n - 1) = 2p^{3n} - 4p^n - 2.$$

(vii) *If G is non-Abelian and $|G|$ is product of two primes p, q such that $p \mid (q - 1)$ then $\mathcal{E}(\Gamma_G) = 2q(p - 1) - 3$.*

In the following theorem we compute energy of commuting graphs of the groups G such that $\frac{G}{Z(G)}$ is isomorphic to $\mathbf{Z}_p \times \mathbf{Z}_p$ and D_{2m}.

Theorem 3. *Let G be a finite group.*

(i) *If p is a prime and $\frac{G}{Z(G)} \cong \mathbf{Z}_p \times \mathbf{Z}_p$, then $\mathcal{E}(\Gamma_G) = 2((p^2 - 1)|Z(G)| - p - 1)$.*

(ii) *If $m\,(\geq 2)$ is a natural number and $\frac{G}{Z(G)} \cong D_{2m}$, then $\mathcal{E}(\Gamma_G) = 2((2m - 1)|Z(G)| - m - 1)$.*

As applications of Theorem 3 we also have the following two corollaries.

Corollary 1. *Let p be a prime.*

(i) If G is a non-Abelian group and $|G| = p^3$, then $\mathcal{E}(\Gamma_G) = 2(p^3 - 2p - 1)$.

(ii) If G is a 4-centralizer finite group, then $\mathcal{E}(\Gamma_G) = 6(|Z(G)| - 1)$.

(iii) If G is a $(p+2)$-centralizer finite p-group, then

$$\mathcal{E}(\Gamma_G) = 2((p^2 - 1)|Z(G)| - p - 1).$$

(iv) If G is a 5-centralizer finite group, then

$$\mathcal{E}(\Gamma_G) \in \Big\{8(2|Z(G)| - 1), 10|Z(G)| - 8\Big\}.$$

Proof. (i) In this case $|Z(G)| = p$ and $\frac{G}{Z(G)} \cong \mathbf{Z}_p \times \mathbf{Z}_p$. Hence the result follows from Theorem 3(i).
(ii) The result follows from Theorem 3(i), noting that $\frac{G}{Z(G)} \cong \mathbf{Z}_2 \times \mathbf{Z}_2$ (see [5, Theorem 2]).
(iii) We have $\frac{G}{Z(G)} \cong \mathbf{Z}_p \times \mathbf{Z}_p$ (see [2, Lemma 2.7]). Hence the result follows from Theorem 3(i).
(iv) By [5, Theorem 4] we have $\frac{G}{Z(G)} \cong \mathbf{Z}_3 \times \mathbf{Z}_3$ or D_6. If $\frac{G}{Z(G)} \cong \mathbf{Z}_3 \times \mathbf{Z}_3$, then by Theorem 3(i) we have $\mathcal{E}(\Gamma_G) = 8(2|Z(G)| - 1)$. In the case that $\frac{G}{Z(G)} \cong D_6$, by Theorem 3(ii) we have $\mathcal{E}(\Gamma_G) = 10|Z(G)| - 8$. □

Corollary 2. *If a group G is isomorphic to any of the following groups*

(i) $\mathbf{Z}_2 \times D_8$

(ii) $\mathbf{Z}_2 \times Q_8$

(iii) $G_{16} = \langle x, y \mid x^8 = y^2 = 1, yxy = x^5 \rangle$

(iv) $H_{16} = \langle x, y \mid x^4 = y^4 = 1, yxy^{-1} = x^{-1} \rangle$

(v) $D_8 * \mathbf{Z}_4 = \langle x, y, c \mid x^4 = y^2 = c^2 = 1, xy = yx, xc = cx, yc = x^2cy \rangle$

(vi) $SG(16, 3) = \langle x, y \mid x^4 = y^4 = 1, xy = y^{-1}x^{-1}, xy^{-1} = yx^{-1} \rangle$,

then $\mathcal{E}(\Gamma_G) = 18$.

Proof. We have $|G| = 16$ and $|Z(G)| = 4$. Therefore, $\frac{G}{Z(G)} \cong \mathbf{Z}_2 \times \mathbf{Z}_2$. Hence, the result follows from Theorem 3(i). $\square$

Note that all the groups considered in this section so far are non-Abelian AC-groups. We conclude this section by computing the energy of commuting graph of a finite non-Abelian AC-group in general. The following lemma is useful.

Lemma 1. *Let $C_1, \dots, C_n$ be the centralizers of non-central elements of a finite non-Abelian group G. Then*

$$\sum_{i=1}^{n} |C_i| = |G| + (n-1)|Z(G)|.$$

Proof. By [7, Lemma 2.1] we have $\Gamma_G = \bigsqcup_{i=1}^{n} K_{|C_i|-|Z(G)|}$. Therefore, the number of vertices in Γ_G is given by

$$\sum_{i=1}^{n} |C_i| - n|Z(G)| = |G| - |Z(G)|.$$

Hence the lemma follows. $\square$

Theorem 4. *Let G be a finite non-Abelian AC-group with n distinct centralizers of non-central elements. Then*

$$\mathcal{E}(\Gamma_G) = 2(|G| - |Z(G)| - n).$$

Proof. Let $C_1, \dots, C_n$ be the distinct centralizers of non-central elements of G. Then by [7, Theorem 2.1] we have

$$\mathcal{E}(\Gamma_G) = 2\sum_{i=1}^{n} |C_i| - 2n(|Z(G)| + 1).$$

Hence, the result follows using Lemma 1. $\square$

Corollary 3. *Let A be any finite Abelian group and G a finite non-Abelian AC-group. Then the energy of the commuting graph of $G \times A$ is given by*

$$\mathcal{E}(\Gamma_{G\times A}) = 2(|G||A| - |Z(G)||A| - n),$$

where n is as given in Theorem 4.

A finite graph Γ is called hyperenergetic and borderenergetic if $\mathcal{E}(\Gamma) > \mathcal{E}(K_{|v(\Gamma)|})$ and $\mathcal{E}(\Gamma) = \mathcal{E}(K_{|v(\Gamma)|})$ respectively, where $v(\Gamma)$ is the set of vertices of Γ. The study of hyperenergetic graph was initiated by Walikar et al. [21] and Gutman [13] in 1999. However, the concept of borderenergetic graph was introduced by Gong et al. [11] in the year 2015. Since then, it becomes an interesting question to determine whether a given graph is hyperenergetic or borderenergetic. As consequences of above results, we now show that the commuting graphs of the groups considered above are neither hyperenergetic nor borderenergetic. We begin with the following result.

Proposition 1. *If $G = M_{2mn}, D_{2m}, Q_{4m}$ and U_{6n} then Γ_G is neither hyperenergetic nor borderenergetic.*

Proof. If $G = M_{2mn}$ then $|v(\Gamma_G)| = \begin{cases} 2mn - n, & \text{if } n \text{ is odd} \\ 2mn - 2n, & \text{if } n \text{ is even.} \end{cases}$

Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = \begin{cases} 4mn - 2n - 2, & \text{if } n \text{ is odd} \\ 4mn - 4n - 2, & \text{if } n \text{ is even.} \end{cases}$

We have

$$4mn - 2m - 2n - 2 < 4mn - 2n - 2 \text{ and } 4mn - 4n - m - 2 < 4mn - 4n - 2.$$

Hence, by Theorem 1(i), $\Gamma_{M_{2mn}}$ is neither hyperenergetic nor borderenergetic.

If $G = D_{2m}$ then $|v(\Gamma_G)| = \begin{cases} 2m - 1, & \text{if } m \text{ is odd} \\ 2m - 2, & \text{if } m \text{ is even.} \end{cases}$

Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = \begin{cases} 4m - 4, & \text{if } m \text{ is odd} \\ 4m - 6, & \text{if } m \text{ is even.} \end{cases}$

We have

$$2m - 4 < 4m - 4 \text{ and } 3m - 6 < 4m - 6.$$

Hence, by Theorem 1(ii), $\Gamma_{D_{2m}}$ is neither hyperenergetic nor borderenergetic.

If $G = Q_{4m}$ then $|v(\Gamma_G)| = 4m - 2$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 8m - 6$. We have

$$6m - 3 < 8m - 6.$$

Hence, by Theorem 1(iii), $\Gamma_{Q_{2m}}$ is neither hyperenergetic nor borderenergetic.

If $G = U_{6n}$ then $|v(\Gamma_G)| = 5n$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 10n - 2$. We have

$$10n - 8 < 10n - 2.$$

Hence, by Theorem 1(iv), $\Gamma_{U_{6n}}$ is neither hyperenergetic nor borderenergetic. $\square$

Proposition 2. *If $G = QD_{2^n}, PSL(2, 2^k), GL(2, q), A(n, \vartheta)$ and $A(n, p)$ then Γ_G is neither hyperenergetic nor borderenergetic.*

Proof. If $G = QD_{2^n}$ then $|v(\Gamma_G)| = 2^n - 2$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2.2^n - 6$. We have

$$2^n + 2^{n-1} - 6 < 2.2^n - 6.$$

Hence, by Theorem 2(i), $\Gamma_{QD_{2^n}}$ is neither hyperenergetic nor borderenergetic.

If $G = PSL(2, 2^k)$ then $|v(\Gamma_G)| = 2^k(2^{2k} - 1)$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2^{3k+1} - 2^{k+1} - 2$. We have

$$2^{3k+1} - 2^{2k+1} - 2.2^{k+1} - 4 < 2^{3k+1} - 2^{k+1} - 2.$$

Hence, by Theorem 2(ii), $\Gamma_{PSL(2,2^k)}$ is neither hyperenergetic nor borderenergetic.

If $G = GL(2, q)$ then $|v(\Gamma_G)| = (q^2 - 1)(q^2 - q) - q - 1 = q^4 - q^3 - q^2 - 1$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2q^4 - 2q^3 - 2q^2 - 4$. We have

$$2q^4 - 2q^3 - 4q^2 - 2q < 2q^4 - 2q^3 - 2q^2 - 4.$$

Hence, by Theorem 2(iii), $\Gamma_{GL(2,q)}$ is neither hyperenergetic nor borderenergetic.

If $G = A(n, \vartheta)$ then $|v(\Gamma_G)| = 2^n(2^n - 1)$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2.2^{2n} - 2.2^n - 2$. We have

$$2.2^{2n} - 4.2^n + 2 < 2.2^{2n} - 2.2^n - 2.$$

Hence, by Theorem 2(v), $\Gamma_{A(n,\vartheta)}$ is neither hyperenergetic nor borderenergetic.

If $G = A(n, p)$ then $|v(\Gamma_G)| = (p^{2n} - p^n)(p^n + 1)$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2.p^{3n} - 2.p^n - 2$. We have

$$2.p^{3n} - 4.p^n - 2 < 2.p^{3n} - 2.p^n - 2.$$

Hence, by Theorem 2(vi), $\Gamma_{A(n,p)}$ is neither hyperenergetic nor borderenergetic. $\square$

Proposition 3. *If G is a non-Abelian group and $|G|$ is product of two primes p, q such that $p \mid (q-1)$ then Γ_G is neither hyperenergetic nor borderenergetic.*

Proof. We have $|v(\Gamma_G)| = pq - 1$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2pq - 4$. Also

$$2pq - 2q - 3 < 2pq - 4.$$

Hence, by Theorem 2(vii), the result follows. □

Proposition 4. *Γ_G is neither hyperenergetic nor borderenergetic if $\frac{G}{Z(G)} \cong Sz(2), \mathbf{Z}_p \times \mathbf{Z}_p$ and D_{2m}.*

Proof. If $\frac{G}{Z(G)} \cong Sz(2)$ then $|v(\Gamma_G)| = 19|Z(G)|$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 38|Z(G)| - 2$. We have

$$38|Z(G)| - 12 < 38|Z(G)| - 2.$$

Hence, by Theorem 2(iv), Γ_G is neither hyperenergetic nor borderenergetic.

If $\frac{G}{Z(G)} \cong \mathbf{Z}_p \times \mathbf{Z}_p$ then $|v(\Gamma_G)| = |Z(G)|p^2 - |Z(G)|$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2|Z(G)|p^2 - 2|Z(G)| - 2$. We have

$$2|Z(G)|p^2 - 2|Z(G)| - p - 1 < 2|Z(G)|p^2 - 2|Z(G)| - 2.$$

Hence, by Theorem 3(i), Γ_G is neither hyperenergetic nor borderenergetic.

If $\frac{G}{Z(G)} \cong D_{2m}$ then $|v(\Gamma_G)| = (2m-1)|Z(G)|$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2(2m-1)|Z(G)| - 2$. We have

$$2(2m-1)|Z(G)| - 2m - 2 < 2(2m-1)|Z(G)| - 2.$$

Hence, by Theorem 3(ii), Γ_G is neither hyperenergetic nor borderenergetic. □

Corollary 4. *Let G be a finite non-Abelian group and p be any prime. Then Γ_G is neither hyperenergetic nor borderenergetic if*

1. *G is of order p^3.*
2. *G is a 4-centralizer group.*
3. *G is a 5-centralizer group.*
4. *G is a $(p+2)$-centralizer p-group.*

Proof. The result follows from Proposition 4 since $\frac{G}{Z(G)}$ is isomorphic to either $\mathbf{Z}_p \times \mathbf{Z}_p$ or D_6. $\square$

Corollary 5. *Γ_G is neither hyperenergetic nor borderenergetic if G is given by $\mathbf{Z}_2 \times D_8$, $\mathbf{Z}_2 \times Q_8$, G_{16}, H_{16}, $D_8 * \mathbf{Z}_4$ and $SG(16, 3)$.*

Proof. The result follows from Proposition 4 since $\frac{G}{Z(G)}$ is isomorphic to $\mathbf{Z}_2 \times \mathbf{Z}_2$. $\square$

In general, we have the following theorem.

Theorem 5. *If G is a finite non-Abelian AC-group then Γ_G is neither hyperenergetic nor borderenergetic.*

Proof. We have $|v(\Gamma_G)| = |G| - |Z(G)|$. Therefore, $\mathcal{E}(K_{|v(\Gamma_G)|}) = 2|G| - 2|Z(G)| - 2$. We have

$$2|G| - 2|Z(G)| - 2n < 2|G| - 2|Z(G)| - 2.$$

Hence, by Theorem 4, Γ_G is neither hyperenergetic nor borderenergetic. $\square$

We conclude this paper with a question given below.

Question 1. *Is it true that the commuting graph of any finite non-Abelian group is neither hyperenergetic nor borderenergetic?*

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