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On Randić energy of graphs

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Abstract

Let d_i be the degree of vertex v_i of G then Randić matrix $R(G) = [r_{ij}]$ is defined as $r_{ij} = 1/\sqrt{d_i d_j}$, if the vertices v_i and v_j are adjacent in G or $r_{ij} = 0$, otherwise. The Randić energy is the sum of absolute values of the eigenvalues of $R(G)$. In this paper we have investigated Randić energy of m-Splitting and m-Shadow graphs. We also have constructed a sequence of graphs having same Randić energy.

Keywords: *Eigenvalues, Graph Energy, Randić Matrix, Randić Energy.*

2020 Mathematics Subject Classification: 05C50, 05C76.

1. Introduction

Let G be a graph on vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and edge set $E(G)$. Let d_i be the degree of a vertex v_i, for $i = 1, 2, ..., n$. The adjacency matrix $A(G) = [a_{ij}]$ of a graph G is a square matrix of order n, where

$$a_{ij} = \begin{cases} 1 & ; \text{ if vertices } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & ; \text{ otherwise} \end{cases}$$

As $A(G)$ is a symmetric matrix so, its eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ are all real numbers with their sum is zero. The concept of graph energy was introduced by Gutman [8]. According to him energy of graph $\mathcal{E}(G)$ is defined as

$$\mathcal{E}(G) = \sum_{i=1}^{n} |\lambda_i|$$

A brief account of graph energy can be found in Balakrishnan [2], Li et al. [14] and Cvetković et al. [6].

In [8], Gutman investigated the energy of complete graph K_n and conjectured that among all graphs on n vertices, the energy of complete graph is maximum. But Walikar et al. [21] disproved it by showing the existence of graphs other than K_n whose energy is greater than that of complete graph K_n. Gong et al. [7] has introduced a new concept called borderenergetic graph. According to him a graph on n vertices is called borderenergetic if $\mathcal{E}(G) = \mathcal{E}(K_n) = 2(n-1)$.

In 1975, Milan Randić [15] has defined one topological index and termed is as Randić index which is denoted as R and defined by

$$R = \sum_{i \sim j} \frac{1}{\sqrt{d_i d_j}}$$

where the summation is taken over all pairs of adjacent vertices v_i and v_j. Randić index is certainly the most widely applicable in chemistry and pharmacology, in particular for designing quantitative structure property and structure activity relations. A brief account on Randić index can be found in [9, 12, 13, 16].

In 2010, Bozkurt et al. [3, 4] pointed out that the Randić index is purposeful to produce a graph matrix of order n named as Randić matrix

$R(G) = [r_{ij}]$, where

$$r_{ij} = \begin{cases} \dfrac{1}{\sqrt{d_i d_j}} & ; \text{ if } v_i \text{ and } v_j \text{ are adjacent} \\ 0 & ; \text{ otherwise} \end{cases}$$

The connection between the Randić matrix and the Randić index is obvious: The sum of all elements of $R(G)$ is equal to $2R$.

Let $R(G)$ be the Randić matrix with $\mu_1, \mu_2, ..., \mu_n$ are eigenvalues of matrix $R(G)$ then the Randić energy [3, 4] is defined as the sum of absolute values of Randić eigenvalues of graph G which is denoted as $RE(G)$. That is,

$$RE = RE(G) = \sum_{i=1}^{n} |\mu_i|$$

The Randić energy of some standard graph families is given as:

Graph	Randić Energy	Reference
K_n	2	[1]
$K_{m,n}$	2	[1]
$K_{1,n}$	2	[1]
F_n	$n+1$	[1]
D_4^n	$2+(n-1)\sqrt{2}$	[1]
C_{2n}	$\dfrac{2\sin((\lfloor \frac{n}{2} \rfloor + \frac{1}{2})\frac{\pi}{n})}{\sin \frac{\pi}{2n}}$	[17]
P_n	$2+\frac{1}{2}\mathcal{E}(P_{n-2})$	[10]

Table 1.1: Randić Energy of Standard Graph Families

From the above Table 1.1, one can observe that among all the graphs, path P_n is the graph with maximum Randić energy. The Randić energy of graphs obtained by means of various graph operations has been explored in [19, 20].

The next section is aimed to discuss the Randić energy of m-Splitting graph, m-Shadow graph and also to construct a sequence of graphs having same Randić energy.

2. Randić Energy of m-Splitting Graph

Definition 2.1. *[18] The m-Splitting graph $Spl_m(G)$ of a graph G is obtained by adding to each vertex v a new m vertex $v_1, v_2, ..., v_m$, such that v_i, $1 \leq i \leq m$ is adjacent to every vertex that is adjacent to v in G.*

Definition 2.2. *[11] For the matrices $A = [a_{ij}]_{m\times n}$, $B = [b_{ij}]_{p\times q}$ the Kronecker product of A and B is defined as the matrix*

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}$$

Proposition 2.3. *[11] If λ is an eigenvalue of matrix $A = [a_{ij}]_{m\times m}$ with corresponding eigenvector x, and μ is an eigenvalue of matrix $B = [b_{ij}]_{n\times n}$ with corresponding eigenvector y. Then $\lambda\mu$ is an eigenvalue of $A \otimes B$ with corresponding eigenvector $x \otimes y$.*

Z. Chu et al.[5] have proved the following result

Lemma 2.4. *For a graph G,*

$$RE(S'(G)) = \frac{3}{2} RE(G).$$

We prove the following result for m-Splitting graph of given graph G and above result is consequence of our result.

Theorem 2.5. *For any graph G, $RE(Spl_m(G)) = \dfrac{\sqrt{1 + 4m(m+1)}}{m+1} RE(G)$.*

Proof. Let G be a graph with $v_1, v_2, \cdots, v_n$ as vertices of then its Randić matrix $R(G)$ is given by

$$R(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & \cdots & v_n \end{array} \\ \begin{bmatrix} 0 & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & 0 & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & 0 & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & r_{n3} & \cdots & 0 \end{bmatrix} \end{array}$$

Now, consider m-copies of vertex v_i for $1 \leq i \leq n$, say $v_i^1, v_i^2, ..., v_i^m$ and then join each vertex v_i^k, for $1 \leq k \leq m$ to neighbors of vertex v_i to obtain m-Splitting of given graph G. Then the Randić matrix $R(Spl_m(G))$ can be written as follows

$$R(Spl_m(G)) = \begin{bmatrix} \frac{1}{m+1}R(G) & \frac{1}{\sqrt{m+1}}R(G) & \cdots & \frac{1}{\sqrt{m+1}}R(G) \\ \frac{1}{\sqrt{m+1}}R(G) & O & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{m+1}}R(G) & O & \cdots & O \end{bmatrix}$$

That is,

$$R(Spl_m(G)) = \begin{bmatrix} \frac{1}{m+1} & \frac{1}{\sqrt{m+1}} & \cdots & \frac{1}{\sqrt{m+1}} \\ \frac{1}{\sqrt{m+1}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{m+1}} & 0 & \cdots & 0 \end{bmatrix} \otimes R(G) = A \otimes R(G),$$

where $A = \begin{bmatrix} \frac{1}{m+1} & \frac{1}{\sqrt{m+1}} & \cdots & \frac{1}{\sqrt{m+1}} \\ \frac{1}{\sqrt{m+1}} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{m+1}} & 0 & \cdots & 0 \end{bmatrix}$

is of order $m+1$. Since, A is a matrix of rank 2 so, it means that matrix A has only two non-zero eigenvalues, say ρ_1 and ρ_2.
Also, we know that

$$\rho_1 + \rho_2 = tr(A) = \frac{1}{m+1} \tag{2.1}$$

Now, consider the matrix

$$A^2 = \begin{bmatrix} \frac{1}{(m+1)^2} + \frac{m}{m+1} & \frac{1}{(m+1)\sqrt{m+1}} & \cdots & \frac{1}{(m+1)\sqrt{m+1}} \\ \frac{1}{(m+1)\sqrt{m+1}} & \frac{1}{m+1} & \cdots & \frac{1}{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{(m+1)\sqrt{m+1}} & \frac{1}{m+1} & \cdots & \frac{1}{m+1} \end{bmatrix}$$

Here,

$$\rho_1^2 + \rho_2^2 = tr(A^2) = \frac{1}{(m+1)^2} + \frac{2m}{m+1} \tag{2.2}$$

by solving equations (2.1) and (2.2), we have

$$\rho_1 = \frac{1+\sqrt{1+4m(m+1)}}{2(m+1)} \quad \text{and} \quad \rho_2 = \frac{1-\sqrt{1+4m(m+1)}}{2(m+1)}$$

Hence,

$$Spec(A) = \begin{pmatrix} 0 & \frac{1+\sqrt{1+4m(m+1)}}{2(m+1)} & \frac{1-\sqrt{1+4m(m+1)}}{2(m+1)} \\ m-1 & 1 & 1 \end{pmatrix}$$

Since, $R(Spl_m(G)) = A \otimes R(G)$, it follows that if $\mu_1, \mu_2, ..., \mu_n$ are eigenvalues of $R(G)$, then by proposition 2.3, we have

$R\text{-}Spec(Spl_m(G)) =$

$$\begin{pmatrix} 0 & \frac{1+\sqrt{1+4m(m+1)}}{2(m+1)}\mu_1 & \cdots & \frac{1+\sqrt{1+4m(m+1)}}{2(m+1)}\mu_n \\ n(m-1) & 1 & \cdots & 1 \\ \frac{1-\sqrt{1+4m(m+1)}}{2(m+1)}\mu_1 & \frac{1-\sqrt{1+4m(m+1)}}{2(m+1)}\mu_2 & \cdots & \frac{1-\sqrt{1+4m(m+1)}}{2(m+1)}\mu_n \\ 1 & 1 & \cdots & 1 \end{pmatrix}$$

Hence,

$$\begin{aligned} RE(Spl_m(G)) &= \sum_{i=1}^{n} \left| \frac{1 \pm \sqrt{1+4m(m+1)}}{2(m+1)} \mu_i \right| \\ &= \sum_{i=1}^{n} |\mu_i| \left(\frac{1+\sqrt{1+4m(m+1)}}{2(m+1)} + \frac{\sqrt{1+4m(m+1)}-1}{2(m+1)} \right) \end{aligned}$$

Therefore,

$$RE(Spl_m(G)) \quad = \frac{\sqrt{1+4m(m+1)}}{m+1} RE(G) \quad \square$$

Illustration 2.6. *Consider cycle C_4 and its 2-spitting graph $Spl_2(C_4)$. We know that R-Spec(C_4) =* $\begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$ *and so, $RE(C_4) = 2$.*

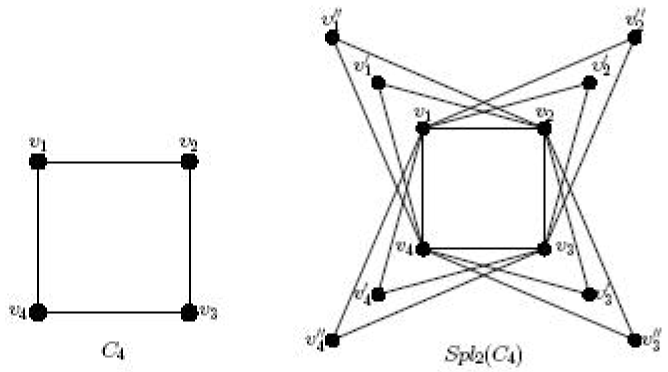

Figure 1: *Cycle C_4 and its 2-Splitting graph $Spl_2(C_4)$*

The Randić matrix of C_4 can be written as

$$R(C_4) = \begin{bmatrix} 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{bmatrix}$$

The Randić matrix of $Spl_2(C_4)$ can be written as

$$R(Spl_2(C_4) = \begin{bmatrix} \frac{1}{3}R(C_4) & \frac{1}{\sqrt{3}}R(C_4) & \frac{1}{\sqrt{3}}R(C_4) \\ \frac{1}{\sqrt{3}}R(C_4) & 0 & 0 \\ \frac{1}{\sqrt{3}}R(C_4) & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 & 0 \\ \frac{1}{\sqrt{3}} & 0 & 0 \end{bmatrix} \otimes R(C_4)$$

Therefore, $R\text{-}Spec(Spl_2(C_4))= \begin{pmatrix} 1 & -1 & \frac{2}{3} & \frac{-2}{3} & 0 \\ 1 & 1 & 1 & 1 & 8 \end{pmatrix}$

Hence,

$$RE(Spl_2(C_4)) = \frac{10}{3} = \frac{5}{3} RE(C_4)$$

3. Randić Energy of m-Shadow Graph

Definition 3.1. *[18] The m-Shadow graph $D_m(G)$ of a connected graph G is constructed by taking m copies of G say $G_1, G_2, ..., G_m$. Then Join each vertex u in G_i to the neighbors of the corresponding vertex v in G_j, $1 \leq i, j \leq m$.*

We prove the following result for m-Shadow graph of graph G.

Theorem 3.2. *For any graph G, $RE(D_m(G)) = RE(G)$.*

Proof. Let G be a graph with $v_1, v_2, \cdots, v_n$ as vertices of then its Randić matrix $R(G)$ is given by

$$R(G) = \begin{array}{c|c} & \begin{matrix} v_1 & v_2 & v_3 & \cdots & v_n \end{matrix} \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ \vdots \\ v_n \end{matrix} & \begin{bmatrix} 0 & r_{12} & r_{13} & \cdots & r_{1n} \\ r_{21} & 0 & r_{23} & \cdots & r_{2n} \\ r_{31} & r_{32} & 0 & \cdots & r_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & r_{n3} & \cdots & 0 \end{bmatrix} \end{array}$$

Now, consider m-copies $G_1, G_2, ..., G_m$ of graph G and then join each vertex of u of graph G_i to the neighbors of the corresponding vertex v in graph G_j, $1 \leq i, j \leq m$ to obtain m-Shadow $D_m(G)$. Then the Randić matrix of graph $D_m(G)$ can be written as

$$R(D_m(G)) = \begin{bmatrix} \frac{1}{m}R(G) & \frac{1}{m}R(G) & \cdots & \frac{1}{m}R(G) \\ \frac{1}{m}R(G) & \frac{1}{m}R(G) & \cdots & \frac{1}{m}R(G) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{m}R(G) & \frac{1}{m}R(G) & \cdots & \frac{1}{m}R(G) \end{bmatrix}$$

That is,

$$R(D_m(G)) = \begin{bmatrix} \frac{1}{m} & \frac{1}{m} & \cdots & \frac{1}{m} \\ \frac{1}{m} & \frac{1}{m} & \cdots & \frac{1}{m} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{m} & \frac{1}{m} & \cdots & \frac{1}{m} \end{bmatrix} \otimes R(G)$$

Therefore, $R(D_m(G)) = B_m \otimes R(G)$
Since, we know that the spectrum of B_m is

$$\begin{pmatrix} 0 & 1 \\ m-1 & 1 \end{pmatrix}$$

Hence, by Proposition 2.3

$$R - Spec(D_m(G)) = \begin{pmatrix} 0\mu_1 & 0\mu_2 & ... & 0\mu_n & 1\mu_1 & 1\mu_2 & ... & 1\mu_n \\ m-1 & m-1 & ... & m-1 & 1 & 1 & ... & 1 \end{pmatrix}$$

where $\mu_i,\ i = 1, 2, ..., n$ are eigenvalues of $R(G)$.
Therefore,

$$RE(D_m(G)) = \sum_{i=1}^{n} |\mu_i| = RE(G)$$

□

Illustration 3.3. *Consider cycle C_4 and its shadow graph $D_3(C_4)$. It is obvious that $RE(C_4) = 2$ as*

$$R - Spec(C_4) = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$$

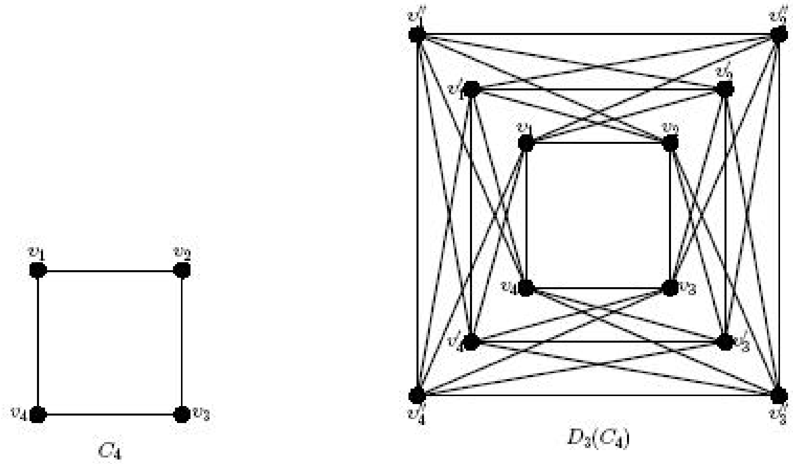

Figure 2: *Cycle C_4 and its 3-Shadow graph $D_3(C_4)$*

Now, the Randić matrix of $D_3(C_4)$) is given as follow

$$R(D_3(C_4)) = \begin{bmatrix} \frac{1}{3}R(C_4) & \frac{1}{3}R(C_4) & \frac{1}{3}R(C_4) \\ \frac{1}{3}R(C_4) & \frac{1}{3}R(C_4) & \frac{1}{3}R(C_4) \\ \frac{1}{3}R(C_4) & \frac{1}{3}R(C_4) & \frac{1}{3}R(C_4) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \otimes R(C_4) = B_3 \otimes R(C_4)$$

Since, the spectrum of B_3 is $\begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$.

So, R-Spec$(D_3(C_4)) = \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 10 \end{pmatrix}$.

Hence,

$$RE(D_3(C_4)) = 2 = RE(C_4).$$

In the following section we constructed a sequence of graphs having same Randić energy.

4. A Sequence of Graphs having same Randić Energy

Let G be a graph of order n. Consider an infinite sequence of graphs $\mathcal{F} = \{G^{(0)}, G^{(1)}, ..., G^{(k)}, ...\}$ such that

$$G^{(0)} = G,\ G^{(1)} = D_m(G^{(0)}),\ G^{(2)} = D_m(G^{(1)}), ..., G^{(k)} = D_m(G^{(k-1)}), ...$$

Let $\mu_1, \mu_2, ..., \mu_n$ be a Randić eigenvalues of graph G, then for $G^{(1)} = D_m(G^{(0)}) \in \mathcal{F}$, the Randić spectrum of $G^{(1)}$ is

$$R\text{-}Spec(G^{(1)}) = \begin{pmatrix} 0 & 1\mu_1 & 1\mu_2 & ... & 1\mu_n \\ n(m-1) & 1 & 1 & ... & 1 \end{pmatrix}$$

Now, we have $G^{(2)} = D_m(G^{(1)})$ and so, Randić spectra of $G^{(2)}$ can be given as

$$R\text{-}Spec(G^{(2)}) = \begin{pmatrix} 0 & 1\mu_1 & 1\mu_2 & ... & 1\mu_n \\ n(m^2-1) & 1 & 1 & ... & 1 \end{pmatrix}$$

By continuing this process we have for any $G^{(k)} = D_m(G^{(k-1)}) \in \mathcal{F}$, for $k1$, the Randić spectra of $G^{(k)}$ is given as,

$$R\text{-}Spec(G^{(k)}) = \begin{pmatrix} 0 & 1\mu_1 & 1\mu_2 & ... & 1\mu_n \\ n(m^k-1) & 1 & 1 & ... & 1 \end{pmatrix}$$

Therefore, Randić energy of $G^{(k)}$ is

$$\begin{aligned} RE(G^{(k)}) &= \sum_{i=1}^{nm^k} |\mu_i| \\ &= \sum_{i=1}^{n} |\mu_i| = RE(G^{(0)}) = RE(G) \end{aligned}$$

Hence, $\mathcal{F}$ is a set of infinite sequence of graphs having same Randić energy.

5. Concluding Remarks

The concept of Randić energy is defined in the context of Randić matrix. We have obtained Randić energy of m-Splitting and m-Shadow graphs. An iterative sequence of graphs having same Randić energy has been investigated. It is worth to note that, unlike in the concept of equienergetic graphs, the graphs whose Randić energies are shown to be equal are of different order. It is a salient feature of our contribution.

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