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On locating chromatic number of Möbius ladder graphs

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Abstract:

In this paper, we are dealing with the study of locating chromatic number of Möbius-ladders. We prove that Möbius-ladders Mn with n even has locating chromatic number 4 if n≠6 and 6 if n=6.

Keywords: Möbius-ladders; Color code; Locating-chromatic number; Locating coloring.

MSC (2020): 05C12, 05C15.

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1. Introduction

Let $G = (V, E)$ be a finite, simple and connected graph. The distance $d(u, v)$ between vertices u and v in G is the length of the shortest path connecting u and v in G and for a subset S of $V(G)$, the distance between u and S is given by $d(u, S) = min\{d(u, x)|x \in S\}$. The eccentricity $\epsilon(v)$ of a vertex v is the greatest distance between v and any other vertex. The diameter $diam(G)$ of the graph is the maximum eccentricity of any vertex in the graph.

A k-coloring c of a graph $G = (V, E)$ is a k-partition $\Pi = (V_1, V_2, \dots, V_k)$ of $V(G)$ into independent sets, called colors. The color code of vertex v of G, with respect to Π is defined to be the ordered k-tuple $c_{\Pi}(v) = (d(v, V_1), d(v, V_2), \dots, d(v, V_k))$. A k-coloring c of G is a locating coloring (or a locating k-coloring) of G if any two distinct vertices of G have distinct color codes with respect to c. The locating-chromatic number of G, denoted by $\chi_L(G)$, is the smallest k such that G admits a locating k-coloring.

The concept of locating coloring was first introduced by Erwin David et al. [1]. They established some bounds for the locating chromatic number of connected graph classes: paths, cycles, complete multipartite graphs and double stars. This concept has been also called resolving coloring and independent resolving partition [2].

For a certain locating-chromatic number, Chartrand et al. [3] characterized all graphs of order n with locating-chromatic number $n-1$. Asmiati and Baskoro [4, 5] determined all graphs with locating-chromatic number 3. In [6], Asmiati et al. derived the locating-chromatic number for some class of trees, especially a class of trees obtained as an amalgamation of n stars. Behtoei and Omoomi [7] gave the locating chromatic number for the cartesian product of any two graphs and gave the following definition and observation :

Definition 1. *[7] Let G be a connected graph. A vertex is called colorful if all of the colors appear in its closed neighborhood.*

Observation 1. *[7] In a locating coloring of G, there are no two colorful vertices that are assigned to the same color. Therefore, if there is a locating k-coloring of G, then there are at most k colorful vertices.*

The following theorem was proved in [1].

Theorem 2. *[1] Let G be a connected graph of order $n \geq 3$. Then $\chi_L(G) = n$ if and only if G is a complete multipartite graph.*

The following observation will be useful later.

Observation 3. *To show that a given coloring c is a locating coloring, it suffices to show that $c_{\Pi}(u) \neq c_{\Pi}(v)$ for vertices u, v with $c(u) = c(v)$.*

The Möbius ladder was originally introduced by Richard Guy and Frank Harary in 1967 [8]. The Möbius ladder on n vertices M_n is constructed by connecting vertices v_i and v_j in the cycle C_n if $d(v_i, v_j) = diam(C_n)$ (see Figure 1b). Some authors considred the case when n is even [9, 10]. The Möbius ladder graph M_n for even positive integer n is a graph can be obtained from the ladder $P_{\frac{n}{2}} \times P_2$ by joining the opposite endpoints of the two copies of $P_{\frac{n}{2}}$ (see Figure 1c). It can be also obtained by introducing a twist in a prism graph of order n. From Figure 1a, it is easy to see why this family is called the Möbius-ladders. Three different views of Möbius-ladders M_{20} are shown on Figure 1. We call $P_1 = v_1 v_2 \ldots v_{\frac{n}{2}}$ the inner path and $P_2 = v_{\frac{n+2}{2}} v_{\frac{n+4}{2}} \ldots v_n$ the outer path of the Möbius ladder graphs M_n.

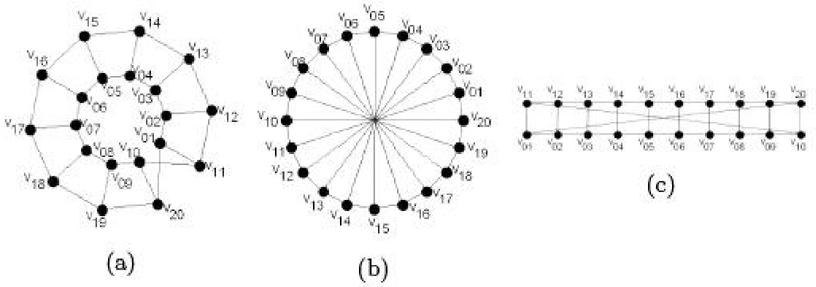


Figure 1: Three views of Möbius ladder M_{20}

Some parameters of Möbius ladders have been studied: strong metric dimension [11], H-antimagic covering [12], local metric dimension [13], metric dimension [14, 15], distance labelings [16] and skew chromatic index [17]. In the next section, we study the locating chromatic number of Möbius-ladders M_n for n even and we prove that $\chi_L(M_n) = 4$ if $n \neq 6$ and n even.

2. Main result

We start by proving that the graph M_n does not has a locating coloring with three colors and there is no 4-coloring or 5-coloring of M_6.

Lemma 1. *$\chi_L(M_6) = 6$.*

Proof. It is easy to see that M_6 is isomorphic to $K_{3,3}$, the complete bipartite graph on 2 sets of 3 vertices each. Thus, again by theorem 2, we obtain that the locating chromatic number of M_6 is 6. □

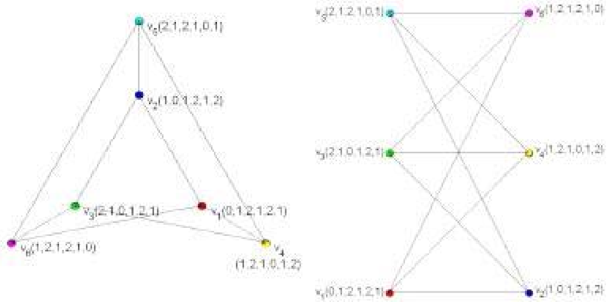


Figure 2: Locating 6-coloring of M_6

Theorem 2. *If $n \geq 6$ and n even, then $\chi_L(M_n) \geq 4$.*

Proof. We will prove that $\chi_L(M_n) \geq 4$, by showing that there is no locating 3-coloring of M_n. Suppose that M_n has locating 3-coloring, there exists an induced cycle of order 4 with 3 colors. Hence, there are two colorful vertices on this cycle with the same color, which is a contradiction with Observation 1. Therefore $\chi_L(M_n) \geq 4$. □

Lemma 3. *Let M_n be the Möbius ladder graph, v_i be a vertex of M_n, $V_3 = \{v_1, v_{\frac{n}{2}}\}$ and $V_4 = \{v_n, v_{\frac{n+2}{2}}\}$. If $n \equiv 0 \mod 4$ and $n \geq 8$ then:*

$$d(v_i, V_4) = \begin{matrix} (v_i, V_3) + 1 \ if \ i \in [1, \frac{n}{2}], \\ d(v_i, V_3) - 1 \ if \ i \in [\frac{n+2}{2}, n], \end{matrix}$$

Proof. We have $d(v_i, V_3) = min(d(v_i, v_1), d(v_i, v_{\frac{n}{2}}))$. We distinguish two cases, depending on whether $d(v_i, V_3) = d(v_i, v_1)$ *or* $d(v_i, V_3) = d(v_i, v_{\frac{n}{2}})$.

Case 1. $i \in [1, \frac{n}{4}] \bigcup [\frac{n+2}{2}, \frac{3n}{4}]$.
In this case, we have $d(v_i, V_3) = d(v_i, v_1)$ and $d(v_i, V_4) = d(v_i, v_{\frac{n+2}{2}})$.

If $i \in [1, \frac{n}{4}]$ *then* $d(v_i, V_4) = d(v_i, v_{\frac{n+2}{2}}) = d(v_i, v_1) + d(v_1, v_{\frac{n+2}{2}}) = d(v_i, V_3) + 1$.
If $i \in [\frac{n+2}{2}, \frac{3n}{4}]$ *then* $d(v_i, V_3) = d(v_i, v_1) = d(v_i, v_{\frac{n+2}{2}}) + d(v_{\frac{n+2}{2}}, v_1)) = d(v_i, V_4) + 1$.

Case 2. $i \in [\frac{n+4}{4}, \frac{n}{2}] \bigcup [\frac{3n+4}{4}, n]$.
In this case, we have $d(v_i, V_3) = d(v_i, v_{\frac{n}{2}})$ *and* $d(v_i, V_4) = d(v_i, v_n)$.
If $i \in [\frac{n+4}{4}, \frac{n}{2}]$ *then* $d(v_i, V_4) = d(v_i, v_n) = d(v_i, v_{\frac{n}{2}}) + d(v_{\frac{n}{2}}, v_n) = d(v_i, V_3) + 1$.
If $i \in [\frac{3n+4}{4}, n]$ *then* $d(v_i, V_3) = d(v_i, v_{\frac{n}{2}}) = d(v_i, v_n) + d(v_n, v_{\frac{n}{2}}) = d(v_i, V_4) + 1$.
Combining the two previous cases we get the following.
If $i \in [1, \frac{n}{4}] \bigcup [\frac{n+4}{4}, \frac{n}{2}] = [1, \frac{n}{2}]$, *then* $d(v_i, V_4) = d(v_i, V_3) + 1$.
If $i \in [\frac{n+2}{2}, \frac{3n}{4}] \bigcup [\frac{3n+4}{4}, n] = [\frac{n+2}{2}, n]$, *then* $d(v_i, V_3) = d(v_i, V_4) + 1$, *so we have* $d(v_i, V_4) = d(v_i, V_3) - 1$.

□

Lemma 4. *Let* M_n *be the Möbius ladder graph with* $n \geq 10$, v_i *a vertex of* M_n, $V_3 = \{v_2, v_{\frac{n}{2}}\}$ *and* $V_4 = \{v_n, v_{\frac{n+4}{2}}\}$. *If* $n \equiv 2 \ mod\ 4$ *and* $n \geq 10$ *then:*

$$d(v_i, V_4) = \begin{array}{l} (v_i, V_3) + 1, \quad if\, \imath \in [2, \frac{n}{2}], \\ d(v_i, V_3) - 1, \quad if\, \imath \in [\frac{n+4}{2}, n], \\ d(v_i, V_3), \quad if\, \imath \in \{1, \frac{n+2}{2}\}, \end{array}$$

Proof. We have $d(v_i, V_3) = min(d(v_i, v_2), d(v_i, v_{\frac{n}{2}}))$. We distinguish three cases,

Case 3. $i \in [2, \frac{n+2}{4}] \bigcup [\frac{n+4}{2}, \frac{3n+2}{4}]$.
In this case, we have $d(v_i, V_3) = d(v_i, v_2)$ *and* $d(v_i, V_4) = d(v_i, v_{\frac{n+4}{2}})$.
If $i \in [2, \frac{n+2}{4}]$, *then* $d(v_i, V_4) = d(v_i, v_{\frac{n+4}{2}}) = d(v_i, v_2) + d(v_2, v_{\frac{n+4}{2}}) = d(v_i, V_3) + 1$.
If $i \in [\frac{n+4}{2}, \frac{3n+2}{4}]$, *then* $d(v_i, V_3) = d(v_i, v_2) = d(v_i, v_{\frac{n+2}{2}}) + d(v_{\frac{n+2}{2}}, v_1)) = d(v_i, V_4) + 1$.

Case 4. $i \in [\frac{n+6}{4}, \frac{n}{2}] \bigcup [\frac{3n+6}{4}, n]$.
In this case, we have $d(v_i, V_3) = d(v_i, v_{\frac{n}{2}})$ *and* $d(v_i, V_4) = d(v_i, v_n)$.

If $i \in [\frac{n+6}{4}, \frac{n}{2}]$ *then* $d(v_i, V_4) = d(v_i, v_n) = d(v_i, v_{\frac{n}{2}}) + d(v_{\frac{n}{2}}, v_n) = d(v_i, V_3) + 1$.
If $i \in [\frac{3n+6}{4}, n]$ *then* $d(v_i, V_3) = d(v_i, v_{\frac{n}{2}}) = d(v_i, v_n) + d(v_n, v_{\frac{n}{2}}) = d(v_i, V_4) + 1$.
Combining the two previous cases we get the following:
If $i \in [1, \frac{n}{4}] \bigcup [\frac{n+4}{4}, \frac{n}{2}] = [1, \frac{n}{2}]$ *then* $d(v_i, V_4) = d(v_i, V_3) + 1$.
If $i \in [\frac{n+2}{2}, \frac{3n}{4}] \bigcup [\frac{3n+4}{4}, n] = [\frac{n+2}{2}, n]$ *then* $d(v_i, V_3) = d(v_i, V_4) + 1$, *so we have* $d(v_i, V_4) = d(v_i, V_3) - 1$.

Case 5. $i \in \{1, \frac{n+2}{2}\}$.
We have:
$d(v_1, V_3) = d(v_1, v_2) = 1$ *and* $d(v_1, V_4) = d(v_1, v_n) = 1$, *so* $d(v_1, V_3) = d(v_1, V_4)$.
$d(v_{\frac{n+2}{2}}, V_3) = d(v_{\frac{n+2}{2}}, v_{\frac{n}{2}}) = 1$ *and* $d(v_{\frac{n+2}{2}}, V_4) = d(v_{\frac{n+2}{2}}, v_{\frac{n+4}{2}}) = 1$, *so* $d(v_{\frac{n+2}{2}}, V_3) = d(v_{\frac{n+2}{2}}, V_4)$.
So, for $i \in \{1, \frac{n+2}{2}\}$ *we have* $d(v_i, V_3) = d(v_i, V_4)$.

□ The following two lemmas will be used to prove our main theorem.

Lemma 5. *If* $n \geq 8$ *and* $n \equiv 0$ *mod* 4 *then* $\chi_L(M_n) = 4$.

Proof. We define 4-coloring c of M_n as follows (see figure 3a).

$$\mathrm{c}(\mathrm{v}_i) = \begin{cases} 1 & ,(i \text{ odd and } i \in]1, \frac{n}{2}[) \text{ or } (i \text{ even and } i \in]\frac{n+2}{2}, n[), \\ 2 & ,(i \text{ even and } i \in]1, \frac{n}{2}[) \text{ or } (i \text{ odd and } i \in]\frac{n+2}{2}, n[), \\ 3 & ,i = 1 \text{ or } i = \frac{n}{2}, \\ 4 & ,i = \frac{n+2}{2} \text{ or } i = n. \end{cases}$$

It suffices to show that the proper coloring c is a locating coloring of M_n. Let be v_i, v_j two distinct vertices at same distance from V_3. Without loss of generality, we can assume that $i < j$. We consider three cases depending on the distance of each vertex of M_n from the color class V_3.

Case 6. $i \in]1, \frac{n}{2}[$ and $j \in]\frac{n+2}{2}, n[$.
In this case, the vertex v_i *is in inner path* P_1 *and the vertex* v_j *is in outer path* P_2. *By Lemma 5, we have* $d(v_i, V_4) = d(v_i, V_3) + 1$ *and* $d(v_j, V_4) = d(v_j, V_3) - 1$. *Using the fact that* $d(v_i, V_3) = d(v_j, V_3)$, *we have* $d(v_i, V_4) \neq d(v_j, V_4)$, *so* $c_\pi(v_i) \neq c_\pi(v_j)$.

Case 7. $i, j \in]1, \frac{n}{2}[$.
In this case, the veritces v_i, v_j are in inner path P_1. We have $j = \frac{n-2i+2}{2}$; we can easily verify that: $d(v_i, V_3) = d(v_{\frac{n-2i+2}{2}}, V_3) = i - 1$ and by Lemma 5, $d(v_i, V_4) = d(v_{\frac{n-2i+2}{2}}, V_4) = i$. Note that i and $\frac{n-2i+2}{2}$ have different parity. The vertices v_i, $v_{\frac{n-2i+2}{2}}$ must be in different color classes under c. so, we have $c_\pi(v_i) \neq c_\pi(v_{\frac{n-2i+2}{2}})$.

Case 8. $i, j \in]\frac{n+2}{2}, n[$.
In this case, the veritces v_i, v_j are in outer path P_2. We have $j = \frac{3n-2i+2}{2}$. We can easily verify that: $d(v_i, V_3) = d(v_{\frac{3n-2i+2}{2}}, V_3) = i - \frac{n}{2}$ and by Lemma 5, $d(v_i, V_4) = d(v_{\frac{3n-2i+2}{2}}, V_4) = i - \frac{n}{2} - 1$. Note that i and $\frac{3n-2i+2}{2}$ have different parity, the vertices v_i, $v_{\frac{3n-2i+2}{2}}$ must be in different color classes under c, we have $c_\pi(v_i) \neq c_\pi(v_{\frac{3n-2i+2}{2}})$.

Case 9. $i, j \in \{1, \frac{n}{2}, \frac{n+2}{2}, n\}$.
The vertex v_1 is adjacent to vertex with color 2 and the vertex $v_{\frac{n}{2}}$ is not adjacent to any vertex with color 2 thus $d(v_i, V_2) = 1$ and $d(v_{\frac{n}{2}}, V_2) \neq 1$. So we have $c_\pi(v_1) \neq c_\pi(v_{\frac{n}{2}})$.
The vertex v_n is adjacent to vertex with color 2 and $v_{\frac{n+2}{2}}$ is not adjacent to any vertex with color 2, thus $d(v_{\frac{n+2}{2}}, V_2) = 1$ and $d(v_n, V_2) \neq 1$, so we have $c_\pi(v_n) \neq c_\pi(v_{\frac{n+2}{2}})$.
The vertices v_1, v_n are adjacent, they must have different color,so we have $c_\pi(v_1) \neq c_\pi(v_n)$.
It's easy to see that $c_\pi(v_1) \neq c_\pi(v_n) \neq (v_{\frac{n}{2}}) \neq c_\pi(v_{\frac{n+2}{2}})$.

From all previous cases, we can see that all vertices in M_n have different color codes, so $\chi_L(M_n) \leq 4$. Using Theorem 2, we have $\chi_L(M_n) = 4$ when $n \equiv 0 \; mod \; 4$. So the lemma is proved. □

Lemma 6. *If $n \geq 8$ and $n \equiv 2 \; mod \; 4$ then $\chi_L(M_n) = 4$.*

Proof. We define 4-coloring c of M_n as follows (see figure 3a).

$$\mathrm{c}(\mathrm{v}_i) = \begin{cases} 1 & , i \text{ odd and } i \notin \{\frac{n}{2}, \frac{n+4}{2}\}, \\ 2 & , i \text{ even and } i \notin \{2, n\}, \\ 3 & , i \in \{2, \frac{n}{2}\}, \\ 4 & , i \in \{\frac{n+4}{2}, n\}. \end{cases}$$

It suffices to show that the proper coloring c is a locating coloring of M_n. We analyse two cases depending on the distance of each vertex of M_n

from the color class V_3. Let v_i, v_j be two distinct vertices at same distance from V_3, without loss of generality we can assume that $i < j$.

Case 10. $i \in]2, \frac{n}{2}[$ *and* $j \in]\frac{n+4}{2}, n[$.
In this case, the vertex v_i is in inner path P_1 and the vertex v_j is in outer path P_2. By lemma 6 we have $d(v_i, V_4) = d(v_i, V_3) + 1$ and $d(v_j, V_4) = d(v_j, V_3) - 1$. Using the fact that $d(v_i, V_3) = d(v_j, V_3)$, we have $d(v_i, V_4) \neq d(v_j, V_4)$, thus $c_\pi(v_i) \neq c_\pi(v_j)$.

Case 11. $i, j \in]2, \frac{n}{2}[$
In this case, the vertices v_i, v_j are in inner path P_1, and we have $j = \frac{n-2i+4}{2}$. We can verify that $d(v_i, V_3) = d(v_{\frac{n-2i+2}{2}}, V_3) = i - 2$. By lemma 4, we have $d(v_i, V_4) = d(v_{\frac{n-2i+4}{2}}, V_4) = i - 1$. Note that i and $\frac{n-2i+4}{2}$ have different parity, the vertices v_i, $v_{\frac{n-2i+4}{2}}$ must be in different color classes under c. So we have $c_\pi(v_i) \neq c_\pi(v_{\frac{n-2i+4}{2}})$.

Case 12. $i, j \in]\frac{n+4}{2}, n[$.
In this case, the veritces v_i, v_j are in outer path P_2, and we have $j = \frac{3n-2i+4}{2}$. We can easily verify that $d(v_i, V_3) = d(v_{\frac{3n-2i+4}{2}}, V_3) = \frac{2i-n-2}{2}$ By lemma 6 $d(v_i, V_4) = d(v_{\frac{3n-2i+4}{2}}, V_4) = \frac{2i-n-4}{2}$. Note that i and $\frac{3n-2i+4}{2}$ have different parity, the vertices v_i, $v_{\frac{3n-2i+4}{2}}$ must be in different color classes under c. So we have $c_\pi(v_i) \neq c_\pi(v_{\frac{3n-2i+4}{2}})$.

Case 13. $i, j \in \{2, \frac{n}{2}, \frac{n+4}{2}, n\}$.
The vertex v_2 is adjacent to vertex with color 1 and the vertex $v_{\frac{n}{2}}$ is not adjacent to any vertex with color 1, thus $d(v_1, V_1) = 1$ and $d(v_{\frac{n}{2}}, V_1) \neq 1$. So we have $c_\pi(v_1) \neq c_\pi(v_{\frac{n}{2}})$.
The vertex v_n is adjacent to vertex with color 1 and $v_{\frac{n+4}{2}}$ is not adjacent to any vertex with color 1, thus $d(v_{\frac{n+4}{2}}, V_1) \neq 1$ and $d(v_n, V_1) = 1$. So we have $c_\pi(v_n) \neq c_\pi(v_{\frac{n+4}{2}})$.
The vertices v_2, $v_{\frac{n+4}{2}}$ are adjacent; they must have different color, so we have $c_\pi(v_2) \neq c_\pi(v_{\frac{n+4}{2}})$.
It's easy to deduce that $c_\pi(v_2) \neq c_\pi(v_n) \neq (v_{\frac{n}{2}}) \neq c_\pi(v_{\frac{n+4}{2}})$.

Case 14. $i, j \in \{1, \frac{n+2}{2}\}$.
The vertices v_1, $v_{\frac{n+2}{2}}$ are adjacent, they must have different color, $c_\pi(v_1) \neq c_\pi(v_{\frac{n+2}{2}})$.

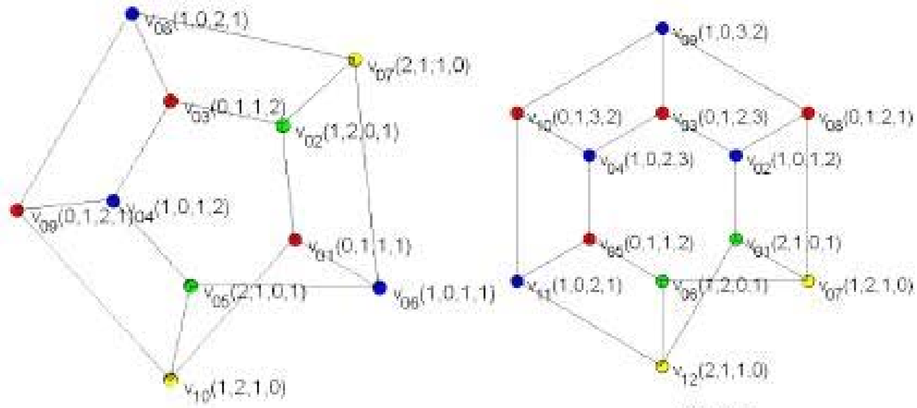
From all cases, we can see that all vertices in M_n have different color codes, so $\chi_L(M_n) \leq 4$. By using Theorem 2, we have $\chi_L(M_n) = 4$ when $n \equiv 2 \ mod \ 4$. So the lemma is proved. □

Combining Lemma 1, Lemma 5 and Lemma 6, we get the following theorem.

Theorem 7. *If $n \geq 6$ and n is even then*

$$\chi_L(M_n) = \begin{cases} 4 & n \neq 6 \\ 6 & n = 6 \end{cases}$$

Proof. For $n \geq 8$, we consider two cases depending on the parity of $\frac{n}{2}$.
If $\frac{n}{2}$ is even then $n \equiv 0 \ mod \ 4$. By lemma 5, we have $\chi_L(M_n) = 4$.
If $\frac{n}{2}$ is odd then $n \equiv 2 \ mod \ 4$ and, by lemma 6, we have $\chi_L(M_n) = 4$.
Then, again by lemma 1, we have $\chi_L(M_6) = 6$. So, the theorem is proved. □



(a) M_{10} (b) M_{12}

Figure 3: 4-Locating Coloring of M_{10} and M_{12}

Conclusion

We considered the Möbius ladders graphs M_n and we proved that the locating chromatic number of M_n with n even is 4 if $n = 6$ and 6 if $n = 6$. In the future, we will study the Möbius ladders graphs M_n for n odd and consider more families of graph in the context of locating chromatic number.

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