On independent position sets in graphs

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Abstract:

An independent set S of vertices in a graph G is an independent position set if no three vertices of S lie on a common geodesic. An independent position set of maximum size is an ip-set of G . The cardinality of an ip-set is the independent position number, denoted by $ip(G)$. In this paper, we introduce and study the independent position number of a graph. Certain general properties of these concepts are discussed. Graphs of order n having the independent position number 1 or $n - 1$ are characterized. Bounds for the independent position number of Cartesian and Lexicographic product graphs are determined and the exact value for Corona product graphs are obtained. Finally, some realization results are proved to show that there is no general relationship between independent position sets and other related graph invariants.

Keywords: General position set; Independent set; Independent number; Independent position number.

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1. Introduction

The general position problem in a graph G is to find the maximum size of a set S of vertices in G such that no three vertices of S lie on a common shortest path. The general position problem in graph theory was introduced and studied in [1] after getting inspiration from the century old Dudeney's no-three-in-line problem [2] and by the general position subset selection problem [3,4] from discrete geometry. But the same concept has already been studied two years earlier in [5] under the name *geodetic irredundant sets*. Refer [6,7,8,9,10,11] to understand the recent developments on general position number.

Graphs in this paper are finite, undirected and simple. The *distance* between two vertices u and v in a connected graph G is the length of a shortest u, v -path in G , denoted as $d(u, v)$. A u, v -*geodesic* is a u, v -path of length $d(u, v)$. Obviously the distance is a metric on the vertex set V . The maximum distance between all pairs of vertices of G is the diameter, $diam(G)$ of G . The *interval* $I_G[u, v]$ between vertices u and v of a graph G is the set of vertices x such that there exists a u, v -geodesic which contains x . For a positive integer n , let $[n] = 1, \dots, n$.

A set of vertices $S \subseteq V(G)$ is a *general position set* of G if no three vertices of S lie on a common geodesic. A gp-set is thus a largest general position set. The *general position number* (*gp-number* for short) of G is the cardinality of a gp-set of G . An independent set or a stable set S , is a set of vertices in a graph such that no two vertices of S are adjacent. The vertex independence number of a graph (or simply, independence number) is the cardinality of the largest independent vertex set. The independence number is most commonly denoted by $\alpha(G)$.

In this paper, we investigate the properties of a graph invariant named independent position number which admits both the properties of general position set and independence set. In the following section definitions and preliminary observations are listed. In Section 3, we prove some bounds for independent position number on Cartesian product, lexicographic product and an exact value for corona product of graphs. In section 4, we prove some realization results which shows that the independent position number is different from other related invariants.

In this paper, we make use of the following results.

Theorem 1.1. [5] *For a connected graph G of order n and diameter d , $gp(G) \leq n - d + 1$.*

Theorem 1.2. [5] For a connected graph G , $gp(G) = 2$ if and only if $G = P_n$ or $G = C_4$.

Theorem 1.3. [5] For any tree T with k end vertices, $gp(T) = k$.

2. Independent general position sets in graphs

A set S of vertices in a graph G is considered to be an independent position set if S is both an independent set and a general position set in G . An independent position set of maximum size is called an *ip*-set of G . The cardinality of an *ip*-set is called the independent position number, denoted by $ip(G)$. It is clear that every independent position set is a general position set and so we have the following observation.

Observation 2.1. For any connected graph G of order n , $1 \leq ip(G) \leq gp(G) \leq n$.

Example 2.2. For the graph G in Figure 1, the set $S = \{v_1, v_2, v_3, v_4\}$ is a maximum general position set of G . Then $gp(G) = 4$. On the other hand, any independent position set of G contains at most one vertex from the set $\{v_1, v_2, v_3, v_4\}$ and so $ip(G) \leq 3$. Now, since the set $T = \{v_5, v_1, v_6\}$ is an independent position set, it follows that $ip(G) = 3$.

Figure 1

Proposition 2.3. For a connected graph G of order n and diameter d , $ip(G) \leq n - d + 1$.

Proof. This follows from Theorem 1.1 and Observation 2.1. ■

Proposition 2.4. . For any connected graph G of order n , $1 \leq ip(G) \leq \alpha(G)$.

Proof. The left inequality is trivial and the right inequality follows from the fact that every independent position set is an independent set. The independent position number can be significantly small while compared to the independent number. To certain instant, consider the path $P_n (n \geq 5)$. Then it is clear from Theorem 1.2 and Observation 2.1 that $ip(G) = 2$, where as $\alpha(G) = \lceil \frac{n}{2} \rceil$. ■

Theorem 2.5. Let G be a connected graph with diameter atmost 3. Then $ip(G) = \alpha(G)$.

Proof. By Proposition 2.4, it remains to prove that $ip(G) \geq \alpha(G)$. For, let S be a maximum independent set of G . Assume the contrary that there exists three vertices x, y and z in S such that $y \in I_G[x, z]$. Let $P : x = x_0, x_1, \dots, x_i = y, x_{i+1}, \dots, x_n = z$ be an x, z - geodesic in G containing the vertex y . Now, since S is an independent set, it follows that $2 \leq i \leq n - 2$. This shows that $d_G(x, z) \geq 4$, a contradiction to the fact that $diam(G) \leq 3$. Hence S must be a general position set of G and so $ip(G) \geq |S| = \alpha(G)$. ■

The converse need not be true. Consider the graph in Figure 2. Then $diam(G) = 4$ whereas, the set $\{v_2, v_3, v_5, v_6\}$ is a maximum independent set which is also a general position set. Hence $ip(G) = 4 = \alpha(G)$.

Figure 2

In view of Theorem 2.5, we have the following open problem.

Problem 2.6. Characterize the classes of graphs in which $ip(G) = \alpha(G)$.

Theorem 2.7. For any tree T with k end vertices, $ip(T) = k$.

Proof. Since the set of all end vertices of T is an independent position set, it follows that $ip(T) \geq k$. Now, it follows from Theorem 1.3 and Observation 2.1 that $ip(T) \leq gp(T) = k$. ■

Theorem 2.8. Let G be a connected graph of order n . Then

1. $ip(G) = 1$ if and only if $G \cong K_n$.
2. $ip(G) = n - 1$ if and only if $G \cong K_{1,n-1}$.

Proof.

1. If $G \cong K_n$, then it is obvious that $ip(G) = 1$. On the other hand, let $ip(G) = 1$. If G is non-complete, then there exists vertices u and v in G such that $u, v \notin E(G)$. Then $\{u, v\}$ is an independent position set and so $ip(G) \geq 2$, impossible. Hence $G \cong K_n$.
2. First, suppose that $ip(G) = n - 1$. Then it follows from Proposition 2.3 that $diam(G) \leq 2$. Since, the complete graph K_n has independent position number 1, it follows that $diam(G) = 2$. Let $S = \{v_1, v_2, \dots, v_{n-1}\}$ be an independent set of size $n - 1$ and $v \in V(G)$ such that $v \notin S$. Now, since S is an independent set and G is connected, we have that v must be adjacent to all the vertices of S . Then $G \cong K_{1,n-1}$. On the other hand, if $G \cong K_{1,n-1}$ then it is evident that $ip(G) = n - 1$. ■

Theorem 2.9. If G is a bipartite graph, then $ip(G) = gp(G)$.

Proof. If $G = P_n$ or $G = C_4$, then one can easily verify that $ip(G) = gp(G) = 2$. So in the following we may assume that G is neither a path nor a C_4 , so that $gp(G) \geq 3$. Now, let S be a maximum general position set of G . Let $S_1, S_2, \dots, S_k$ ($k \geq 1$) be the components of the induced subgraph of S in G . Since G is bipartite and each S_i ($1 \leq i \leq k$) induces a clique, it follows that $|S_i| \leq 2$ for all $i \in [k]$. This shows that $k \geq 2$. In the following, we claim that $|S_i| = 1$ for all $i \in [k]$. Assume the contrary that S_1 contains two vertices, say u and v . Since S_1 induces a clique in G , we have that u and v are adjacent in G . Let w be any vertex in S_2 . Since S is a general position

set, it follows that $d(u, w) = d(v, w)$. Now, let $P : w = x_1, x_2, \dots, x_r = u$ be a w, u -geodesic in G and let $Q : w = y_1, y_2, \dots, y_r = v$ be a w, v -geodesic in G . Let t be the largest suffix such that $x_t \in V(Q)$. Since $d(u, w) = d(v, w)$, it follows that $x_t = y_t$ and hence the x_t, u -subpath of P and y_t, v -subpath of Q together with the edge uv forms an odd cycle in G . This is impossible in a bipartite graph. This shows that $|S_i| = 1$ for all $i \in [k]$. Hence S is an independent general position set in G and so $ip(G) \geq |S| = gp(G)$. Now, Observation 2.1 implies that $ip(G) = gp(G)$. ■

The converse of the above theorem need not be true. For, consider the non-bipartite graph shown in the Figure 3. Then the set $S = \{u_2, u_3, u_5, u_6\}$ is an independent position set in G . On the other hand, any general position set of G contains at most two vertices from each component of $G \setminus u_4$. This shows that S is a maximum independent position set as well as a maximum general position set of G . Thus $gp(G) = ip(G)$.

Figure 3

3. Independent position number on some graph operations

In the following section, we determine some bounds of independent position number on cartesian product, lexicographic product and an exact value for corona product of graphs. For more on the product graphs see [12].

3.1. Cartesian products

The Cartesian product $G \square H$ of graphs G and H has the vertex set $V(G) \times V(H)$ and the edge set $E(G \square H) = \{(g, h)(g', h') : gg' \in E(G) \text{ and } h = h', \text{ or, } g = g' \text{ and } hh' \in E(H)\}$. If $(g, h) \in V(G \square H)$, then the G -layer G^h through the vertex (g, h) is the subgraph of $G \square H$ induced by the vertices $(g', h) : g' \in V(G)$. Similarly, the H -layer H^g through (g, h) is the subgraph of $G \square H$ induced by the vertices $(g, h') : h' \in V(H)$. It is well-known that for given vertices $u = (g_1, h_1)$ and $v = (g_2, h_2)$ of $G \square H$ we have $d_{G \square H}(u, v) = d_G(g_1, g_2) + d_H(h_1, h_2)$.

The announced lower bound reads as follows.

Theorem 3.1. *For any connected graphs G and H , $ip(G \square H) \geq ip(G) + ip(H) - 2$.*

Proof. Let S and T be the maximum independent position sets of G and H respectively. Let $g \in S$ and $h \in T$ and $S_1 = \{(x, h) | x \in S\}$ and $T_1 = \{(g, y) | y \in T\}$. We claim that $W = (S_1 \cup T_1) \setminus \{(g, h)\}$ is an independent position set in $G \square H$. First, we prove that W is an independent set in $G \square H$. Since S and T are independent set in G and H respectively, it follows that both S_1 and T_1 are independent sets in $G \square H$. Let $(x, h) \in S_1$ and $(g, y) \in T_1$. Then it is clear that $x \neq g$ and $y \neq h$ and so by definition it follows that (x, h) is not adjacent to (g, y) . Thus there is no adjacency between the vertices of S_1 and T_1 in $G \square H$. This shows that W is an independent set in $G \square H$. Next, we claim that W is a general position set in $G \square H$. Since layers are convex sets in $G \square H$, it follows that both S_1 and T_1 are general position sets in $G \square H$. Now, suppose there exist vertices $(x, h), (u, v), (g, y) \in W$ such that $(u, v) \in I_{G \square H}[(x, h), (g, y)]$. Without loss of generality, we may assume that $(u, v) \in S_1$. Then $(u, v) \neq (g, h)$ and $v = h$. Let $P : (x, h) = (x_1, y_1), (x_2, y_2), \dots, (x_i, y_i) = (u, h), (x_{i+1}, y_{i+1}), \dots, (x_n, y_n) = (g, y)$ geodesic in $G \square H$ containing the vertex (u, v) . Since G -layer G^h is a convex set in $G \square H$, it follows that $y_1 = y_2 = \dots = y_i = h$. Thus $x_j \neq g$ for all $j = [i]$. Let k be the largest suffix such that $y_k = h$. Thus $x_{k+1} = x_{k+2} \dots = x_n = g$. This shows that $x = x_1, x_2, \dots, x_i = u, \dots, x_n = g$ is a x, g -geodesic in G containing the vertex u with $x, u, g \in S$. This is a contradiction to the fact that S is a general position set. Thus W must be a general position set in $G \square H$ and $ip(G \square H) \geq |W| = ip(G) + ip(H) - 2$. ■

3.2. Lexicographic products

The lexicographic product $G[H]$ of two graphs G and H has vertex set $V(G) \times V(H)$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent whenever $u_1 u_2 \in E(G)$, or $u_1 = u_2$ and $v_1 v_2 \in E(H)$. To prove the bound on Lexicographic products, we may make use of the following Lemma.

Lemma 3.2. [13] *Let G and H be graphs. Then*

1. *The graph $G[H]$ is connected if and only if G is connected.*
2. *The lexicographic product is associative but not commutative.*
3. *If G is connected, then $d_{G[H]}((g, h), (g', h')) = d_G(g, g')$ if $g \neq g'$
 $d_{G[H]}((g, h), (g, h')) = 2$ if $hh' \notin E(H)$
 $d_{G[H]}((g, h), (g, h')) = 1$ if $hh' \in E(H)$*

Theorem 3.3. *For any connected graphs G and H , $ip(G[H]) \geq ip(G)ip(H)$.*

Proof. Let S be an ip -set of G and T be an ip -set of H . We first show that $S \times T$ is an independent position set in $G[H]$. For, suppose that the vertices (g_1, h_1) and (g_2, h_2) in $S \times T$ are adjacent in $G[H]$, then we have either $g_1 g_2 \in E(G)$ or $h_1 h_2 \in E(H)$. This is impossible. Thus $S \times T$ is an independent set in $G[H]$. Next, we claim that $S \times T$ is a general position set in $G[H]$. Assume on contrary that there exist $(g_1, h_1), (g_2, h_2), (g_3, h_3) \in S \times T$ such that $(g_2, h_2) \subseteq I_{G[H]}[(g_1, h_1), (g_3, h_3)]$. First, suppose that $(g_1, h_1), (g_3, h_3)$ belongs to the same H -layer H^g . Thus $g_1 = g_3 = g$ and $d_{G[H]}[(g, h_1), (g, h_3)] = 2$. In this case (g_2, h_2) is adjacent with both (g_1, h_1) and (g_3, h_3) , a contradiction to the fact that $S \times T$ is an independent set. Hence $g_1 \neq g_3$. Let $(g_1, h_1) = (x_1, y_1), (x_2, y_2), \dots, (x_i, y_i) = (g_2, h_2), \dots, (x_n, y_n) = (g_3, h_3)$ be a $(g_1, h_1), (g_3, h_3)$ - geodesic contains the vertex (g_2, h_2) . Then by Lemma 3.2 $d_G(g_1, g_3) = d_{G[H]}[(g_1, h_1), (g_3, h_3)]$ and so $g_1 = x_1, x_2, \dots, x_i = g_2, \dots, x_n = g_3$ is a g_1, g_3 -geodesic in G containing the vertex g_2 . This is a contradiction. Hence $S \times T$ is an independent position set in $G[H]$. So $ip(G[H]) \geq |S \times T| = ip(G)ip(H)$. ■

The bound is sharp. Consider the graph $C_4[K_2]$ shown in the Figure 4.

Figure 4

Clearly from the Figure 4, we can choose atmost two vertices v_1, v_3 or v_2, v_4 or u_1, u_3 or u_2, u_4 from the every layer. If we choose only one vertex from first layer say, v_1 then the possible choice of vertex from the second layer is u_3 only, since v_1 is adjacent to v_2, v_4, u_1, u_2, u_4 . Therefore, $ip(C_4[K_2]) = |S \times T| = 2 = |S| \times |T| = ip(C_4)ip(K_2)$.

3.3. Corona of graphs

Let G and H be graphs where $V(G) = \{v_1, \dots, v_{n(G)}\}$. The corona $G \circ H$ of graphs G and H is obtained from the disjoint union of G and $n(G)$ disjoint copies of H , say $H_1, \dots, H_{n(G)}$, where for all $i \in [n(G)]$, the vertex $v_i \in V(G)$ is adjacent to each vertex of H_i .

Theorem 3.4. For any graphs G and H , $ip(G \circ H) = n(G)\alpha(H)$.

Proof. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and let $H_1, H_2, \dots, H_n$ be the corresponding copies of H in $G \circ H$. For each $i \in [n]$, let S_i denote the maximum independent set in H . Then the set $S = S_1 \cup S_2 \dots \cup S_n$ is an independent position set in $G \circ H$. Hence $ip(G \circ H) \geq |S| = n(G)\alpha(H)$. On the other hand, let T be the maximum general position set in $G \circ H$. Then $|T| \geq n(G)\alpha(H)$.

Claim: $T \subseteq \bigcup_{i=1}^n H_i$.

Suppose that there exists a vertex $v_i \in T$ for some i with $1 \leq i \leq n$. Then since T is an independent set, we have that $V(H_i) \cap T = \emptyset$. Moreover, the set $(T \setminus \{v_i\}) \cup S_i$ is an independent position set of size atleast the size of T . Hence without loss of generality, we may assume that $T \subseteq \bigcup_{i=1}^n H_i$. Moreover, since T is an independent set, we have that $T \cap H_i$ is also an independent subset of H_i . Then $|T \cap H_i| \leq \alpha(H_i)$. Hence $|T| = \sum_{i=1}^n |T \cap H_i| \leq \sum_{i=1}^n \alpha(H_i) \leq n(G)\alpha(H)$. This shows that $ip(G \circ H) = n(G)\alpha(H)$. ■

4. Some Realization Results

Here, we prove some realization results which shows that the independent position number is different from other related invariants.

Theorem 4.1. *For integers k and n with $1 \leq k < n$, there is a connected graph G of order n such that $ip(G) = k$.*

Proof. If $k = 1$ or $k = n - 1$, obviously the graphs K_n and $K_{1,n-1}$ respectively having the desired properties. If $k = n - 2$, then it follows from Theorem 2.7 that $ip(T) = n - 2$ for any tree T with diameter 3. So, assume that $2 \leq k \leq n - 3$. Let G be the graph obtained from the cycle $C_{(n-k+2)} : v_1, v_2, \dots, v_{(n-k+2)}, v_1$ by adding $k - 2$ new vertices $u_1, u_2, \dots, u_{(k-2)}$ and joining each u_i to both v_1 and $v_{(n-k+2)}$. The graph G is shown in the Figure 5.

Figure 5

From the graph G in Figure 5, it is clear that $|G| = n - k + 2 + k - 2 = n$. We claim that $ip(G) = k$, it is clear that any independent position set contains at most two vertices from the cycle. Thus $ip(G) \leq n - (n - k + 2) + 2 = k$. Now, on the other hand, the set $M = \{u_1, u_2, \dots, u_{(k-2)}, v_i, v_m\}$ with $i, m \in \{1, n - k + 2\}$ is an ip -set in G , we have that $ip(G) = k$. ■

Theorem 4.2. For every pair of positive integers a, b with $2 \leq a \leq b < n$, there exists a connected graph G with $ip(G) = a$ and $gp(G) = b$.

Proof. We consider the following two cases.

Case 1. $a = b$. In this case, the graph G of order n with desired properties is constructed as follows. Let G be the graph obtained from the path $P : v_1, v_2, \dots, v_{(n-a+2)}$ by adding $(a - 2)$ new vertices $\{u_1, u_2, \dots, u_{(a-2)}\}$ and joining each u_i to the vertex v_3 . The graph G is shown in the Figure 6. Then, from Theorem 2.7 and Theorem 1.3 we get that $ip(G) = gp(G) = a$.

Figure 6

Case 2. $2 < a < b < n$

Let H be the graph obtained from the complete graph $K_{(b-a+2)} : w_1, w_2, \dots, w_{(b-a+2)}$ and a vertex disjoint path $P : u_0, u_1, u_2, \dots, u_{(n-b-1)}$ by identifying the vertices w_1 and u_0 . Let G the graph obtained from the graph H by adding $a - 1$ new vertices $v_1, v_2, \dots, v_{(a-1)}$ and join each v_i to the vertex $u_{(n-b-1)}$. The graph G of order n is shown in the Figure 7.

Figure 7

Let S be a maximum general position set of G . Then S contains at most two vertices from the set $\{w_1, u_1, u_2, \dots, u_{(n-b-1)}, v_1\}$. This shows that $|S| \leq b$. Now, on the other hand, since the set $\{w_2, w_3, \dots, w_{(b-a+2)}, v_1, v_2, \dots, v_{(a-1)}\}$ is a maximum general position set in G , it follows that $gp(G) = |S| = a - 1 + (b - a + 1) = b$.

Next, we prove that $ip(G) = a$. Let T be any independent position set. Then T contains at most one vertex from the the set $\{w_1, w_2, \dots, w_{(b-a+2)}\}$. Also, T contains at most two vertices from the set $\{w_1, u_1, u_2, \dots, u_{(n-b-1)}, v_1\}$. Hence $|S| \leq a$. Now, since the set $\{w_1, v_1, v_2, \dots, v_{(a-1)}\}$ is an independent position set, we have that $ip(G) = a$. ■

Theorem 4.3. For every pair of positive integers a, b with $2 \leq a \leq b$, there exists a connected graph G with $ip(G) = a$ and $\alpha(G) = b$.

Proof. We consider the following two cases.

Case 1. $a = b$. Then the the complete bipartite graph $K_{a,a}$ has the required properties.

Case 2. $2 < a < b$.

Let G be the graph obtained from the cycle $C_{2(b-a+2)} : v_1, v_2, \dots, v_{2(b-a+2)}$ by adding $a - 2$ new vertices $u_1, u_2, \dots, u_{(a-2)}$ and joining each u_i to the vertex v_1 . The graph G thus obtained is shown in the Figure 8.

Figure 8

Let I be a maximum independent set in G . Then I contains at most $b-a+2$, vertices form the cycle $C_{2(b-a+2)}$. If $v_1 \in I$, then $|I| = b-a+2 \leq b$. So assume that $v_1 \notin I$. Then $|I| \leq b$. On the other hand, since the set $\{v_2, v_4, \dots, v_{2(b-a+2)}, u_1, u_2, \dots, u_{(a-2)}\}$ is an independent set of size b , we have that $|I| = \alpha(G) = b-a+2+a-2 = b$.

Next, we prove that $ip(G) = a$. Note that any independent position set S contains at most two vertices from the cycle $C_{2(b-a+2)}$. This shows that $ip(G) \leq a$. Now, since the set $\{v_{2(b-a+2)}, v_2, u_1, u_2, \dots, u_{(a-2)}\}$ is an independent position set of size a , we have that $ip(G) = |S| = 2+a-2 = a$. ■

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