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Some pairwise weakly fuzzy mappings

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Abstract:

The aim of this paper is to introduce some pairwise weakly fuzzy mappings, called pairwise weakly fuzzy δ-semi-pre-continuous mappings and pairwise weakly fuzzy δ-semi pre-open mappings in fuzzy bitopological spaces. The concept of pairwise weakly fuzzy δ-semi-precontinuous mappings is to be introduced in fuzzy bitopological spaces with the help of the concept of (i, j)-fuzzy pre-open and (i, j)-fuzzy δ-semi pre-open set. Then some of its basic properties and characterization theorems are to be investigated. Also the notion of pairwise weakly fuzzy δ-semi-pre-open mappings is to be introduced in fuzzy bitopological spaces with the help of the concept of (i, j)-fuzzy open set and (i, j)-fuzzy δ-semi pre-interior. Some of its basic properties and its relationship with other known mappings are also to be studied.

Keywords: Fuzzy bitopological space; Pairwise weakly fuzzy; Fuzzy δ-semi pre-continuous; Fuzzy δ-semi pre-open mappings.

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1. Introduction

Zadeh [25] introduced the fundamental concept of fuzzy sets in his classical paper. Chang [4] introduced the concept of fuzzy topological spaces as a generalization of topological spaces. Since then many topologists have contributed to the theory of fuzzy topological spaces. Thakur and Singh [21] introduced the concept of fuzzy semi pre-open sets and fuzzy semi pre-continuity. Ganguly and Saha [7] introduced the concept of δ-continuity and δ-connected set in fuzzy set theory. The concept of weakly fuzzy δ-semi-pre-continuous mappings and pairwise weakly fuzzy δ-semi pre-open mappings in fuzzy topological spaces were studied by Mukherjee and Dhar [12, 13]. Dutta and Tripathy [6] introduced fuzzy b-θ-open sets in fuzzy topological spaces. Also Tripathy and Ray [22, 23] introduced δ-continuity and weakly continuous functions on mixed fuzzy topological spaces. Dubey, Panwar and Tiwari [5] introduced weakly pairwise irresolute mappings. Kandil [8] introduced and studied the notion of fuzzy bitopological spaces (a system (X, τ_1, τ_2) consisting of a non-empty set X with two arbitrary topologies τ_1 and τ_2 on X is called a fuzzy bitopological space) as a natural generalization of fuzzy topological spaces. Also D. Sarma and B.C. Tripathy [17] and B.C. Tripathy and S. Debnath [24] introduced and studied different concepts in bitopological spaces. In this paper, the concept of weakly fuzzy δ-semi- pre-continuous mappings and pairwise weakly fuzzy δ-semi pre-open mappings in fuzzy bitopological spaces are to be introduced. Throughout the present study, the spaces X, Y and Z always represent fuzzy bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, Γ_1, Γ_2) respectively. As to the notations and terminologies $cl(A)$ and $int(A)$ to be denote the closure of A and the interior of A, respectively in a fuzzy topological space (X, τ). Also $\tau_i - int(\lambda)$ and $\tau_j - cl(\lambda)$ to be mean respectively the interior and closure of a fuzzy set λ with respect to the fuzzy topologies τ_i and τ_j in a fuzzy bitopological space (X, τ_1, τ_2).

2. Preliminaries and Definitions

In this section, some preliminary results and definitions have been procured.

Definition 2.1. *[25] Let X be a crisp set and A and B be two fuzzy subsets of X with membership functions μ_A and μ_B respectively. Then*
(a) A is equal to B, i.e., $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, for all $x \in X$,
(b) A is called a subset of B if and only if $\mu_A(x) \le \mu_B(x)$, for all $x \in X$,
(c) the Union of two fuzzy sets A and B is denoted by $A \vee B$ and its

membership function is given by $\mu_{A\vee B} = \max(\mu_A, \mu_B)$,
(d) the Intersection of two fuzzy sets A *and* B *is denoted by* $A \wedge B$ *and its membership function is given by* $\mu_{A\wedge B} = \min(\mu_A, \mu_B)$,
(e) the Complement of a fuzzy set A *is defined as the negation of the specified membership function. Symbolically it can be written as* $\mu_A^c = 1 - \mu_A$.

Definition 2.2. *[16] A fuzzy point* x_p *in* X *is a fuzzy set in* X *defined by*

$$x_p(y) = \begin{cases} p \quad (0 < p \leq 1), & \text{for } y = x; \\ 0, & \text{for } y \neq x (y \in X). \end{cases}$$

x *and* p *are respectively the support and the value of* x_p.

A fuzzy point x_p is said to belong to a fuzzy set A of X if and only if $p \leq A(x)$. A fuzzy set A in X is the union of all fuzzy points which belong to A.

Definition 2.3. *[3] Let* A *be a fuzzy subset of a fuzzy topological space* (X, τ). *Then* A *is called fuzzy* δ*-pre-open if* $A \leq int(\delta cl(A))$. *The complement of a fuzzy* δ*-pre-open set is called fuzzy* δ*-pre-closed .*

Definition 2.4. *A fuzzy subset* λ *in a fuzzy topological space* X *is called*
(a) [1] fuzzy semi-open if $\lambda \leq cl(int(\lambda))$
(b) [2] fuzzy pre-open if $\lambda \leq int(cl(int(\lambda)))$
(c) [21] fuzzy semi pre-open if there exists a fuzzy pre-open set μ *such that* $\mu \leq \lambda \leq cl(\mu)$.

Definition 2.5. *[18] A fuzzy subset* γ *in* (X, τ) *is said to be fuzzy* δ*-semi pre-open if there exists a fuzzy* δ*-pre-open set* μ *such that* $\mu \leq \gamma \leq \delta - cl(\mu)$ *or equivalently* $\gamma \leq \delta - cl(int\ \delta - cl(\gamma))$.

Definition 2.6. *[1] A fuzzy subset* A *of a fuzzy topological space* (X, τ) *is called*
(a) a fuzzy regular open set of (X, τ) *if* $int\ cl(A) = A$ *and*
(b) a fuzzy regular closed set of (X, τ) *if* $cl\ int(A) = A$.

Definition 2.7. *[14] A fuzzy subset* A *of a fuzzy topological space* (X, τ) *is said to be fuzzy* δ*-semi-open if* $A \leq clint_\delta(A)$.

Definition 2.8. *[11] A mapping $f : (X, \tau) \to (Y, \sigma)$ from a fuzzy topological space (X, τ) to another fuzzy topological space (Y, σ) is said to be weakly fuzzy δ-semi-pre-continuous mapping if $f^{-1}(\alpha) \in f\delta spo(X)$ for each $\alpha \in fpo(Y)$, where $f\delta spo(X)$ (respectively $fpo(X)$) denotes the family of all fuzzy δ-semi pre-open (respectively fuzzy pre-open) sets of X.*

Definition 2.9. *[12] The function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly fuzzy δ-semi pre-open if $f(\lambda) \leq \delta spint(fpcl\lambda)$ for each fuzzy open set λ of X.*

Definition 2.10. *[12] A function $f : (X, \tau) \to (Y, \sigma)$ is said to be weakly fuzzy δ-semi pre-closed if $\delta spcl(f(pint(\lambda))) \leq f(\lambda)$ for each fuzzy pre-closed subset λ of X.*

Definition 2.11. *[10] Let (X, τ_1, τ_2) be a fuzzy bitopological space. The (i, j) - fuzzy semi-closure (denoted by (i, j)-$scl(A)$) and (i, j)-fuzzy semi-interior (denoted by (i, j)-$sint(A)$) of a fuzzy set A in (X, τ_1, τ_2) are defined respectively as follows :*
(a) (i, j)-$scl(A) = inf\{B : B \geq A, B$ is (i, j)-fuzzy semi-closed $\}$,
(b) (i, j)-$sint(A) = sup\{B : B \leq A, B$ is (i, j)-fuzzy semi-open $\}$.

Definition 2.12. *Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a mapping from a fuzzy bitopological space (X, τ_1, τ_2) to another fuzzy bitopological space (Y, σ_1, σ_2). Then f is called*
(a) [19] pairwise fuzzy continuous (respectively pairwise fuzzy open) if $f : (X, \tau_1) \to (Y, \sigma_1)$ and $f : (X, \tau_2) \to (Y, \sigma_2)$ are fuzzy continuous (respectively fuzzy open),
(b) [20] pairwise fuzzy irresolute if the inverse image of each (i, j)-fuzzy semi-open set in Y is (i, j)-fuzzy semi-open set in X, $i \neq j$, $i, j = 1, 2$,
(c) [10] pairwise fuzzy semi-continuous if the inverse image of each σ_i-fuzzy open set in Y is (i, j)-fuzzy semi-open set in X, $i \neq j$,$i, j = 1, 2$,
(d) [10] pairwise fuzzy semi-open if the image of every τ_i-fuzzy open set in X is (i, j)-fuzzy semi-open set in Y, $i \neq j$, $i, j = 1, 2$,
(e) [9] pairwise fuzzy pre-continuous if the inverse image of each σ_i-fuzzy open set in Y is (i, j)-fuzzy pre-open set in X, $i \neq j$, $i, j = 1, 2$,
(f) [9] pairwise fuzzy pre-open if the image of every τ_i-fuzzy open set in X is (i, j)-fuzzy pre-open set in Y, $i \neq j$,$i, j = 1, 2$,
(g) [15] pairwise fuzzy semi-pre-continuous if the inverse image of each σ_i-fuzzy open set in Y is (i, j)-fuzzy semi pre-open set in X, $i \neq j$,$i, j = 1, 2$,
(h) [15] pairwise fuzzy semi pre-open if the image of every τ_i-fuzzy open set in X is (i, j)-fuzzy semi pre-open set in Y, $i \neq j$,$i, j = 1, 2$.

Definition 2.13. *[13] A subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be (i, j)-fuzzy δ-semi-open if $A \leq \tau_j - cl(\tau_i - int_\delta(A))$. The complement of (i, j)-fuzzy δ-semi-open set is called (i, j)-fuzzy δ-semi-closed.*

Definition 2.14. *[13] Let A be a subset of a fuzzy bitopological space (X, τ_1, τ_2). Then*
(a) the intersection of all (i, j)-fuzzy δ-semi-closed sets containing A is called the (i, j)-fuzzy δ-semi-closure of A and is denoted by (i, j)-$scl_\delta(A)$,
(b) the union of all (i, j)-fuzzy δ-semi-open sets contained in A is called the (i, j)-fuzzy δ-semi-interior of A and is denoted by (i, j)-$sint_\delta(A)$.

Definition 2.15. *[13] A subset A of a fuzzy bitopological space (X, τ_1, τ_2) is said to be (i, j)-fuzzy δ-semi pre-open if $A \leq ((i, j)$ -$spint_\delta(i, j)$-$spcl_\delta(A))$. The complement of (i, j)-fuzzy δ-semi pre-open set is called (i, j)-fuzzy δ-semi pre-closed.*

Definition 2.16. *[13] Let A be a subset of a fuzzy bitopological space (X, τ_1, τ_2). Then*
(a) the intersection of all (i, j)-fuzzy δ-semi pre-closed sets containing A is called the (i, j)-fuzzy δ-semi pre-closure of A and is denoted by (i, j)-$spcl_\delta(A)$,
(b) the union of all (i, j)-fuzzy δ-semi pre-open sets contained in A is called the (i, j)-fuzzy δ-semi pre-interior of A and is denoted by (i, j)-$spint_\delta(A)$.

3. Pairwise weakly fuzzy δ - semi precontinuous mappings

In this section the concept of weakly fuzzy δ-semi-pre-continuous mapping is introduced. Some characterization theorems and its basic properties are studied.

Definition 3.1. *For any two fuzzy bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2), a mapping $f : X \to Y$ is said to be pairwise weakly fuzzy δ-semi-pre-continuous if for each x_p in X and each (i, j)-fuzzy-δ-semi pre-open set V containing $f(x_p)$, there is an (i, j)-fuzzy pre-open set U in X such that $x_p \in U$ and $f(U) \leq (i, j)$-$spcl_\delta(V)$, $i \neq j$ and $i, j = 1, 2$.*

Theorem 3.2. *For any mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following conditions are equivalent:*
(i) For any subset A of Y, (i, j)-$spcl_\delta(f^{-1}(i, j)$-$spint_\delta(i, j)spcl_\delta(A)))) \leq f^{-1}((i, j)$-$spcl_\delta(A))$.

(ii) For any (i,j) - fuzzy pre-open set G in Y, (i,j)-$spcl_\delta(f^{-1}(G)) \leq f^{-1}((i,j)$-$spcl_\delta(G))$.
(iii) For any (i,j) - fuzzy pre-open set H in G in Y, (i,j)-$spcl_\delta(f^{-1}(i,j))$-$spint_\delta(H)) \leq f^{-1}(H)$, where $i \neq j$ and $i,j = 1,2$.

Proof. (i)⇒(ii). Let G be any (i,j)-fuzzy pre-open set in Y. Then, by (i), (i,j)-$spcl_\delta(f^{-1}(i,j) - spint_\delta(i,j)$-$spcl_\delta(G)))) \leq f^{-1}((i,j)$-$spcl_\delta(G))$. Since G is (i,j)-fuzzy pre-open, $G \leq ((i,j)$-$spint_\delta(i,j)$-$spcl_\delta(G))$.Consequently, (i,j)-$spcl_\delta(f^{-1}(G)) \leq f^{-1}$ $((i,j)$-$spcl_\delta(G))$.
(ii) ⇒ (iii). For any (i,j)-fuzzy δ-semi pre-closed set H in Y, (i,j)-$spint_\delta(H)$ is (i,j)-fuzzy δ- semi pre-open set in Y. Therefore, by (ii), (i,j)-$spcl_\delta(f^{-1}(i,j)$-$spint_\delta(H)) \leq f^{-1}((i,j)$-$spcl_\delta(i,j)$-$spint_\delta(H)))$. Since H is (i,j)-fuzzy δ-semi pre-closed, (i,j)-$spcl_\delta((i,j)$-$spint_\delta(H)) \leq H$. Therefore, (i,j)-$spcl_\delta(f^{-1}(i,j))$-$spint_\delta(H)) \leq f^{-1}(H)$.
(iii) ⇒ (i). Let A be any fuzzy subset of Y. Let $H = (i,j)$-$spcl_\delta(A)$. Then for the (i,j)-fuzzy δ-semi pre-closed set H, by (iii), (i,j)-$spcl_\delta(f^{-1}(i,j)$-$spint_\delta(i,j)$-$spcl_\delta(A)))) \leq f^{-1}((i,j)$-$spcl_\delta(A))$.

Theorem 3.3. *A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise weakly fuzzy δ- semi-pre -continuous if and only if for any (i,j)-fuzzy pre-open set V in Y, $f^{-1}(V) \leq (i,j)$-$spint_\delta(f^{-1}((i,j)$-$spcl_\delta(V)), i \neq j$ and $i,j = 1,2$.*

Proof. Let f be pairwise weakly fuzzy δ- semi-pre-continuous mapping and let V be any (i,j)-fuzzy pre-open set in Y. Then for any $x_p \in X$ with $x_p \in f^{-1}(V)$, there exists some (i,j)-fuzzy δ- semi pre-open set U in X such that $x_p \in U$ and $f(U) \leq ((i,j)$-$spcl_\delta(V)$. Hence $x_p \in U \leq f^{-1}((i,j)$-$spcl_\delta(V))$. Consequently, $x_p \in (i,j)$-$spint_\delta(f^{-1}(i,j)$-$spcl_\delta(V))$ and $f^{-1}(V) \leq (i,j)$-$spint_\delta(f^{-1}((i,j)$-$spcl_\delta(V))$.

Conversely, let $x_p \in X$ and V be any (i,j) - fuzzy pre-open set in Y with $f(x_p) \in V$. Then by hypothesis $f^{-1}(V) \leq (i,j)$-$spint_\delta(f^{-1}((i,j)$-$spcl_\delta(V))$. Put $U = (i,j)$-$spint_\delta(f^{-1}((i,j)$-$spcl_\delta(V))$. Then (i,j)-fuzzy δ -semi pre-open subset U is such that $x_p \in U \leq f^{-1}((i,j) - spcl_\delta(V))$. Therefore, $f(U) \leq (i,j) - spcl_\delta(V)$.

Theorem 3.4. *A mapping $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is pairwise weakly fuzzy δ- semi-pre-continuous if and only if for any fuzzy subset A of Y, (i,j)-$spcl_\delta(f^{-1}(i,j)$-$spint_\delta(((i,j)$-$spcl_\delta(A)))) \leq f^{-1}((i,j)$-$spcl_\delta(A))$, $i \neq j$ and $i,j = 1,2$.*

Proof. **Sufficiency.** Let A be any subset of Y and $x_p \in X$ be such that $x_p \notin f^{-1}((i,j)\text{-}spcl_\delta(A))$. Then $f(x_p) \notin f^{-1}((i,j)\text{-}spcl_\delta(A))$ and so there exists some (i,j)-fuzzy pre-open set W in Y such that $f(x_p) \in W$ and $W \cap A = 0_X$. Since f is pairwise weakly fuzzy δ-semi-pre -continuous, there exists some (i,j) - fuzzy δ- semi pre-open set U in X such that $x_p \in U$ and $f(U) \leq (i,j)\text{-}spcl_\delta(W)$. Further, $W \cap (i,j)\text{-}spcl_\delta(A) = 0_X$ and $(i,j)\text{-}spcl_\delta(Y - (i,j)\text{-}spcl_\delta(A)) = (Y - (i,j)\text{-}spint_\delta(((i,j)\text{-}spcl_\delta(A))$. Therefore, $f(U) \leq (Y - (i,j)\text{-}spint_\delta((i,j)\text{-}spcl_\delta(A))$ and hence, $f(U) \cap (i,j)\text{-}spint_\delta(i,j)\text{-}spcl_\delta(A) = 0_X$. Consequently, $U \cap f^{-1}((i,j)\text{-}spint_\delta(i,j)\text{-}spcl_\delta(A)) = 0_X$. It follows that $x_p \notin (i,j)\text{-}spcl_\delta(f^{-1}(i,j)\text{-}spint_\delta(i,j)\text{-}spcl_\delta(A)))$. Hence $(i,j)\text{-}spcl_\delta(f^{-1}(i,j)\text{-}spint_\delta(((i,j)\text{-}spcl_\delta(A)))) \leq f^{-1}((i,j)\text{-}spcl_\delta(A))$.

Necessity. Let $x_p \in X$ and V be any (i,j)-fuzzy pre-open set in Y with $f(x_p) \in V$. Then $V \cap (Y - (i,j)\text{-}spcl - \delta(V)) = 0_X$. Therefore, $f(x_p) \notin (i,j)\text{-}spcl_\delta(Y - (i,j)\text{-}spcl_\delta(V))$ and hence $x_p \notin f^{-1}((i,j)\text{-}spcl_\delta(Y - (i,j)\text{-}spcl_\delta(V)))$. Now, $(Y - (i,j)\text{-}spcl - \delta(V)) \leq (i,j)\text{-}spint_\delta(i,j)\text{-}spcl_\delta(Y - (i,j)\text{-}spcl_\delta(V)))$ and by hypothesis, $(i,j)\text{-}spcl_\delta(f^{-1}((i,j)\text{-}spint_\delta((i,j)\text{-}spcl_\delta(Y - (i,j)\text{-}spcl_\delta(V)))) \leq f^{-1}(i,j)\text{-}spcl_\delta(Y - (i,j)\text{-}spcl_\delta(V))$. Therefore, $x_p \notin ((i,j)\text{-}spcl_\delta(f^{-1}(Y - (i,j)\text{-}spcl_\delta(V))))$. Therefore, there exists some (i,j)-fuzzy-δ-semi pre-open set U in X such that $x_p \in U$ and $U \cap f^{-1}((i,j)\text{-}spcl_\delta(V)) = 0_X$. Consequently, $U \leq X - f^{-1}(Y\text{-}(i,j)\text{-}spcl_\delta(V)) = f^{-1}((i,j)\text{-}spcl_\delta(V))$. Therefore, it follows that $f(U) \leq (i,j)\text{-}spcl_\delta(V)$.

Theorem 3.5. *Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be any two fuzzy bitoplogical spaces and let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a mapping. Then the following statements are equivalent:*
(i) The mapping f is pairwise weakly fuzzy δ-semi-pre-continuous.
(ii) For each $A \leq Y$,$(i,j)\text{-}spcl_\delta(f^{-1}(i,j)\text{-}spint_\delta(i,j)\text{-}spcl_\delta(A)))) \leq f^{-1}((i,j)\text{-}spcl_\delta(A))$.
(iii) For each (i,j)-fuzzy pre-open set G in Y, $(i,j)\text{-}spcl_\delta(f^{-1}(G)) \leq f^{-1}((i,j)\text{-}spcl_\delta(G))$.
(iv) For each (i,j)-fuzzy pre-open set G in Y, $(i,j)\text{-}spcl_\delta(f^{-1}((i.j)\text{-}spint_\delta(H))) \leq f^{-1}(H)$.
(v) For each (i,j)-fuzzy pre-open set G in Y, $f^{-1}(G) \leq ((i.j)\text{-}spint_\delta(f^{-1}((i,j)\text{-}spcl_\delta(G)))$ where $i \neq j$ and $i,j = 1,2$.

Proof. The proof follows from the Theorem 3.2., Theorem 3.3. and Theorem 3.4. and hence omitted.

Theorem 3.6. *For any three fuzzy bitopological spaces* (X, τ_1, τ_2), (Y, σ_1, σ_2) *and* (Z, Γ_1, Γ_2), *if the mapping* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *and* $g : (Y, \sigma_1, \sigma_2) \to (Z, \Gamma_1, \Gamma_2)$ *are pairwise weakly fuzzy* δ*-semi-pre-continuous, then* $g \circ f : (X, \tau_1, \tau_2) \to (Z, \Gamma_1, \Gamma_2)$ *is pairwise weakly fuzzy* δ*-semi-pre-continuous.*

Proof. Let $x_p \in X$ and W be any (i, j)-fuzzy pre-open subset of Z such that $(g \circ f)(x_p) \in W$. Since g is pairwise weakly fuzzy δ-semi-pre-continuous, there exists an (i, j)-fuzzy pre-open set V in Y containing $f(x_p)$ such that $V \le g^{-1}((i, j)\text{-}spcl_\delta(W))$. Further f being pairwise weakly fuzzy δ-semi-pre-continuous, there exists an (i, j)-fuzzy - semi pre-open set U in X such that $x_p \in U \le f^{-1}((i, j)\text{-}spcl_\delta(V))$. Thus, $x_p \in U \le f^{-1}((i, j)\text{-}spcl_\delta(g^{-1}((i, j)\text{-}spcl_\delta(W))))$. But g being pairwise weakly fuzzy δ-semi-pre-continuous, $(i, j)\text{-}spcl_\delta(g^{-1}(W)) \le g^{-1}((i, j)\text{-}spcl_\delta(W))$. Therefore, $x_p \in U \le (g \circ f)^{-1}((i, j)\text{-}spcl_\delta(W))$. Consequently, $g \circ f$ is pairwise weakly fuzzy δ-semi- pre-continuous. □

Theorem 3.7. *Let* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *be a mapping and* $g : X \to X \times Y$ *be the graph mapping of* f*, given by* $g(x_p) = (x_p, f(x_p))$ *for* $x_p \in X$*. If* $g : X \to X \times Y$ *is pairwise weakly fuzzy* δ*-semi-pre-continuous, then* f *is pairwise weakly fuzzy* δ*-semi-pre-continuous.*

Proof. Let $x_p \in X$ and V be an (i, j) - fuzzy pre-open set containing $f(x_p)$ in Y. Then $X \times Y$ is (i, j) - fuzzy pre-open set in $X \times Y$ containing $g(x_p)$. Since g is pairwise weakly fuzzy δ-semi-pre-continuous, there exists an (i, j)-fuzzy δ-semi pre-open set U containing x_p in X such that $g(U) \le (i, j)\text{-}spcl_\delta(X \times Y) \le X \le (i, j)\text{-}spcl_\delta(V)$. Since g is the graph mapping of f, we have $f(U) \le (i, j)\text{-}spcl_\delta(V)$. This shows that f is pairwise weakly fuzzy δ-semi-pre-continuous.

4. Pairwise weakly fuzzy δ-semi pre-open mappings

In this section, the concept of pairwise weakly fuzzy δ-semi pre-open (pre-closed) mapping is to be introduced. Some characterization theorems and basic properties of them are also to be studied.

Definition 4.1. *A mapping* $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be pairwise weakly fuzzy-*δ*-semi pre-open if* $f(\lambda) \le (i, j)\text{-}spint_\delta(f(pcl\lambda))$*, for each* (i, j)*-fuzzy open set* λ *of* X*.*

Definition 4.2. *A mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be pairwise weakly fuzzy -δ-semi pre-closed if (i, j)-$spcl_\delta(f(pint(\lambda))) \leq f(\lambda)$ for each (i, j)-fuzzy pre-closed subset λ of X.*

Theorem 4.3. *For a mapping $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following conditions are equivalent :*
(i) f is pairwise weakly fuzzy δ-semi pre-open.
(ii) (i, j)-$spcl_\delta(f(\lambda)) \leq f(pcl\lambda)$ for each (i, j)-fuzzy pre-open set λ of X.
(iii) (i, j)-$spcl_\delta(f(pint\gamma)) \leq f(\gamma)$ for each (i, j)-fuzzy pre-closed set γ of X.

Proof. (i) $\Rightarrow$ (iii). Let γ be a (i, j)-fuzzy pre-closed set of X. Then $f(1 - \gamma) = 1 - f(\gamma) \leq (i, j)$-$spint_\delta(f(pcl(1 - \gamma))$, by (i).
$\Rightarrow 1 - f(\gamma) \leq 1 - (i, j)$-$spcl_\delta(f(pint(\gamma))$.
Hence (i, j)-$spcl_\delta(f(pint\gamma)) \leq f(\gamma)$.
(iii) $\Rightarrow$ (ii). Let λ be a (i, j)-fuzzy pre-open set of X . Since (i, j)-$pcl(\lambda)$ is a fuzzy pre -closed set and $\lambda \leq (i, j)$-$int(pcl\lambda)$, by (iii)
(i, j)-$spcl_\delta(f(\lambda)) \leq (i, j)$-$spcl_\delta f(intpcl(\lambda)) \leq f(pcl(\lambda))$.
(ii) $\Rightarrow$ (iii). Let γ be a (i, j)-fuzzy pre-closed set in X. Since (i, j)-$pint\gamma$ is a (i, j) - fuzzy pre-open set and by (ii), (i, j)-$spcl_\delta(f(\lambda)) \leq f(pcl\lambda) \leq f(pcl\gamma) = f(\gamma)$ [since (i, j)-$pcl\gamma = \gamma$]. Therefore (i, j)-$scl_\delta(f(pint\gamma)) \leq f(\gamma)$.
(iii) $\Rightarrow$ (i). By (iii), (i, j)-$spcl_\delta(f(pint\gamma)) \leq f(\gamma)$ for each (i, j)-fuzzy pre-closed set γ of X.
$\Rightarrow 1 - f(\gamma) \leq 1 - (i, j)$-$spcl_\delta(f(pint\gamma))$
$\Rightarrow f(1 - \gamma) \leq (i, j)$-$spint_\delta(f(pcl(1 - \gamma))$
$\Rightarrow f(\alpha) \leq (i, j)$-$spint_\delta(f(pcl\alpha)$
where $\alpha = 1 - \gamma$ is any (i, j) - fuzzy pre-open set in X.

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