



Extended results on sum divisor cordial labeling

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Abstract:

A sum divisor cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ such that an edge uv assigned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise. Further the number of edges labeled with 0 and the the number of edges labeled with 1 differ by atmost 1. A graph with sum divisor cordial labeling is called a sum divisor cordial graph. In this paper we prove that the graphs $P_n + P_n$ (n is odd), $P_n @ K_{1,m}$, $C_n @ K_{1,m}$ (n is odd), $W_n * K_{1,m}$ (n is even), $\langle K_{1,n,n}^1 \triangle K_{1,n,n}^2 \rangle$, $\langle Fl_n^1 \triangle Fl_n^2 \rangle$ are sum divisor cordial graphs.

Keywords: Divisor cordial labeling; Sum divisor cordial labeling.

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1. Introduction

By a graph, we mean a finite undirected graph without loops or multiple edges. For standard terminology and notations related to graph theory, we refer to Harary[2]. A labeling of graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of edges, then we speak about edge labeling. If the labels are assigned to both vertices and edges, then the labeling is called total labeling. Cordial labeling is further extended to divisor cordial labeling, prime cordial labeling, total cordial labeling, Fibonacci cordial labeling etc.,

Varatharajan et al. [11] introduced the concept of divisor cordial labeling. For dynamic survey of various graph labeling, we refer to Gallian [1]. Lourdusamy and Patrick [5] introduced the concept of sum divisor cordial labeling. Sugumaran and Rajesh [6] have proved that Swastik graph Sw_n , path union of finite copies Swastik graph Sw_n , cycle of k copies of Swastik graph Sw_n (k is odd), Jelly fish $J(n, n)$ and Petersen graph are sum divisor cordial graphs. Sugumaran and Rajesh [7] have proved that the Theta graph and some of the graph operations of Theta graph are sum divisor cordial graphs. Sugumaran and Rajesh [8] have proved that the Herschel graph and some of the graph operations of Herschel graph are sum divisor cordial graphs. Sugumaran and Rajesh [9] have proved that H_n (n is odd), $C_3 @ K_{1,n}$, $< F_n^1 \Delta F_n^2 >$, open star of Swastik graph $S(t.Sw_n)$, when t is odd are sum divisor cordial graphs. Sugumaran and Rajesh [10] have proved that some graph operations related to H -graph are sum divisor cordial graphs. In this paper we investigate sum divisor cordial labeling of the graphs such as $P_n + P_n$ (n is odd), $P_n @ K_{1,m}$, $C_n @ K_{1,m}$ (n is odd), $W_n * K_{1,m}$ (n is even), $< K_{1,n,n}^1 \Delta K_{1,n,n}^2 >$ and $< Fl_n^1 \Delta Fl_n^2 >$.

Definition 1.1. *If the vertices are assigned values subject to certain conditions then it is known as graph labeling.*

Definition 1.2. *A mapping $f : V(G) \rightarrow \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f .*

Notation. If for an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$, then we denote $v_f(i)$ = number of vertices of G having label i under f ,

$e_f(i)$ = number of vertices of G having label i under f^* .

Definition 1.3. A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits a cordial labeling.

Definition 1.4. [11] Let $G = (V(G), E(G))$ be a simple graph and $f : V \rightarrow \{1, 2, \dots, |V(G)|\}$ be a bijection. For each edge uv , assign the label 1 if either $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a divisor cordial labeling is called a divisor cordial graph.

Definition 1.5. Let $G = (V(G), E(G))$ be a simple graph and let $f : V \rightarrow \{1, 2, \dots, |V(G)|\}$ be a bijection. For each edge uv , assign the label 1 if $2|(f(u) + f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.

Definition 1.6. Let G be a simple and connected graph. The graph $G @ K_{1,n}$ is obtained from attaching the apex vertex of a copy of $K_{1,n}$ at each vertex of G .

Definition 1.7. $W_n * K_{1,m}$ is the graph obtained from the wheel W_n by attaching the apex vertex of a copy of $K_{1,m}$ at each rim vertex of W_n .

Definition 1.8. Let G be a simple and connected graph. Consider two copies of graph G namely G_1 and G_2 . Then the graph $G' = \langle G_1 \triangle G_2 \rangle$ is the graph obtained by joining the apex/central vertices of G_1 and G_2 by an edge as well as to a new vertex x .

2. Main Results

Theorem 2.1. The connected graph $P_n + P_n$ is a sum divisor cordial labeling, where n is odd.

Proof. Let $G = P_n + P_n$ be a connected graph with vertex set $\{v_1, v_2, \dots, v_n, v_{n+1}, \dots, v_{2n}\}$. Then G has $2n$ vertices and $n^2 + 2n - 2$ edges. We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ as follows:
 $f(v_i) = i, i \equiv 1(mod 4)$.

$$f(v_i) = i, i \equiv 2(\text{mod}4).$$

$$f(v_i) = i + 1, i \equiv 3(\text{mod}4).$$

$$f(v_i) = i - 1, i \equiv 0(\text{mod}4).$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence G is a sum divisor cordial graph.

□

Example 2.2. The sum divisor cordial labeling of bistar $P_5 + P_5$ is shown in Figure 2.1

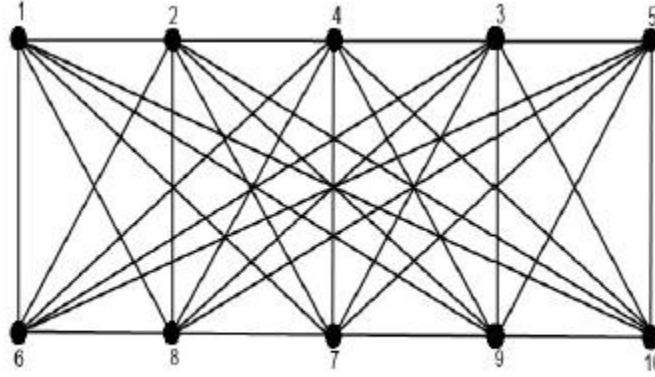


Figure 2.1

Theorem 2.3. The graph $P_n @ K_{1,m}$ is a sum divisor cordial labeling.

Proof. Let $G = P_n @ K_{1,m}$ and let u_1, u_2, \dots, u_n be the vertices of P_n . Let $V(G) = \{v, u_i, v_j^i : 1 \leq i \leq n; 1 \leq j \leq m\}$ where v is the apex vertex and v_j^i is the j^{th} pendant vertex of the i^{th} copy of $K_{1,m}$ ($1 \leq i \leq n; 1 \leq j \leq m$). In G , $v = u_i$. Then G has $n(m+1)$ vertices and $n(m+1) - 1$ edges. We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ as follows:

Case 1: n is even

$$f(u_i) = i, i \equiv 1(\text{mod}4).$$

$$f(u_i) = i, i \equiv 2(\text{mod}4).$$

$$f(u_i) = i + 1, i \equiv 3(\text{mod}4).$$

$$f(u_i) = i - 1, i \equiv 0(\text{mod}4).$$

$$f(v_j^i) = m(i-1) + n + j, 1 \leq i \leq n; 1 \leq j \leq m.$$

Case 2: n is odd

$$f(u_i) = i, i \equiv 1(\text{mod}4).$$

$$f(u_i) = i + 1, i \equiv 2(\text{mod}4).$$

$$f(u_i) = i - 1, i \equiv 3(\text{mod}4).$$

$$f(u_i) = i, i \equiv 0(\text{mod}4).$$

$$f(v_j^i) = m(i - 1) + n + j, 1 \leq i \leq n; 1 \leq j \leq m.$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence G is a sum divisor cordial graph.

□

Example 2.4. The sum divisor cordial labeling of bistar $P_4 @ K_{1,5}$ is shown in Figure 2.2

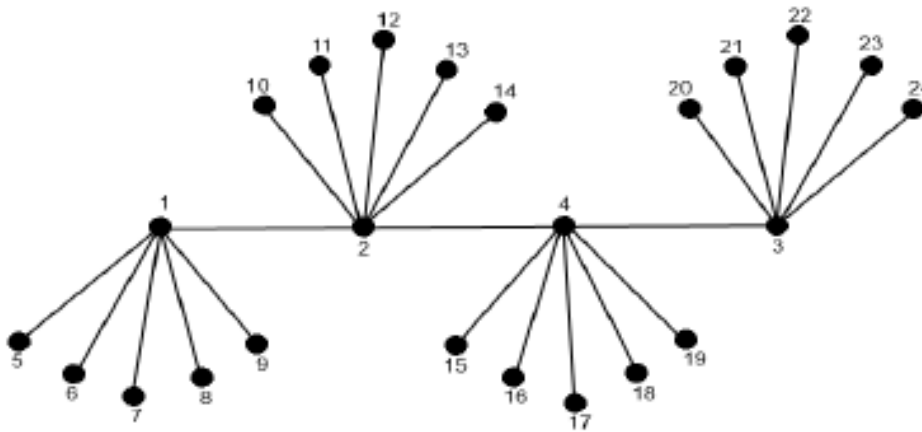


Figure 2.2

Theorem 2.5. The graph $C_n @ K_{1,m}$ is a sum divisor cordial labeling, where n is odd and m is any positive integer.

Proof. Let $G = C_n @ K_{1,m}$. We denote v_j^i is the j^{th} pendant vertex of the i^{th} copy of $K_{1,m}$ ($1 \leq i \leq n; 1 \leq j \leq m$). Let $V(G) = \{u, u_i, v_j^i : 1 \leq i \leq n; 1 \leq j \leq m\}$ where v is the apex vertex of $K_{1,m}$ and in G , $v = u_i$ ($1 \leq i \leq n$). Then G has $n(m + 1)$ vertices and $n(m + 1)$ edges.

We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ as follows:

$$f(u_i) = i, i \equiv 1(\text{mod}4).$$

$$f(u_i) = i + 1, i \equiv 2(\text{mod}4).$$

$$f(u_i) = i - 1, i \equiv 3(\text{mod}4).$$

$$f(u_i) = i, i \equiv 0(\text{mod}4).$$

$$f(v_j^i) = m(i - 1) + n + j, 1 \leq i \leq n; 1 \leq j \leq m.$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a sum divisor cordial graph.

□

Example 2.6. The sum divisor cordial labeling of $C_5 @ K_{1,6}$ is shown in Figure 2.3

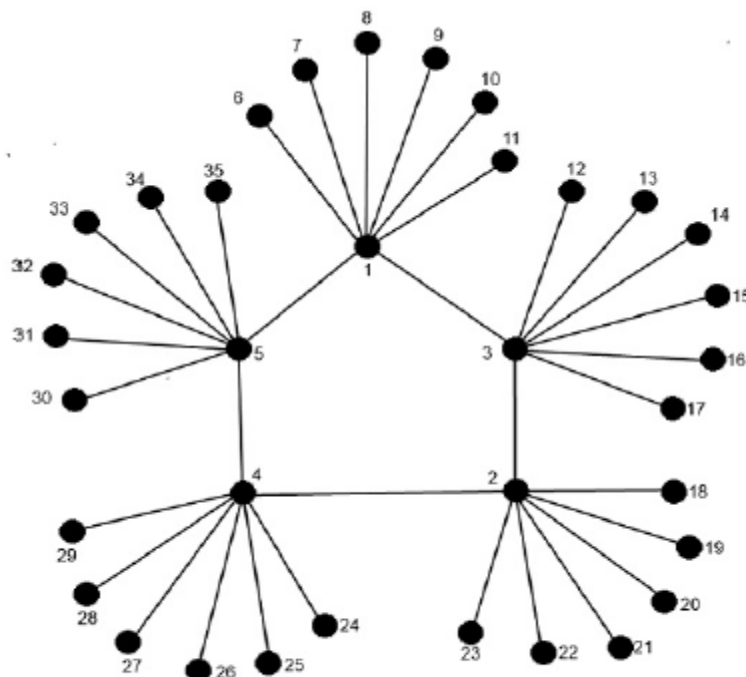


Figure 2.3

Theorem 2.7. The graph $W_n * K_{1,m}$ is a sum divisor cordial labeling, where n is even and m is any positive integer.

Proof. Consider $G = W_n * K_{1,m}$. We denote v_j^i is the j^{th} pendant vertex of the i^{th} copy of $K_{1,m}$. Let $V(G) = \{u, v, u_i, v_j^i : 1 \leq i \leq n; 1 \leq j \leq m\}$ where u, u_i are the apex and rim vertices of W_n and v is the apex vertex of $K_{1,m}$. In G , $v = u_i$ for all $i = 1, 2, \dots, n$.

Then G has $n(m+1) + 1$ vertices and $n(m+2)$ edges.

We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ as follows:

Case 1: When $n \equiv 0(mod 4)$

$$f(u) = 1,$$

$$f(u_i) = i + 1, i \equiv 1(mod 4).$$

$$f(u_i) = i + 2, i \equiv 2(mod 4).$$

$$f(u_i) = i, i \equiv 3(mod 4).$$

$$f(u_i) = i + 1, i \equiv 0(mod 4).$$

$$f(v_j^i) = m(i - 1) + n + j + 1, 1 \leq i \leq n; 1 \leq j \leq m.$$

Case 2: when $n \equiv 2(mod 4)$

$$f(u) = 2,$$

$$f(u_i) = i, i \equiv 1(mod 4).$$

$$f(u_i) = i + 1, i \equiv 2(mod 4).$$

$$f(u_i) = i + 1, i \equiv 3(mod 4).$$

$$f(u_i) = i + 2, i \equiv 0(mod 4).$$

$$f(v_j^i) = m(i - 1) + n + j + 1, 1 \leq i \leq n; 1 \leq j \leq m.$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence G is a sum divisor cordial graph.

□

Example 2.8. The sum divisor cordial labeling of $W_6 * K_{1,4}$ is shown in Figure 2.4

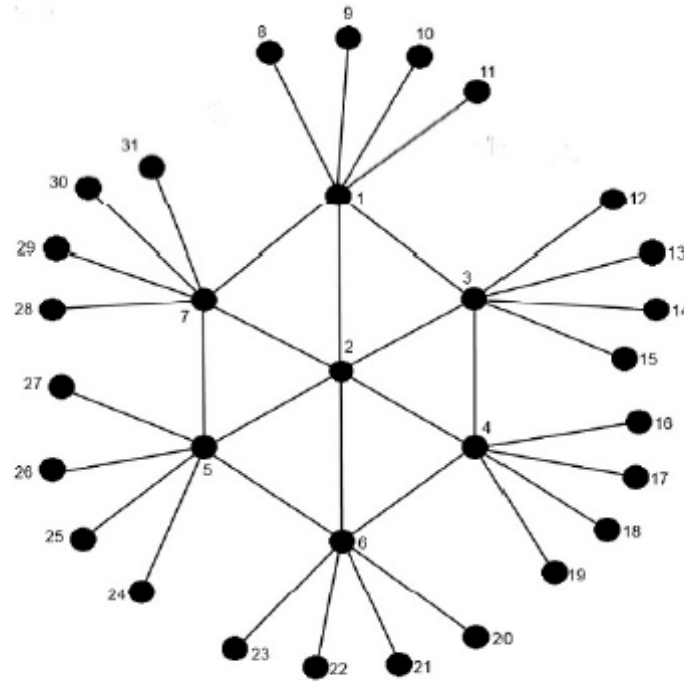


Figure 2.4

Theorem 2.9. The graph $\langle K_{1,n,n}^1 \triangle K_{1,n,n}^2 \rangle$ is a sum divisor cordial labeling.

Proof. Let $G = \langle K_{1,n,n}^1 \triangle K_{1,n,n}^2 \rangle$. Let $V(G) = \{x, u, v, u_i, v_i, u'_i, v'_i : 1 \leq i \leq n\}$, where u and v are the apex vertices, u_i and v_i ($1 \leq i \leq n$) are the first level vertices, u'_i and v'_i ($1 \leq i \leq n$) are the second level vertices in $K_{1,n,n}^1$ and $K_{1,n,n}^2$ respectively. Also, let x be the new vertex added in G . Then G has $4n + 3$ vertices and $4n + 3$ edges.

We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ as follows:

$$f(x) = 3,$$

$$f(u) = 2,$$

$$f(v) = 1,$$

$$f(u_i) = 2i + 2, 1 \leq i \leq n.$$

$$f(u'_i) = 2i + 3, 1 \leq i \leq n.$$

$$f(v_i) = 2n + 2i + 3, 1 \leq i \leq n.$$

$$f(v'_i) = 2n + 2i + 2, 1 \leq i \leq n.$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.

Hence G is a sum divisor cordial graph.

□

Example 2.10. The sum divisor cordial labeling of $\langle K_{1,3,3}^1 \triangle K_{1,3,3}^2 \rangle$ is shown in Figure 2.5

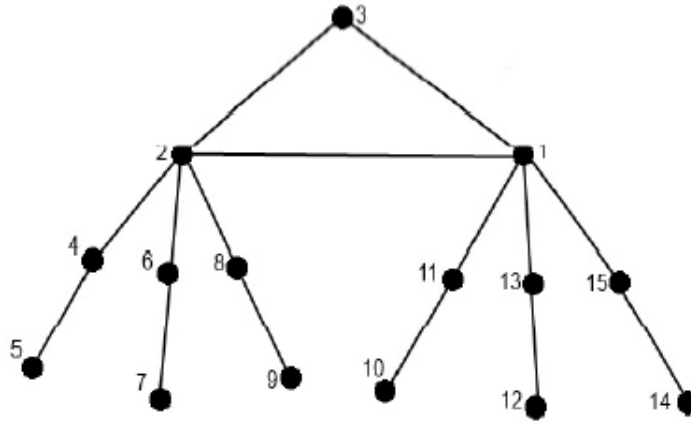


Figure 2.5

Theorem 2.11. The graph $\langle Fl_n^1 \triangle Fl_n^2 \rangle$ is a sum divisor cordial labeling.

Proof. Let $G = \langle Fl_n^1 \triangle Fl_n^2 \rangle$. Let $V(G) = \{x, u, v, u_i, v_i, u'_i, v'_i : 1 \leq i \leq n\}$, where u and v are the central vertices, u_i and v_i ($1 \leq i \leq n$) are the rim vertices, u'_i and v'_i ($1 \leq i \leq n$) are the newly added vertices in F_n^1 and F_n^2 respectively. Also, let x be the new vertex added in G . Then G has $4n + 3$ vertices and $8n + 3$ edges.

We define the vertex labeling $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ as follows:

$$\begin{aligned} f(u) &= 1, \\ f(v) &= 2n + 3, \\ f(x) &= 2n + 2, \\ f(u_i) &= 2i, 1 \leq i \leq n. \\ f(u'_i) &= 2i + 1, 1 \leq i \leq n. \\ f(v_i) &= 2n + 2i + 2, 1 \leq i \leq n. \\ f(v'_i) &= 2n + 2i + 3, 1 \leq i \leq n. \end{aligned}$$

From the above labeling pattern, we have $|e_f(0) - e_f(1)| \leq 1$.
Hence G is a sum divisor cordial graph.

Example 2.12. The sum divisor cordial labeling of $\langle Fl_3^1 \triangle Fl_3^2 \rangle$ is shown in Figure 2.6

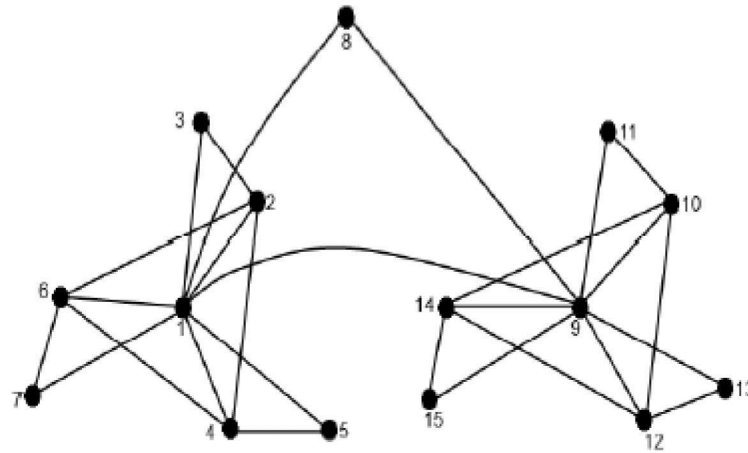


Figure 2.6

Conclusion

In this paper, we proved that some of the graphs such as $P_n + P_n$ (n is odd), $P_n @ K_{1,m}$, $C_n @ K_{1,m}$ (n is odd), $W_n * K_{1,m}$ (n is even), $\langle K_{1,n}^1 \triangle K_{1,n}^1 \rangle$, $\langle Fl_n^1 \triangle Fl_n^2 \rangle$ are sum divisor cordial graphs. It is an interesting open area of research to identify new graphs which admit sum divisor cordial labeling.

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