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On ideal convergence of triple sequences in intuitionistic Fuzzy normed space defined by compact operator

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Abstract

The main purpose of this article is to introduce and study some new spaces of I-convergence of triple sequences in intuitionistic fuzzy normed space defined by compact operator i.e ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ *and* ${}_3\mathcal{S}^I_{0(\mu,\nu)}(T)$ *and examine some fundamental properties, fuzzy topology and verify inclusion relations lying under these spaces.*

Key words: *Ideal, Filter, T-norm, T-conorm, Intuitionistic fuzzy normed spaces.*

Introduction

In 1986, Atanassov K.T.[13] introduced the idea of Intuitionistic fuzzy set theory which is a generalization of fuzzy set. Fuzzy set theory is a powerful tool for modelling uncertainty and vagueness by imputing the degree of membership to the elements so that individuals can be distinguished in a given set. Immense number of research papers recently surfaced in scientific discipline showing that the fuzzy set theory strangely has turned into the todays norm for young scientists or researchers. Many authors have made the concept of fuzzy topology as a very significant tool towards their work. The idea of intuitionistic fuzzy normed space[15] and intuitionistic fuzzy 2-normed space[20] are the most recent evolutions in fuzzy topology.

In the beginning, the idea of statistical convergence was presented independently by Fast[11] and Schoenberg[12]. I-convergence is a generality of statistical convergence which was introduced by Salat et al. [18]. Later on the idea of statistical convergence for double sequences have been defined by Edely and Mursaleen[16] and Tripathy [6] independently and for fuzzy numbers by Mursaleen and Savaş[10]. Related to this, there are infact two quite different types of convergence i.e I and I^*-convergence for double sequences [17].

In 2007 Gurdal, Sahiner and Duden[3] introduced the concept of convergence of triple sequences. This concept has been further investigated by many authors, see([1],[2],[4],[5]). Tripathy and Goswami used the idea of I-convergence of triple sequences in probabilistic normed spaces. Tripathy and Shiner[4] studied the properties associated with I-convergence in triple sequence spaces and showed some useful results.

Preliminaries and definitions

Here, we recall some basic definitions and examples associated to this article.

Definition 1.1:[3] A triple sequence $x = (x_{nkl})$ is said to be convergent to a number ξ in pringsheim's sense if for every $\varepsilon_0 > 0$, $\exists\ m_{(\varepsilon_0)} \in \mathbf{N}$ such that

$$|x_{nkl} - \xi| < \varepsilon_0 \text{ whenever } n \geq m_{(\varepsilon_0)},\ \ k \geq m_{(\varepsilon_0)},\ \ l \geq m_{(\varepsilon_0)}.$$

Example:[3] Let

$$x_{nkl} = \begin{cases} kl, & n = 3, \\ nl, & k = 5, \\ nk, & l = 7, \\ 8, & \text{otherwise.} \end{cases}$$

Then $(x_{nkl}) \to 8$ in Pringsheim's sense.

Definition 1.2:[3] A triple sequence $x = (x_{nkl})$ is said to be statistically convergent to a number ξ if for each $\varepsilon_0 > 0$,

$$\delta_3(\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : |x_{nkl} - \xi| \geq \varepsilon_0\}) = 0$$

and to be symbolized as, $st \lim_{n,k,l \to \infty} = \xi$.

Definition 1.3:[22] A family of sets $I \subseteq 2^X$ is said to be an ideal for a non-empty set X, if it satisfies:

(i) $\phi \in I$;

(ii) if $A, B \in I \Rightarrow A \cup B \in I$;

(iii) if $A \in I$ and $B \subseteq A \Rightarrow B \in I$.

An ideal I is called a non-trivial ideal if $X \notin I$.

Definition 1.4:[22] A family of sets $\mathcal{F} \subseteq 2^X$ is said to be a filter for a non-empty set X, if it satisfies:

(i) $\phi \notin \mathcal{F}$;

(ii) if $A, B \in \mathcal{F} \Rightarrow A \cap B \in \mathcal{F}$;

(iii) if $A \in \mathcal{F}$ and $A \subseteq B \Rightarrow B \in \mathcal{F}$.

For each ideal I there is a filter $\mathcal{F}(I)$ corresponding to I.

$$\mathcal{F}(I) = \{A \subseteq N : A^c \in I\}, \text{ where } A^c = N - A.$$

Definition 1.5:[5] A triple sequence $x = (x_{nkl})$ is said to be I convergent to a number ξ if for every $\varepsilon_0 > 0$, such that

$$\left\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : |x_{nkl} - \xi| \geq \varepsilon_0\right\} \in I$$

and to be symbolized as, $I \lim x_{nkl} = \xi$.

Definition 1.6:[23] A triangular norm $(t-norm)$ is defined as a binary operation

$$*: [0,1] \times [0,1] \longrightarrow [0,1]$$

if $*$ satisfies :

(i) $*$ is commutative and associative,

(ii) $*$ is continuous,

(iii) $\alpha * 1 = \alpha \ \forall \ \alpha \in [0,1]$,

(iv) $\alpha_1 * \beta_1 \leq \alpha_2 * \beta_2$ whenever $\alpha_1 \leq \alpha_2$ and $\beta_1 \leq \beta_2 \ \forall \ \alpha_i, \beta_i \in [0,1]$ $(i = 1, 2)$.

Examples: (i) $\alpha * \beta = \alpha\beta$ (ii) $\alpha * \beta = \min\{\alpha, \beta\}$.

Definition 1.7:[23] A triangular conorm $(t-conorm)$ is defined as a binary operation

$$\diamond : [0,1] \times [0,1] \longrightarrow [0,1]$$

if $\diamond$ satisfies:

(i) $\diamond$ is commutative and associative,

(ii) $\diamond$ is continuous,

(iii) $\alpha \diamond 0 = \alpha \ \forall \ \alpha \in [0,1]$,

(iii) $\alpha_1 \diamond \beta_1 \leq \alpha_2 \diamond \beta_2$ whenever $\alpha_1 \leq \alpha_2$ and $\beta_1 \leq \beta_2 \ \forall \ \alpha_i, \beta_i \in [0,1]$ $(i = 1, 2)$.

Examples: (i) $\alpha \diamond \beta = \min\{\alpha + \beta, 1\}$ (ii) $\alpha \diamond \beta = \max\{\alpha, \beta\}$.

Definition 1.8[23] Functions $\mu, \nu : X \times (0, \infty) \rightarrow [0,1]$ are said to be fuzzy norms on a linear space X over a field $\mathbf{R}(\mathbf{C})$, if for every $x_1, x_2 \in X$ and $t_1, t_2 > 0$, they satisfy:

(a) $\mu(x_1, t_1) + \nu(x_1, t_1) \leq 1$,

(b) $\mu(x_1, t_1) > 0$,

(c) $\mu(x_1, t_1) = 1 \iff x_1 = 0$,

(d) $\mu(\alpha x_1, t_1) = \mu\left(x_1, \frac{t_1}{|\alpha|}\right) \ \forall\ \alpha \neq 0$,

(e) $\mu(x_1, t_1) * \mu(x_2, t_2) \leq \mu(x_1 + x_2, t_1 + t_2)$,

(f) $\mu(x_1, .) : (0, \infty) \to [0, 1]$ is continuous,

(g) $\lim_{t_1 \to \infty} \mu(x_1, t_1) = 1$ and $\lim_{t_1 \to 0} \mu(x_1, t_1) = 0$,

(h) $\nu(x_1, t_1) < 1$,

(i) $\nu(x_1, t_1) = 0 \iff x_1 = 0$,

(j) $\nu(\alpha x_1, t_1) = \nu(x_1, \frac{t_1}{|\alpha|}) \ \forall\ \alpha \neq 0$,

(k) $\nu(x_1, t_1) \diamond \nu(x_2, t_2) \geq \nu(x_1 + x_2, t_1 + t_2)$,

(l) $\nu(x_1, .) : (0, \infty) \to [0, 1]$ is continuous,

(m) $\lim_{t_1 \to \infty} \nu(x_1, t_1) = 0$ and $\lim_{t_1 \to 0} \nu(x_1, t_1) = 1$.

Then the 5-tuple $(X, \mu, \nu, *, \diamond)$ entirely is said to be an intuitionistic fuzzy normed space (abbreviated as IFNS).

Definition 1.9[23] Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space. Then a sequence $x = (x_k)$ is said to be convergent to a number ξ with respect to the intuitionistic norm (μ, ν) if for every $\varepsilon_0 > 0$ and $t_1 > 0$, $\exists\ k_o \in \mathbf{N}$ such that $\mu(x_k - \xi, t_1) > 1 - \varepsilon_0$ and $\nu(x_k - \xi, t_1) < \varepsilon_0\ \forall\ k \geq k_o$ and to be symbolized as, $(\mu, \nu) \lim x = \xi$.

Definition 1.10:[23] Let $(x, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space. Then a sequence $x = (x_k)$ is said to be statistically convergent to a number ξ with respect to the intuitionistic norm (μ, ν) if for every $\varepsilon_0 > 0$ and $t_1 > 0$, we have

$$\delta(\{k \in \mathbf{N} : \mu(x_k - \xi, t_1) \leq 1 - \varepsilon_0 \ \text{ or } \ \nu(x_k - \xi, t_1) \geq \varepsilon_0\}) = 0$$

and to be symbolized as, $st_{(\mu,\nu)} - \lim x_k = \xi$.

Definition 1.11:[23] Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space. Then a sequence $x = (x_k)$ is said to be I-convergent to a number ξ with respect to the intuitionistic norm (μ, ν) if for every $\varepsilon_0 > 0$ and $t_1 > 0$, we have

$$\{k \in \mathbf{N} : \mu(x_k - \xi, t_1) \leq 1 - \varepsilon_0 \text{ or } \nu(x_k - \xi, t_1) \geq \varepsilon_0\} \in I.$$

and to be symbolized as, $I_{(\mu,\nu)} \lim x_k = \xi$.

The idea of statistical convergence and statistical Cauchy for double sequences in intuitionistic fuzzy normed spaces have been studied by Mursaleen and Mohiuddine. Recently, Khan and Yasmeen[25] introduced intuitionistic Zweier I-convergent double sequence spaces defined by modulus function:

$$_2\mathcal{Z}^I_{(\mu,\nu)}(f) = \{(x_{nk}) \in {}_2\omega : \{(n,k) \in \mathbf{N} \times \mathbf{N} : f(\mu(x''_{nk} - \xi, t_1)) \leq 1 - \varepsilon_0$$
$$\text{or } f(\nu(x''_{nk} - \xi, t_1)) \geq \varepsilon_0\} \in I_2\};$$
$$_2\mathcal{Z}^I_{0(\mu,\nu)}(f) = \{(x_{nk}) \in {}_2\omega : \{(n,k) \in \mathbf{N} \times \mathbf{N} : f(\mu(x''_{nk}, t_1)) \leq 1 - \varepsilon_0$$
$$\text{or } f(\nu(x''_{nk}, t_1)) \geq \varepsilon_0\} \in I_2\}.$$

I_3-convergence of triple sequences in IFNS

Definition 2.1: Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space. Then a triple sequence $x = (x_{nkl})$ is said to be statistically convergent to a number ξ with respect to the intuitionistic norm (μ, ν) if for every $\epsilon_o > 0$ and $t_1 > 0$, we have

$$\delta(\{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(x_{nkl}-\xi, t_1) \leq 1-\varepsilon_0 \text{ or } \nu(x_{nkl}-\xi, t_1) \geq \varepsilon_0\}) = 0$$

or equivalently,

$$\lim_{pqr} \frac{1}{pqr} |\{n \leq p, k \leq q, l \leq r : \mu(x_{nkl}-\xi, t_1) \leq 1-\varepsilon_0 \text{ or } \nu(x_{nkl}-\xi, t_1) \geq \varepsilon_0\}| = 0$$

and to be symbolized as, $st^3_{(\mu,\nu)} \lim x_{nkl} = \xi$.

Definition 2.2: Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space. Then a triple sequence $x = (x_{nkl})$ is said to be statistically Cauchy with respect to the intuitionistic norm (μ, ν) if for each $\varepsilon_0 > 0$ and $t_1 > 0$ $\exists$ $p = p_{(\varepsilon_0)}$, $q = q_{(\varepsilon_0)}$, $r = r_{(\varepsilon_0)}$ such that, we have

$$\delta(\{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(x_{nkl}-x_{pqr}, t_1) \leq 1-\varepsilon_0 \text{ or } \nu(x_{nkl}-x_{pqr}, t_1) \geq \varepsilon_0\}) = 0.$$

Definition 2.3: Let $(X, \mu, \nu, *, \diamond)$ be an intuitionistic fuzzy normed space. Then a triple sequence $x = (x_{nkl})$ is said to be I_3-convergent to a number ξ with respect to the intuitionistic norm (μ, ν) if for every $\varepsilon_0 > 0$ and $t_1 > 0$, we have

$$\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(x_{nkl} - \xi, t_1) \leq 1 - \varepsilon_0 \text{ or } \nu(x_{nkl} - \xi, t_1) \geq\} \in I_3.$$

and to be symbolized as, $I_3^{(\mu,\nu)} \lim x_k = \xi$.
where, I_3 is a non trivial ideal of $\mathbf{N} \times \mathbf{N} \times \mathbf{N}$.

A **Compact Linear Operator** is a function $T : V \to W$ which satisfies two properties:

(i) T is linear;

(ii) $T(x_k)$ has a convergent subsequence in W, for every bounded sequence $(x_k) \in V$.

where V and W are normed linear spaces. The set of all bounded linear operators $\mathcal{B}(V, W)$ is normed linear space normed by

$$\|T\| = \sup_{x \in V, \|x\|=1} \|Tx\|$$

Remark: The set of all compact linear operators $\mathcal{C}(V, W)$ is a closed subspace of $\mathcal{B}(V, W)$ and if W is a Banach space then $\mathcal{C}(V, W)$ is also a Banach space.

In this article, we study on I-convergence of triple sequences defined by compact operator in IFNS. We also define an open ball centered at triple sequence with a non-zero radius and study the topology on the defined spaces.

Main results

In this section we introduce the following classes of sequence spaces:

$$ {}_3\mathcal{S}^I_{(\mu,\nu)}(T) = \{(x_{nkl}) \in {}_3\omega : \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(T(x_{nkl})-\xi, t_1) \leq 1-\varepsilon_0 \text{ or } \nu(T(x_{nkl})-\xi, t_1) \geq \varepsilon_0\} \in I_3\}; $$

$$ {}_3\mathcal{S}^I_{0(\mu,\nu)}(T) = \{(x_{nkl}) \in {}_3\omega : \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(T(x_{nkl}), t_1) \leq 1-\varepsilon_0 \text{ or } \nu(T(x_{nkl}), t_1) \geq \varepsilon_0\} \in I_3\}. $$

We also define;

$$ {}_3\mathcal{B}_x(\delta, t_1)(T) = \{(y_{nkl}) \in {}_3\omega : \{(n,k,l) : \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(T(x)-T(y), t_1) > 1-\delta \text{ and } \nu(T(x)-T(y), t_1) < \delta\} \in I_3\} $$

which is an open ball with centre at $x = (x_{nkl})$ and radius δ with respect to t_1.

Theorem 3.1: If a triple sequence $x = (x_{nkl}) \in {}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ is I_3-convergent to a number ξ with respect to the intuitionistic norm (μ,ν), then the limit ξ is unique.

Proof: Let $x = (x_{nkl}) \in {}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ such that

$$ I_3^{(\mu,\nu)} \lim x_{nkl} = \xi_1 \text{ and } I_3^{(\mu,\nu)} \lim x_{nkl} = \xi_2. $$

for a given ε_0, we have $\delta > 0$ s.t

$$ (1-\delta) * (1-\delta) > 1-\varepsilon_0 \text{ and } \delta \diamond \delta < \varepsilon_0 $$

then we define for $t_1 > 0$,

$$ K_1 = \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(T(x_{nkl})-\xi_1, \frac{t_1}{2}) \leq 1-\delta \text{ or } \nu(T(x_{nkl})-\xi_1, \frac{t_1}{2}) > \delta\}, $$

$$ K_2 = \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu(T(x_{nkl})-\xi_2, \frac{t_1}{2}) \leq 1-\delta \text{ or } \nu(T(x_{nkl})-\xi_2, \frac{t_1}{2}) > \delta\}, $$

since

$$ I_3^{(\mu,\nu)} \lim x_{nkl} = \xi_1 \text{ and } I_3^{(\mu,\nu)} \lim x_{nkl} = \xi_2 $$

$\Rightarrow K_1 \in I_3$ and $K_2 \in I_3 \ \forall \ t_1 > 0$
Let $K = K_1 \cup K_2$, $\Rightarrow K \in I_3$ and $K^c \in F(I_3)$.
If, $(p, q, r) \in K^c$, then

$$\mu(\xi_1 - \xi_2, t_1) \geq \mu(Tx_{pqr} - \xi_1, \frac{t_1}{2}) * \mu(Tx_{pqr} - \xi_2, \frac{t_1}{2})$$

$$> (1 - \delta) * (1 - \delta)$$

$$> 1 - \varepsilon_0$$

$\varepsilon_0 > 0$ being arbitrary,

$$\Rightarrow \mu(\xi_1 - \xi_2, t_1) = 1 \ \forall \ t_1 > 0 \Rightarrow \xi_1 = \xi_2.$$

Similarly it can be proved that,

$$\nu(\xi_1 - \xi_2, t_1) < \varepsilon_0 \ \forall \ t_1 > 0 \Rightarrow \xi_1 = \xi_2.$$

Thereby one can conclude that,

$$I_3^{(\mu,\nu)} \lim x_{nkl} = \xi \ \text{ is unique.}$$

Theorem 3.2: Let $x = (x_{nkl})$ be a triple sequence in ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$, then following assertions are equivalent:

(a) If $I_3^{(\mu,\nu)} \lim x_{nkl} = \xi$,

(b) $\{(n, k, l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{nkl}) - \xi, t_1) \leq 1 - \varepsilon_0$ or $\nu(T(x_{nkl}) - \xi, t_1) \geq \varepsilon_0\} \in I_3$

(c) $\{(n, k, l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{nkl}) - \xi, t_1) > 1 - \varepsilon_0$ or $\nu(T(x_{nkl}) - \xi, t_1) < \varepsilon_0\} \in F(I_3)$

(d) $I_3 \lim \mu(T(x_{nkl}) - \xi, t_1) = 1$ and $I_3 \lim \nu(T(x_{nkl}) - \xi, t_1) = 0$.

Theorem 3.3: The spaces ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ and ${}_3\mathcal{S}^I_{0(\mu,\nu)}(T)$ are linear spaces.

Proof: We prove the linearity of the space ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$. For other space it directly follows.

Let $x = (x_{nkl}), y = (y_{nkl}) \in {}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ and α, β be non-zero scalars and $\epsilon_o > 0$ be given, then

$$A_1 = \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu\left(T(x_{nkl}) - \xi_1, \frac{t_1}{2|\alpha|}\right) \leq 1 - \varepsilon_0$$

$$\text{or } \nu\left(T(x_{nkl}) - \xi_1, \frac{t_1}{2|\alpha|}\right) \geq \varepsilon_0\} \in I_3,$$

$$A_2 = \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu\left(T(y_{nkl}) - \xi_2, \frac{t_1}{2|\beta|}\right) \leq 1 - \varepsilon_0$$

$$\text{or } \nu\left(T(y_{nkl}) - \xi_2, \frac{t_1}{2|\beta|}\right) \geq \varepsilon_0\} \in I_3,$$

$$A_1^c = \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu\left(T(x_{nkl}) - \xi_1, \frac{t_1}{2|\alpha|}\right) > 1 - \varepsilon_0$$

$$\text{or } \nu\left(T(x_{nkl}) - \xi_1, \frac{t_1}{2|\alpha|}\right) < \varepsilon_0\} \in \mathcal{F}(I_3).$$

$$A_2^c = \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu\left(T(y_{nkl}) - \xi_2, \frac{t_1}{2|\beta|}\right) > 1 - \varepsilon_0$$

$$\text{or } \nu\left(T(y_{nkl}) - \xi_2, \frac{t_1}{2|\beta|}\right) < \varepsilon_0\} \in \mathcal{F}(I_3).$$

Define $A_3 = A_1 \cup A_2$, so that $A_3 \in I_3 \Rightarrow A_3^c \in \mathcal{F}(I_3)$ is non-empty. Now we show that for each $(x_{nkl}), (y_{nkl}) \in {}_3\mathcal{S}^I_{(\mu,\nu)}(T)$.

$$A_3^c \subset \{(n,k,l) \in \mathbf{N}\times\mathbf{N}\times\mathbf{N} : \mu((\alpha T(x_{nkl}) + \beta T(y_{nkl})) - (\alpha\xi_1 + \beta\xi_2), t_1) > 1 - \varepsilon_0$$

$$\text{or } \nu((\alpha T(x_{nkl}) + \beta T(y_{nkl})) - (\alpha\xi_1 + \beta\xi_2), t_1) < \varepsilon_0\}.$$

Let $(p,q,r) \in A_3^c$. In this case

$$\mu\left(T(x_{pqr}) - \xi_1, \frac{t_1}{2|\alpha|}\right) > 1 - \varepsilon_0 \text{ or } \nu\left(T(x_{pqr}) - \xi_1, \frac{t_1}{2|\alpha|}\right) < \varepsilon_0$$

and

$$\mu\left(T(y_{pqr}) - \xi_2, \frac{t_1}{2|\beta|}\right) > 1 - \varepsilon_0 \text{ or } \nu\left(T(y_{pqr}) - \xi_2, \frac{t_1}{2|\beta|}\right) < \varepsilon_0.$$

We have

$$\mu((\alpha T(x_{pqr}) + \beta T(y_{pqr})) - (\alpha\xi_1 + \beta\xi_2), t_1)$$
$$\geq \mu(\alpha T(x_{pqr}) - \alpha\xi_1, \frac{t_1}{2}) * \mu(\beta T(x_{pqr}) - \beta\xi_2, \frac{t_1}{2})$$
$$= \mu(T(x_{pqr}) - \xi_1, \frac{t_1}{2|\alpha|}) * \mu(T(x_{pqr}) - \xi_2, \frac{t_1}{2|\beta|})$$
$$> (1 - \varepsilon_0) * (1 - \epsilon_o) = 1 - \varepsilon_0.$$

and

$$\nu((\alpha T(x_{pqr}) + \beta T(y_{pqr})) - (\alpha\xi_1 + \beta\xi_2), t_1)$$
$$\leq \nu(\alpha T(x_{pqr}) - \alpha\xi_1, \frac{t_1}{2}) \diamond \nu(\beta T(x_{pqr}) - \beta\xi_2, \frac{t_1}{2})$$
$$= \mu(T(x_{pqr}) - \xi_1, \frac{t_1}{2|\alpha|}) \diamond \mu(T(x_{pqr}) - \xi_2, \frac{t_1}{2|\beta|})$$
$$< \varepsilon_0 \diamond \varepsilon_0 = \varepsilon_0.$$

Hence,

$$A_3^c \subset \{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu((\alpha T(x_{nkl}) + \beta T(y_{nkl})) - (\alpha\xi_1 + \beta\xi_2), t_1) > 1 - \varepsilon_0$$
$$\text{or } \nu((\alpha T(x_{nkl}) + \beta T(y_{nkl})) - (\alpha\xi_1 + \beta\xi_2), t_1) < \varepsilon_0\}.$$

Hence ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ is a linear space.

Theorem 3.4: Let $x = (x_{nkl})$ be a triple sequence in ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ such that $(\mu,\nu)_3 \lim x_{nkl} = \xi$, then $I_3^{(\mu,\nu)} \lim x_{nkl} = \xi$.

Proof: Let $(\mu,\nu)_3 \lim x_{nkl} = \xi$, and $\varepsilon_0 > 0$ be given then $\forall\ t_1 > 0$, $\exists$ $(p,q,r) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N}$ s.t.

$$\mu(T(x_{nkl}) - \xi) > 1 - \epsilon_o \text{ and } \nu(T(x_{nkl}) - \xi) < \varepsilon_0$$

$\forall\ (n,k,l) \geq (p,q,r)$.
Therefore, we get

$$B = \{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu((T(x_{nkl}) - \xi, t_1) \leq 1 - \varepsilon_0 \text{ or } \nu((T(x_{nkl}) - \xi, t_1) \geq \varepsilon_0\}$$
$$\subseteq \{(p',q',r') \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : p' < p-1, q' < q-1, r' < r-1\}$$

But, I_3 being admissible $\Rightarrow B \in I_3$
Hence,

$$I_3^{(\mu,\nu)} \lim x_{nkl} = \xi.$$

Theorem 3.5: Let $x = (x_{nkl})$ be a triple sequence in $_3\omega$. If $y = (y_{nkl})$ in $_3\omega$ is a $I_3^{(\mu,\nu)}$convergent sequence s.t.

$$\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : x_{nkl} \neq y_{nkl}\} \in I_3,$$

then x is also $I_3^{(\mu,\nu)}$convergent.

Proof: Consider the set, $\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : x_{nkl} \neq y_{nkl}\} \in I_3$ and $I_3^{(\mu,\nu)} \lim y_{nkl} = \xi$ then $0 < \varepsilon_0 < 1 \ \forall \ t_1 > 0$, we get

$$\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu((T(y_{nkl}) - \xi, t_1) \leq 1 - \epsilon_o$$

$$\text{or } \ \nu((T(y_{nkl}) - \xi, t_1) \geq \varepsilon_0\} \in I_3.$$

$0 < \varepsilon_0 < 1 \ \forall \ t_1 > 0$,

$$\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu((T(x_{nkl}) - \xi, t_1) \leq 1 - \varepsilon_0$$

$$\text{or } \ \nu((T(x_{nkl}) - \xi, t_1) \geq \varepsilon_0\}.$$

$$\subseteq \{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : x_{nkl} \neq y_{nkl}\}$$

$$\cup \{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu((T(y_{nkl}) - \xi, t_1) \leq 1 - \varepsilon_0$$

$$\text{or } \ \nu((T(y_{nkl}) - \xi, t_1) \geq \varepsilon_0\} \tag{1}$$

Right hand side of (1) is an element of I_3, therefore we get

$$\{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu((T(x_{nkl}) - \xi, t_1) \leq 1 - \varepsilon_0$$
$$\text{or } \ \nu((T(x_{nkl}) - \xi, t_1) \geq \varepsilon_0\} \in I_3.$$

Theorem 3.6: If $x = (x_{nkl}) \in \ _3\mathcal{S}^I_{(\mu,\nu)}(T)$ is a Cauchy sequence with respect to intuitionistic norm (μ, ν) then it is $I_3^{(\mu,\nu)}$-Cauchy with respect to same norm.

Proof: The proof is straight forward thus omitted.

Theorem 3.7: If $x = (x_{nkl}) \in \ {}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ is a Cauchy sequence with respect to intuitionistic norm (μ, ν) then the sequence $x = (x_{nkl})$ has a subsequence which is an ordinary Cauchy sequence with respect to the same norm.

Proof: The proof is straight forward, thus omitted.

Theorem 3.8: Every open ball ${}_3\mathcal{B}_x(\delta, t_1)(T)$ is an open set in ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$.

Proof: Let ${}_3\mathcal{B}_x(\delta, t_1)(T)$ be an open ball with radius δ, centered at x with respect to t_1.
i.e

$$
\begin{aligned}
{}_3\mathcal{B}_x(\delta, t_1)(T) = \{y = (y_{nkl}) \in \ {}_3\omega : \{(n,k,l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{nkl}) \\
-T(y_{nkl}), t_1) > 1 - \delta \quad \text{and } \nu(T(x_{nkl}) - T(y_{nkl}), t_1) < \delta\} \in I_3\}.
\end{aligned}
$$

Let $y \in \ {}_3\mathcal{B}^c_x(\delta, t_1)(T)$.

$$\text{Then,} \ \ \mu(T(x_{nkl}) - T(y_{nkl}), t_1) > 1 - \delta \ \text{ and } \ \nu(T(x_{nkl}) - T(y_{nkl}), t_1) < \delta.$$

$$\text{Since,} \ \ \mu(T(x_{nkl}) - T(y_{nkl}), t_1) > 1 - \delta, \ \exists \ 0 < t_o < t_1$$

s.t.,

$$\mu(T(x_{nkl}) - T(y_{nkl}), t_o) > 1 - \delta \ \text{ and } \ \nu(T(x_{nkl}) - T(y_{nkl}), t_o) < \delta.$$

Now, Consider

$$\delta_o = \mu(T(x_{nkl}) - T(y_{nkl}), t_o), \ \text{ we have } \ \delta_o > 1 - \delta$$

there exists,

$$t_2 \in (0, 1) \quad \text{ such that } \ \delta_o > 1 - t_2 > 1 - \delta$$

For $\delta_o > 1 - t_2$, we have $\delta_1, \delta_2 \in (0, 1)$ such that $\delta_o * \delta_1 > 1 - t_2$ and $(1 - \delta_o) \diamond (1 - \delta_2) \leq t_2$.
let $\delta_3 = \max\{\delta_1, \delta_2\}$.
Here we consider the open ball ${}_3\mathcal{B}^c_y(1 - \delta_3, t_1 - t_o)(T)$ and we show that

$${}_3\mathcal{B}^c_y(1 - \delta_3, t_1 - t_o)(T) \subset \ {}_3\mathcal{B}^c_x(\delta, t_1)(T).$$

Let,

$$z = (z_{nkl}) \in {}_3\mathcal{B}^c_y(1-\delta_3, t_1 - t_o)(T)$$

then

$$\mu(T(y_{nkl}) - T(z_{nkl}), t_1 - t_o) > \delta_3$$

$$\text{and } \nu(T(y_{nkl}) - T(z_{nkl}), t_1 - t_o) < 1 - \delta_3.$$

Therefore,

$$\mu(T(x_{nkl}) - T(z_{nkl}), t_1)$$

$$\geq \mu(T(x_{nkl}) - T(y_{nkl}), t_o) * \mu(T(y_{nkl}) - T(z_{nkl}), t_1 - t_o)$$

$$\geq (\delta_o * \delta_3) \geq (\delta_o * \delta_1) \geq (1 - t_2) \geq (1 - \delta)$$

and

$$\nu(T(x_{nkl}) - T(z_{nkl}), t_1)$$

$$\leq \nu(T(x_{nkl}) - T(y_{nkl}), t_o) \diamond \nu(T(y_{nkl}) - T(z_{nkl}), t_1 - t_o)$$

$$\leq (1 - \delta_o) \diamond (1 - \delta_3) \leq (1 - \delta_o) \diamond (1 - \delta_2) \leq t_2 \geq \delta.$$

Thus

$$z \in {}_3\mathcal{B}^c_x(\delta, t_1)(T)$$

hence

$${}_3\mathcal{B}^c_y(1-\delta_3, t_1 - t_o)(T) \subset {}_3\mathcal{B}^c_x(\delta, t_1)(T).$$

Remark: ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ is an IFNS.
Consider the set

$${}_3\tau^I_{(\mu,\nu)}(T) = \{\mathcal{A} \subset {}_3\mathcal{S}^I_{(\mu,\nu)}(T) : \forall\, x \in \mathcal{A}\, \exists\, t_1 > 0$$

$$\text{and } 0 < \delta < 1 \text{ s.t. } {}_3\mathcal{B}_x(\delta, t_1)(T) \subset \mathcal{A}\}.$$

$\Rightarrow {}_3\tau^I_{(\mu,\nu)}(T)$ is clearly a topology on ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$.

Theorem 3.9: The topology ${}_3\tau^I_{(\mu,\nu)}(T)$ on ${}_3\mathcal{S}^I_{0(\mu,\nu)}(T)$ is first countable.

Proof: $\{{}_3\mathcal{B}_x(\frac{1}{k}, \frac{1}{k})(T) : k = 1, 2, 3, ...\}$ forms a local base at x therefore the topology ${}_3\tau^I_{(\mu,\nu)}(T)$ on ${}_3\mathcal{S}^I_{0(\mu,\nu)}(T)$ is first countable.

Theorem 3.10: ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ and ${}_3S^I_{0(\mu,\nu)}(T)$ are Hausdorff spaces.

Proof: We establish the result for the space ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$.
The result follows directly for ${}_3\mathcal{S}^I_{0(\mu,\nu)}(T)$.
Let,

$$x = (x_{nkl}), y = (y_{nkl}) \in \ {}_3\mathcal{S}^I_{(\mu,\nu)}(T) \text{ s.t. } (x_{nkl}) \neq (y_{nkl}).$$

Then

$$0 < \mu(T(x_{nkl}) - T(y_{nkl}), t_1) < 1 \text{ and } 0 < \nu(T(x_{nkl}) - T(y_{nkl}), t_1) < 1.$$

consider, $\delta_1 = \mu(T(x_{nkl}) - T(y_{nkl}), t_1)$, $\delta_2 = \nu(T(x_{nkl}) - T(y_{nkl}), t_1)$ and $\delta = \max\{\delta_1, 1 - \delta_2\}$.
$\forall\ \delta_o \in (\delta, 1)\ \exists\ \delta_3$ and δ_4 s.t. $\delta_3 * \delta_3 \geq \delta_o$
and

$$(1 - \delta_4) \diamond (1 - \delta_4) \leq (1 - \delta_o).$$

Again for, $\delta_5 = \max\{\delta_3, \delta_4\}$ and we consider ${}_3\mathcal{B}_x(1 - \delta_5, \frac{t_1}{2})$ and ${}_3\mathcal{B}_y(1 - \delta_5, \frac{t_1}{2})$.
Clearly,

$${}_3\mathcal{B}^c_x(1 - \delta_5, \frac{t_1}{2}) \cap \ {}_3\mathcal{B}^c_y(1 - \delta_5, \frac{t_1}{2}) = \emptyset.$$

For, if there exists

$$z = (z_{nkl}) \in \ {}_3\mathcal{B}^c_x(1 - \delta_5, \frac{t_1}{2}) \cap \ {}_3\mathcal{B}^c_y(1 - \delta_5, \frac{t_1}{2})$$

then

$$\delta_1 = \mu(T(x_{nkl}) - T(y_{nkl}), t_1)$$
$$\geq \mu(T(x_{nkl}) - T(z_{nkl}), \frac{t_1}{2}) * \mu(T(z_{nkl}) - T(y_{nkl}), \frac{t_1}{2})$$
$$\geq \delta_5 * \delta_5 \geq \delta_3 * \delta_3 \geq \delta_o > \delta_1$$

and

$$\delta_2 = \nu(T(x_{nkl}) - T(y_{nkl}), t_1)$$
$$\leq \nu(T(x_{nkl}) - T(z_{nkl}), \frac{t_1}{2}) \diamond \nu(T(z_{nkl}) - T(y_{nkl}), \frac{t_1}{2})$$
$$\leq (1 - \delta_5) \diamond (1 - \delta_5)$$
$$\leq (1 - \delta_4) \diamond (1 - \delta_4) \leq (1 - \delta_o) < \delta_2$$

which is not possible.
Therefore, ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ is Hausdorff.

Theorem 3.11: ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$ is an IFNS and ${}_3\tau^I_{(\mu,\nu)}(T)$ is a topology on ${}_3\mathcal{S}^I_{(\mu,\nu)}(T)$.
Then $\exists$ a sequence $(x_{nkl}) \in {}_3\mathcal{S}^I_{(\mu,\nu)}(T)$, such that $x_{nkl} \to x$ iff $\mu((T(x_{nkl}) - T(x), t_1) \to 1$ and $\nu(T(x_{nkl}) - T(x), t_1) \to 0$ as $n, k, l \to \infty$.

Proof: Let $t_1 > 0$, let $x_{nkl} \to x$ and $0 < \delta < 1$, $\exists\ (p, q, r) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N}$ s.t.,

$$(x_{nkl}) \in {}_3\mathcal{B}_x(\delta, t_1)(T) \text{ for all } n \geq p, k \geq q, l \geq r,$$

$${}_3\mathcal{B}_x(\delta, t_1)(T) = \{(n, k, l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{nkl}) - T(x), t_1)\} > 1 - \delta$$
$$\text{and } \nu(T(x_{nkl}) - T(x), t_1) < \delta\} \in I_3.$$

such that,

$${}_3\mathcal{B}^c_x(\delta, t_1)(T) \in \mathcal{F}(I_3).$$

Then,

$$1 - \mu(T(x_{nkl}) - T(x), t_1) < \delta \text{ and } \nu(T(x_{nkl}) - T(x), t_1) < \delta.$$

Hence,

$$\mu(T(x_{nkl}) - T(x), t_1) \to 1 \text{ and } \nu(T(x_{nkl}) - T(x), t_1) \to 0, \text{ as } n, k, l \to \infty.$$

Conversely,

$$\mu(T(x_{nkl}) - T(x), t_1) \to 1 \text{ and } \nu(T(x_{nkl}) - T(x), t_1) \to 0 \text{ as } n, k, l \to \infty,$$

then for,

$$0 < \delta < 1, \exists\ (p, q, r) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N}$$

s.t.,

$$1 - \mu(T(x_{nkl}) - T(x), t_1) < \delta \text{ and } \nu(T(x_{nkl}) - T(x), t_1) < \delta, \ \forall\ n \geq p, k \geq q, l \geq r.$$

It follows that,

$$\mu(T(x_{nkl}) - T(x), t_1) > 1 - \delta \text{ and } \nu(T(x_{nkl}) - T(x), t_1) < \delta \ \forall\ n \geq p, k \geq q, l \geq r.$$

Thus,

$$(x_{nkl}) \in \ _3\mathcal{B}_x^c(\delta, t_1)(T) \text{ for all } n \geq p, k \geq q, l \geq r$$

hence,

$$x_{nkl} \to x.$$

Theorem.3.12: A triple sequence $x = (x_{nkl}) \in \ _3S^I_{(\mu,\nu)}(T)$ is I-convergent iff for every $\varepsilon_0 > 0$, $t_1 > 0$ there exists numbers $p = p(x, \varepsilon_0, t_1)$, $q = q(x, \varepsilon_0, t_1)$ and $r = r(x, \varepsilon_0, t_1)$ such that,

$$\{(p, q, r) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{pqr}) - \xi, \frac{t_1}{2}) > 1 - \varepsilon_0$$

$$\text{or } \nu(T(x_{pqr}) - \xi, \frac{t_1}{2}) < \varepsilon_0\} \in \mathcal{F}(I_3).$$

Proof: Let $I_3^{(\mu,\nu)} \lim x = \xi$ and let $\varepsilon_0 > 0$ and $t_1 > 0$.For $\varepsilon_0 > 0$ to be given, choose $t_2 > 0$
s.t.,

$$(1 - \varepsilon_0) * (1 - \varepsilon_0) > 1 - t_2 \text{ and } \varepsilon_0 \diamond \varepsilon_0 < t_2.$$

we get,

$$x \in \ _3S^I_{(\mu,\nu)}(T),$$

$$P = \{(n, k, l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{nkl}) - \xi, \frac{t_1}{2}) \leq 1 - \varepsilon_0$$

$$\text{or } \nu(T(x_{nkl}) - \xi, \frac{t_1}{2}) \geq \varepsilon_0\} \in I_3,$$

This implies

$$P^c = \{(n, k, l) \in \mathbf{N} \times \mathbf{N} \times \mathbf{N} : \mu(T(x_{nkl}) - \xi, \frac{t_1}{2}) > 1 - \varepsilon_0$$

$$\text{or } \nu(T(x_{nkl}) - \xi, \frac{t_1}{2}) < \varepsilon_0\} \in \mathcal{F}(I_3).$$

Conversely, let $p, q, r \in P$.
Then

$$\mu(T(x_{pqr}) - \xi, \frac{t_1}{2}) > 1 - \varepsilon_0 \text{ or } \nu(T(x_{pqr}) - \xi, \frac{t_1}{2}) < \varepsilon_0.$$

Now, we show that $\exists\, p = p(x, \varepsilon_0, t_1)$ and $q = q(x, \varepsilon_0, t_1)$ and $r = r(x, \varepsilon_0, t_1)$
such that,

$$\{(n, k, l) : \mu(T(x_{nkl}) - T(x_{pqr}), t_1) \leq 1 - t_2 \text{ or } \nu(T(x_{nkl}) - T(x_{pqr}), t_1) \geq t_2\} \in I_3.$$

Thus, for each $x \in {}_3S^I_{(\mu,\nu)}(T)$ we consider

$$Q = \{(n,k,l) : \mu(T(x_{nkl}) - T(x_{pqr}), t_1) \leq 1 - t_2$$
$$\text{or } \nu(T(x_{nkl}) - T(x_{pqr}), t_1) \geq t_2\} \in I_3.$$

Now we show that $Q \subset P$.
Let $Q \not\subset P$ then $\exists$ $(n^{'}, k^{'}, l^{'}) \in Q$ and $(n^{'}, k^{'}, l^{'}) \notin P$.
Therefore we get,

$$\mu(T(x_{n',k',l'}) - T(x_{pqr}), t_1) \leq 1 - t_2 \text{ or } \mu(T(x_{n',k',l'}) - \xi, \frac{t_1}{2}) > 1 - \varepsilon_0.$$

particularly,

$$\mu(T(x_{pqr}) - \xi, \frac{t_1}{2}) > 1 - \varepsilon_0.$$

moreover we get,

$$1 - t_2 \geq \mu(T(x_{n',k',l'}) - T(x_{pqr}), t_1)$$
$$\geq \mu(T(x_{n',k',l'}) - \xi, \frac{t_1}{2}) * \mu(T(x_{pqr}) - \xi, \frac{t_1}{2})$$
$$\geq (1 - \varepsilon_0) * (1 - \epsilon_o) > 1 - t_2,$$

which is not possible.
On the other hand,

$$\nu(T(x_{n',k',l'}) - T(x_{pqr}), t_1) \geq t_2 \text{ or } \nu(T(x_{n',k',l'}) - \xi, \frac{t_1}{2}) < \varepsilon_0$$

particularly,

$$\nu(T(x_{pqr}) - \xi, \frac{t_1}{2}) < \varepsilon_0.$$

Therefore we get,

$$t_2 \leq \nu(T(x_{n',k',l'}) - T(x_{pqr}), t_1)$$
$$\leq \nu(T(x_{n',k',l'}) - \xi, \frac{t_1}{2}) \diamond \nu(T(x_{pqr}) - \xi, \frac{t_1}{2})$$
$$\leq \varepsilon_0 \diamond \varepsilon_0 < t_2,$$

which is not possible.
Hence,

$$Q \subset P$$

and we have,

$$P \in I \Rightarrow Q \in I.$$

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