On minimal λ_{co} -open sets

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Abstract:

We introduce and discuss the notions of minimal λ_{co} -open sets in topological spaces. We establish some of its basic fundamental properties of minimal λ_{co} -open. We show that the notions of minimal open sets and minimal λ_{co} -open are independent and finally we obtain some application of a minimal λ_{co} -open sets.

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1. Introduction

The study of semi open sets in topological spaces was initiated by Levine [12]. The complement of a subset A of X is denoted by $X \setminus A$. In 1937, M. Stone [29], defined regular closed set, a subset A is said to be regular-closed if $A = Cl(Int(A))$. The family of all regular-closed sets of (X, τ) is denoted by $RC(X)$. Sharma J. N. and Chauhan J. P., [28] defined clopen set, a subset A of a topological space (X, τ) is said to be clopen if A is open and closed. The family of all clopen sets of (X, τ) is denoted by $CO(X)$. The concept of operation γ was initiated by Kasahara [4]. He also introduced γ -closed graph of a function. Using this operation, Ogata [26] introduced the concept of γ -open sets and investigated the related topological properties of the associated topology τ_γ and τ . He further investigated general operator approaches of closed graph of mappings. Further Ahmad and Hussain [1] continued studying the properties of γ -open(γ -closed) sets. In 2009, Hussain and Ahmad [3], introduced the concept of minimal γ -open sets. In 2011 [5] (resp. in 2013[7]) Khalaf and Namiq, defined an operation γ called s -operation. They work in operation in topology in [16], [8], [17], [9], [10], [18], [11], [19], [20], [21],[22], [23]. They defined $\lambda_{\beta c}$ -open set[15] by using s -operation and β -closed set and also investigated several properties of $\lambda_{\beta c}$ -derived, $\lambda_{\beta c}$ -interior and $\lambda_{\beta c}$ -closure points in topological spaces. In 2017, Carpintero et al. [2], investigated the notions of minimal open sets in a generalized topologies an investigated its fundamental properties.

In this paper, we introduce and discuss minimal λ_{co} -open sets in topological spaces. We establish some basic properties of minimal λ_{co} -open sets and provide an example to illustrate that minimal λ_{co} -open sets and minimal open sets are independent. Finally we obtain some applications of minimal λ_{co} -open. First, we recall some definitions and results used in this paper.

2. Preliminaries

Throughout, X denotes a topological space. Let A be a subset of X , then the closure and the interior of A are denoted by $Cl(A)$ and $Int(A)$ respectively. A subset A of a topological space (X, τ) is said to be semi open [12] if $A \subseteq Cl(Int(A))$. The complement of a semi open set is called semi closed [12]. The family of all semi open (resp. semi closed) sets in a topological space (X, τ) is denoted by $SO(X, \tau)$ or $SO(X)$ (resp. $SC(X, \tau)$

or $SC(X)$). We consider λ as a function defined on $SO(X)$ into $P(X)$ and $\lambda : SO(X) \rightarrow P(X)$ is called an s -operation if $V \subseteq \lambda(V)$ for each non-empty semi open set V . It is assumed that $\lambda(\emptyset) = \emptyset$ and $\lambda(X) = X$ for any s -operation λ . Let X be a topological space and $\lambda : SO(X) \rightarrow P(X)$ be an s -operation, then a subset A of X is called a λ^* -open set [14] which is equivalent to λ -open set [5] and λ_s -open set [7] if for each $x \in A$ there exists a semi open set U such that $x \in U$ and $\lambda(U) \subseteq A$. The complement of a λ^* -open set is called λ^* -closed. The family of all λ^* -open (resp. λ^* -closed) subsets of a topological space (X, τ) is denoted by $SO_\lambda(X, \tau)$ or $SO_\lambda(X)$ (resp. $SC_\lambda(X, \tau)$ or $SC_\lambda(X)$).

Proposition 2.1. [15] For a topological space X , $SO_\lambda(X) \subseteq SO(X)$.

The following example shows that the contention of the above proposition may be strict..

Example 2.2. [15] Let $X = \{a, b, c\}$, and $\tau = \{\emptyset, \{a\}, X\}$. We define an s -operation $\lambda : SO(X) \rightarrow P(X)$ as $\lambda(A) = A$ if $b \in A$ and $\lambda(A) = X$ otherwise. Here, we have $\{a, c\}$ is semi open but it is not λ^* -open.

Definition 2.3. [15] An s -operation λ on X is said to be s -regular which is equivalent to λ -regular [6] if for every semi open sets U and V of X containing the point $x \in X$, there exists a semi open set W containing x such that $\lambda(W) \subseteq \lambda(U) \cap \lambda(V)$.

The proof of the following two propositions are in [6].

Proposition 2.4. Let $\{A_\alpha\}_{\alpha \in I}$ be any collection of λ^* -open sets in a topological space (X, τ) , then $\cup_{\alpha \in I} A_\alpha$ is a λ^* -open set.

Proposition 2.5. Let λ be semi-regular operation. If A and B are λ^* -open sets in X , then $A \cap B$ is also a λ^* -open set.

Definition 2.6. A λ^* -open [14](λ -open [5], λ_s -open [7]) subset A of a topological space X is called λ_{rc} -open [25](λ_{co} -open [24]) set if for each $x \in A$ there exists a regular closed (clopen) set F such that $x \in F \subseteq A$. The complement of a λ_{rc} -open (λ_{co} -open) set is called λ_{rc} -closed (λ_{co} -closed). The family of all λ_{rc} -open (resp. λ_{rc} -closed, λ_{co} -open, λ_{co} -closed) subsets of a topological space (X, τ) is denoted by $SO_{\lambda_{rc}}(X, \tau)$ or $SO_{\lambda_{rc}}(X)$ (resp. $SC_{\lambda_{rc}}(X, \tau)$ or $SC_{\lambda_{rc}}(X)$, $SO_{\lambda_{co}}(X, \tau)$ or $SO_{\lambda_{co}}(X)$, $SC_{\lambda_{co}}(X, \tau)$ or $SC_{\lambda_{co}}(X)$).

Definition 2.7. Let X be a topological space and $\lambda : SO(X) \rightarrow P(X)$ be an s -operation, then a subset A of X is called a λ_{co} -open neighbourhood of a point $x \in X$ IF A is a λ_{co} -open set and $x \in A$.

Proposition 2.8. [27] For a topological space X , $SO_{\lambda_{rc}}(X) \subseteq SO_{\lambda}(X) \subseteq SO(X)$.

Proposition 2.9. [24] For a topological space X , $SO_{\lambda_{co}}(X) \subseteq SO_{\lambda_{rc}}(X) \subseteq SO(X)$.

The following example shows that the contention of the above proposition may be strict.

Example 2.10. In Example 2.2, we have $\{a, c\}$ is semi open but it is not λ^* -open. And also $\{a, b\}$ is λ^* -open set but it is not λ_{co} -open.

Example 2.11. Let $X = \{a, b, c\}$, and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. We define an s -operation $\lambda : SO(X) \rightarrow P(X)$ as $\lambda(A) = A$ if $A \neq \{a\}, \{b\}$ and $\lambda(A) = \{a, b\}$ if $A = \{a\}$ or $\{b\}$. The λ_{rc} -open sets are $\emptyset, \{a, c\}, \{b, c\}, X$. We have that $\{a, c\}$ is λ_{rc} -open set but it is not λ_{co} -open set.

Definition 2.12. [24] Let A be a subset of X . Then:

1. The λ_{co} -closure of A ($\lambda_{co}Cl(A)$) is the intersection of all λ_{co} -closed sets containing A .
2. The λ_{co} -interior of A ($\lambda_{co}Int(A)$) is the union of all λ_{co} -open sets of X contained in A .

Proposition 2.13. [24] For each point $x \in X$, $x \in \lambda_{co}Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every $V \in SO_{\lambda_{co}}(X)$ such that $x \in V$.

Proposition 2.14. [24] Let $\{A_{\alpha}\}_{\alpha \in I}$ be any collection of λ_{co} -open sets in a topological space (X, τ) , then $\cup_{\alpha \in I} A_{\alpha}$ is a λ_{co} -open set.

Proposition 2.15. [24] Let λ be an s -regular operation. If A and B are λ_{co} -open sets in X , then $A \cap B$ is also a λ_{co} -open set.

Definition 2.16. [13] Let X be a space and $A \subseteq X$ be an open set. Then A is called a minimal open set if $\emptyset$ and A are the only open subsets of A .

3. Minimal λ_{co} -Open Sets

Definition 3.1. Let X be a space and $A \subseteq X$ be a λ_{co} -open set. Then A is called a minimal λ_{co} -open set if $\emptyset$ and A are the only λ_{co} -open subsets of A .

The following example show that every minimal λ_{co} -open set is minimal λ_{rc} -open set but the converse is not necessarily true.

Example 3.2. By Proposition 2.9, $SO_{\lambda_{co}}(X) \subseteq SO_{\lambda_{rc}}(X)$ and Example 2.11, $\{a, c\}$ is λ_{rc} -open set but it is not λ_{co} -open set.

Proposition 3.3. Let A be a nonempty λ_{rc} -open subset of a space X . If $A \subseteq \lambda_{co}Cl(C)$, then $\lambda_{co}Cl(A) = \lambda_{co}Cl(C)$, for any nonempty subset C of A .

Proof. For any nonempty subset C of A , we have $\lambda_{co}Cl(C) \subseteq \lambda_{co}Cl(A)$. On the other hand, by supposition, we see $\lambda_{co}Cl(A) \subseteq \lambda_{co}Cl(\lambda_{co}Cl(C)) = \lambda_{co}Cl(C)$ implies $\lambda_{co}Cl(A) \subseteq \lambda_{co}Cl(C)$. Therefore we have $\lambda_{co}Cl(A) = \lambda_{co}Cl(C)$ for any nonempty subset C of A .

Proposition 3.4. Let A be a nonempty λ_{co} -open subset of a space X . If $\lambda_{co}Cl(A) = \lambda_{co}Cl(C)$, for any nonempty subset C of A , then A is a minimal λ_{co} -open set.

Proof. Suppose that A is not a minimal λ_{co} -open set. Then there exists a nonempty λ_{co} -open set B such that $B \subseteq A$ and hence there exists an element $x \in A$ such that $x \notin B$. Then we have $\lambda_{co}Cl(\{x\}) \subseteq X \setminus B$ implies that $\lambda_{co}Cl(\{x\})$ is a proper subset of $\lambda_{co}Cl(A)$. And the result follows.

Remark 3.5. In the remainder of this section, we suppose that λ is an s -regular operation defined on a topological space X . Observe that if the condition of λ is not an s -regular operation, then the intersection of two λ_{co} -open sets not necessarily is λ_{co} -open set, as we can see in the following Example.

Example 3.6. Let $X = \{a, b, c\}$, and $\tau = P(X)$. We define an s -operation $\lambda : SO(X) \rightarrow P(X)$ as $\lambda(A) = A$ if $A \neq \{b\}, \{c\}$ and $\lambda(A) = \{b, c\}$ if $A = \{b\}$ or $\{c\}$.

$SO(X) = P(X)$.

$SO_{\lambda}(X) = SO_{\lambda_{co}}(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Clearly λ is not a s -regular operation and the intersection of the λ_{co} -open sets $\{a, b\}$ and $\{b, c\}$ is not a λ_{co} -open.

Proposition 3.7. *The following statements are true:*

1. *If A is a minimal λ_{co} -open set and B a λ_{co} -open set. Then $A \cap B = \emptyset$ or $A \subseteq B$.*
2. *If B and C are minimal λ_{co} -open sets. Then $B \cap C = \emptyset$ or $B = C$.*

Proof. (1) Let B be a λ_{co} -open set such that $A \cap B \neq \emptyset$. Since A is a minimal λ_{co} -open set and $A \cap B \subseteq A$, we have $A \cap B = A$. Therefore $A \subseteq B$.

(2) If $A \cap B \neq \emptyset$, then by (1), we have $B \subseteq C$ and $C \subseteq B$. Therefore, $B = C$.

Proposition 3.8. *Let A be a minimal λ_{co} -open set. If $x \in A$, then $A \subseteq B$ for any λ_{co} -open neighborhood B of x .*

Proof. Let B be a λ_{co} -open neighborhood of x such that A is not contained in B . Since λ is a s -regular operation, then $\emptyset \neq A \cap B$ is a λ_{co} -open set. This contradicts our assumption that A is a minimal λ_{co} -open set.

Proposition 3.9. *Let A be a minimal λ_{co} -open set. Then for any $x \in A$, $A = \cap\{B : B \text{ is } \lambda_{co} - \text{open neighborhood of } x\}$.*

Proof. By Proposition 3.4 and the fact that A is λ_{co} -open neighborhood of x , we have $A = \cap\{B : B \text{ is } \lambda_{co} - \text{open neighborhood of } x\} \subseteq A$. Therefore, the result follows.

Proposition 3.10. *If A is a minimal λ_{co} -open set in X not containing $x \in X$. Then for any λ_{co} -open neighborhood C of x , either $C \cap A = \emptyset$ or $A \subseteq C$.*

Proof. Since C is a λ_{co} -open set, we have the result by Proposition 3.3.

Corollary 3.11. *If A is a minimal λ_{co} -open set in X not containing a point $x \in X$. If $A_x = \cap\{B : B \text{ is } \lambda_{co} - \text{open neighborhood of } x\}$. Then either $A_x \cap A = \emptyset$ or $A \subseteq A_x$.*

Proof. If $A \subseteq B$ for any λ_{co} -open neighborhood B of x , then $A \subseteq \cap\{B : B \text{ is } \lambda_{co}\text{-open neighborhood of } x\}$. Therefore $A \subseteq A_x$. Otherwise there exists a λ_{co} -open neighborhood B of x such that $B \cap A = \emptyset$. Then we have $A_x \cap A = \emptyset$.

Corollary 3.12. *If A is a nonempty minimal λ_{co} -open set of X , then for a nonempty subset C of A , $A \subseteq \lambda_{co}Cl(C)$.*

Proof. Let C be any nonempty subset of A . Let $y \in A$ and B be any λ_{co} -open neighborhood of y . By Proposition 3.4, we have $A \subseteq B$ and $C = A \cap C \subseteq B \cap C$. Thus we have $B \cap C \neq \emptyset$ and hence $y \in \lambda_{co}Cl(C)$. This implies that $A \subseteq \lambda_{co}Cl(C)$.

Combining Corollary 3.12 and Propositions 3.3 and 3.4, we have:

Theorem 3.13. *Let A be a nonempty λ_{co} -open subset of space X . Then the following are equivalent:*

1. A is minimal λ_{co} -open set, where λ is s -regular.
2. For any nonempty subset C of A , $A \subseteq \lambda_{co}Cl(C)$.
3. For any nonempty subset C of A , $\lambda_{co}Cl(A) = \lambda_{rc}Cl(C)$.

4. Finite λ_{co} -Open Sets

In this section, we study some properties of minimal λ_{co} -open sets in finite λ_{co} -open sets and λ_{co} -locally finite spaces.

Proposition 4.1. *Let (X, τ) be a topological space and $\emptyset \neq B$ a finite λ_{co} -open set in X . Then there exists at least one (finite) minimal λ_{co} -open set A such that $A \subseteq B$.*

Proof. Suppose that B is a finite λ_{co} -open set in X . Then we have the following two possibilities:

1. B is a minimal λ_{co} -open set.
2. B is not a minimal λ_{co} -open set.

In case (1), if we choose $A = B$, then the proposition is proved. If the case (2) is true, then there exists a nonempty (finite) λ_{co} -open set B_1 which is properly contained in B . If B_1 is minimal λ_{co} -open, we take $A = B_1$. If

B_1 is not a minimal λ_{co} -open set, then there exists a nonempty (finite) λ_{co} -open set B_2 such that $B_2 \subseteq B_1 \subseteq B$. We continue this process and have a sequence of λ_{co} -open sets $\subseteq B_m \subseteq \dots \subseteq B_2 \subseteq B_1 \subseteq B$. Since B is a finite, this process will end in a finite number of steps. That is, for some natural number k , we have a minimal λ_{co} -open set B_k such that $A = B_k$. This completes the proof.

Definition 4.2. A space X is said to be a λ_{co} -locally finite space, if for each $x \in X$ there exists a finite λ_{co} -open set A in X such that $x \in A$.

Example 4.3. Let $X = N$ and $\tau = P(N)$. We define an s -operation $\lambda : SO(X) \rightarrow P(X)$ as $\lambda(A) = A$ for every subset A of N . Then (N, τ) is λ_{co} -locally finite space.

Corollary 4.4. Let X be a λ_{co} -locally finite space and B a nonempty λ_{co} -open set. Then there exists at least one (finite) minimal λ_{co} -open set A such that $A \subseteq B$, where λ is s -regular.

Proof. Since B is a nonempty set, there exists an element x of B . Since X is a λ_{co} -locally finite space, we have a finite λ_{co} -open set B_x such that $x \in B_x$. Since $B \cap B_x$ is a finite λ_{co} -open set, we get by Proposition 4.1, a minimal λ_{co} -open set A such that $A \subseteq B \cap B_x \subseteq B$.

Proposition 4.5. Let X be a space and for any $\alpha \in I$, B_α a λ_{co} -open set and $\emptyset \neq A$ a finite λ_{co} -open set. Then $A \cap (\cap_{\alpha \in I} B_\alpha)$ is a finite λ_{co} -open set, where λ is s -regular.

Proof. We see that there exists an integer n such that $A \cap (\cap_{\alpha \in I} B_\alpha) = A \cap (\cap_{i=1}^n B_{\alpha_i})$ and hence we have the result. Using Proposition 4.5, we can prove the following:

Theorem 4.6. Let X be a space and for any $\alpha \in I$, B_α is a λ_{co} -open set and for any $\beta \in I$, B_β is a nonempty finite λ_{co} -open set. Then $(\cup_{\beta \in I} B_\beta) \cap (\cap_{\alpha \in I} B_\alpha)$ is a λ_{co} -open set, where λ is s -regular.

Remark 4.7. It is important to know that the notions of λ_{co} -locally finite space and locally finite spaces are independent. Since the Definitions of λ_{co} -locally finite space and respectively locally finite spaces depended on λ_{co} respectively open sets, but the family of λ_{co} -open sets and open sets in a topological space (X, τ) equipped with an s -operator are independent.

5. More Properties

Let A be a nonempty finite λ_{co} -open set. It is clear, by Proposition 3.3 and Proposition 4.1, that if λ is s -regular, then there exists a natural number m such that $\{A_1, A_2, \dots, A_m\}$ is the class of all minimal λ_{co} -open sets in A satisfying the following two conditions:

1. For any i, n with $1 \leq i, n \leq m$ and $i \neq n$, $A_i \cap A_n = \emptyset$.
2. If C is a minimal λ_{co} -open set in A , then there exists i with $1 \leq i \leq m$ such that $C = A_i$.

Theorem 5.1. *Let X be a space and $A \neq \emptyset$ be a finite λ_{co} -open set which is not a minimal λ_{co} -open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{co} -open sets in A and $y \in A \setminus (\cup_{i=1}^m A_i)$. Define $A_y = \cap\{B_y\}$, where B_y is a λ_{co} -open neighborhood of y . Then there exists a natural number $k \in \{1, 2, 3, \dots, m\}$ such that A_k is contained in A_y , where λ is s -regular.*

Proof. Suppose on the contrary that for any natural number $k \in \{1, 2, 3, \dots, m\}$, A_k is not contained in A_y . By Proposition 3.7, for any minimal λ_{co} -open set A_k in A , $A_k \cap A_y = \emptyset$. By Proposition 4.5, $\emptyset \neq A_y$ is a finite λ_{co} -open set. Therefore by Proposition 4.1, there exists a minimal λ_{co} -open set C such that $C \subseteq A_y$. Since $C \subseteq A_y \subseteq A$, we have C is a minimal λ_{co} -open set in A . By supposition, for any minimal λ_{co} -open set A_k , we have $A_k \cap C \subseteq A_k \cap A_y = \emptyset$. Therefore, for any natural number $k \in \{1, 2, 3, \dots, m\}$, $C \neq A_k$. This contradicts our assumption. Hence the proof.

Proposition 5.2. *Let X be a space and $A \neq \emptyset$ be a finite λ_{co} -open set which is not a minimal λ_{co} -open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{co} -open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Then there exists a natural number $k \in \{1, 2, 3, \dots, m\}$, such that for any λ_{co} -open neighborhood B_y of y , A_k is contained in B_y , where λ is s -regular.*

Proof. This follows from Theorem 5.1.

Theorem 5.3. *Let X be a space and $A \neq \emptyset$ be a finite λ_{co} -open set which is not a minimal λ_{co} -open set. Let $\{A_1, A_2, \dots, A_m\}$ be a class of all minimal λ_{co} -open sets in A and $y \in A \setminus (A_1 \cup A_2 \cup \dots \cup A_m)$. Then there exists a natural number $k \in \{1, 2, 3, \dots, m\}$, such that $y \in \lambda_{co}Cl(A_k)$. where λ is s -regular.*

Proof. It follows from Proposition 5.2, that there exists a natural number $k \in \{1, 2, 3, \dots, m\}$ such that $A_k \subseteq B$ for any λ_{co} -open neighborhood B of y . Therefore $\emptyset \neq A_k \subseteq A_k \cap B$ and then, $y \in \lambda_{co}Cl(A_k)$. $\square$

Proposition 5.4. *Let $A \neq \emptyset$ be a finite λ_{co} -open set in a space X and for each $k \in \{1, 2, 3, \dots, m\}$, A_k is a minimal λ_{co} -open set in A . If the class $\{A_1, A_2, \dots, A_m\}$ contains all minimal λ_{co} -open sets in A , then for any $\emptyset \neq B_k \subseteq A_k$, $A \subseteq \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, where λ is s -regular.*

Proof. If A is a minimal λ_{co} -open set, then this is the result of Theorem 3.13(2). Otherwise, when A is not a minimal λ_{co} -open set. If x is any element of $A \setminus (A_1 \cup A_2 \cup A_3 \cup \dots \cup A_m)$, then by Theorem 5.3, $x \in \lambda_{co}Cl(A_1) \cup \lambda_{co}Cl(A_2) \cup \dots \lambda_{co}Cl(A_m)$. Therefore, by Theorem 3.13 (3), we obtain that $A \subseteq \lambda_{co}Cl(A_1) \cup \lambda_{co}Cl(A_2) \cup \dots \lambda_{co}Cl(A_m) = \lambda_{co}Cl(B_1) \cup \lambda_{co}Cl(B_2) \cup \dots \lambda_{co}Cl(B_m) = \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$.

Proposition 5.5. *Let $A \neq \emptyset$ be a finite λ_{co} -open set and for each $k \in \{1, 2, 3, \dots, m\}$, A_k is a minimal λ_{co} -open set in A . If for any $\emptyset \neq B_k \subseteq A_k$, $A \subseteq \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, then $\lambda_{co}Cl(A) = \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$.*

Proof. For any $\emptyset \neq B_k \subseteq A_k$, with $k \in \{1, 2, 3, \dots, m\}$, we have $\lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m) \subseteq \lambda_{co}Cl(A)$. Also, we have $\lambda_{co}Cl(A) \subseteq \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m) = \lambda_{co}Cl(B_1) \cup \lambda_{co}Cl(B_2) \cup \lambda_{co}Cl(B_3) \cup \dots \cup \lambda_{co}Cl(B_m)$. Therefore, $\lambda_{co}Cl(A) = \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, for any nonempty subset B_k of A_k with $k \in \{1, 2, 3, \dots, m\}$.

Proposition 5.6. *Let $A \neq \emptyset$ be a finite λ_{co} -open set and for each $k \in \{1, 2, 3, \dots, m\}$, A_k is a minimal λ_{co} -open set in A . If for any $\emptyset \neq B_k \subseteq A_k$, $\lambda_{co}Cl(A_k) = \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, then the class $\{A_1 \cup A_2 \cup A_3 \cup \dots \cup A_m\}$ contains all minimal λ_{co} -open sets in A .*

Proof. Suppose that C is a minimal λ_{co} -open set in A and $C \neq A_k$ for $k \in \{1, 2, 3, \dots, m\}$. Then we have $C \cap \lambda_{co}Cl(A_k) = \emptyset$ for each $k \in \{1, 2, 3, \dots, m\}$. It follows that any element of C is not contained in $\lambda_{co}Cl(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_m)$. This is a contradiction to the fact that $C \subseteq A \subseteq \lambda_{co}Cl(A) = \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$. This completes the proof. Combining Propositions 5.4, 5.5 and 5.6, we have the following theorem:

Theorem 5.7. Let A be a nonempty finite λ_{co} -open set and A_k a minimal λ_{co} -open set in A for each $k \in \{1, 2, 3, \dots, m\}$. Then the following three conditions are equivalent:

1. The class $\{A_1, A_2, \dots, A_m\}$ contains all minimal λ_{co} -open sets in A .
2. For any $\emptyset \neq B_k \subseteq A_k$, $A_k \subseteq \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$.
3. For any $\emptyset \neq B_k \subseteq A_k$, $\lambda_{co}Cl(A_k) = \lambda_{co}Cl(B_1 \cup B_2 \cup B_3 \cup \dots \cup B_m)$, where λ is s -regular.

6. Some Applications

Recall that a topological space (X, τ) is Alexandroff if for any arbitrary intersection of open sets is open, or equivalently, a space (X, τ) is Alexandroff if for each $x \in X$ there exists a minimal open set U containing x . It is well known that this theory of minimal open sets has its applications in the Alexandroff Spaces and in the digital topology. Now in our case, we can define that a topological space (X, τ) is λ_{co} -Alexandroff if for any arbitrary intersection of λ_{co} -open sets is a λ_{co} -open set. It is easily to see that a space (X, τ) is λ_{co} -Alexandroff if for each $x \in X$ there exists a λ_{co} -minimal open set U containing x . There are many examples of λ_{co} -Alexandroff spaces for example: The discrete topological spaces, where $\lambda(A) = A$ for each $A \in P(X)$. In summary, this new theory is being studied by many mathematicians.

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