



Global dynamics of COPD and Tuberculosis among smokers

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Abstract:

Chronic Obstructive Pulmonary Disease (COPD) and tuberculosis among smokers have turned out to be major public health concern, accounting for millions of deaths annually. Smoking is becoming like a trend in human society. This adds in maximizing the risk of people suffering from most dreadful health issues and the reason for extensive morbidity and mortality around the world. People who smoke have approximately twice the risk of suffering from tuberculosis and also COPD patients with the history of tuberculosis writhes in agony. To scrutinize spread of this disease, an understanding of the epidemiological relationship between them are important. The relation between COPD, tuberculosis among smokers and medication is studied in this research work and the formulation for the mathematical model is done using system of non-linear ordinary differential equations. The threshold for smokers suffering from COPD and tuberculosis who needs medication is computed using next generation matrix method. Also, stability analysis and numerical simulations are carried out to validate the data.

Keywords: Mathematical model; Basic reproduction number; Stability.

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1. Introduction

Smoking is considered to be an alarmingly common issue in the society. There are different modes through which people inhale these harmful smokes. It can be in the form of bidis, cigarette, cigar, e-cigarette, hukkah and many more [10]. Among youths, vaping or juuling is usually found which may lead to smoke addiction [2]. Though the modes of smoking are different but each harms the body in the same manner. The reason to consume this poisonous smoke varies person to person, it may be for enjoyment, stress relief, weight control, peer pressure, etc. In some cases, people smoke as it is also considered as a status symbol. Due to which many horrific diseases prevail in one's body. It is also observed that not only smokers but people around them are also affected. So, individuals are affected directly as well as indirectly due to smoking. It mostly affects lungs which disturbs breathing functions. In the beginning it is in the form of coughs, colds, wheezing and asthma but sometimes it leads to pneumonia, emphysema and lung cancer [1, 18]. Also, maximum cases of tuberculosis and COPD are observed. Smoke mainly damages the airways, air sacs and the lining of the lungs. So, in case of COPD, the progression of the condition can only be slowed down by treatment as it permanently damages the lungs. While tuberculosis can be cured if prescribed course is completed with proper precautions. People do smoke despite of being aware of these outcomes. It's very difficult for a person mentally as well as physically to stop this addiction. Science has progressed to a great level to stop them from this urge of smoking. Medication in the form of nicotine chewing gum, nicotine patch, nicotine spray, nicotine nasal etc. are used to quit smoking [4, 8].

A lot of related study is done which gives the idea about issues and problems prevailing in the society due to smoking. Davies *et al.* [5] reviewed an epidemiological relation between smoking and tuberculosis from the UK, India, china and the USA. Also, Zaher *et al.* [19] assessed the burden of illness like COPD, CHD and lung diseases due to smoking. A population-based study was done by Buist *et al.* [3] which aimed to measure the international variation in the prevalence of COPD by age, sex, and smoking status. Lin *et al.* [14] studied a time-based, multiple risk factor and modelling to show the effects of smoking and solid-fuel use on COPD, lung cancer and tuberculosis in china. Also, Salvi and Barnes [16] discussed about chronic obstructive pulmonary disease found in non-smokers. While Lee *et al.* [12] analyzed a nationwide cohort study in Taiwan about the

risk factors for pulmonary tuberculosis in patients with chronic obstructive airway disease. Also, whether age matters on the impact of cigarette smoking on tuberculosis was discussed by Feng *et al.* [7]. Moreover, Jha *et al.* [9] did a nationally representative case-control study of smoking and death in India. Even the colliding epidemics of tuberculosis, tobacco smoking, HIV and COPD were analyzed by van Zyl Smit *et al.* [17] which are the global lung health issues. Leung *et al.* [13] investigated about the relation between smoking and tuberculosis and also how it adversely affects treatment response, outcome and relapse in tuberculosis.

The mathematical model for COPD and tuberculosis among smokers is designed and formulated in section 2. Stability at all equilibrium points is analyzed in section 3 followed by numerical simulation in section 4, then finally concluding the model in section 5.

2. Mathematical Model

In this mathematical model, the transmission of COPD and tuberculosis among smokers is studied. It is based on *SIRS*-model. This model as shown in Figure 1 comprises of four compartments, namely, smokers (S_M) who are likely to get infected by COPD (I_1) and tuberculosis (I_2). In some cases, individuals suffering from COPD may suffer from tuberculosis and vice-versa. To get cured or quit smoking they procure medication (M). After getting cured, they may again start smoking.

Table 1: Model parameters and their interpretation

Notation	Description	Parametric values
B	Birth Rate	0.4
α_1	Rate at which individuals suffers from COPD due to smoking	0.4
α_2	Rate at which smokers get TB infection	0.3
α_3	Rate of TB patients suffering from COPD	0.75
α_4	Rate of COPD patients suffering from TB	0.4
α_5	Rate at which COPD patients starts medication	0.4
α_6	Rate at which TB patients procure medication	0.8
α_7	Rate of individuals after getting cured again starts smoking	0.2
μ_d	Death rate due to disease	0.3
μ	Natural Death rate	0.1

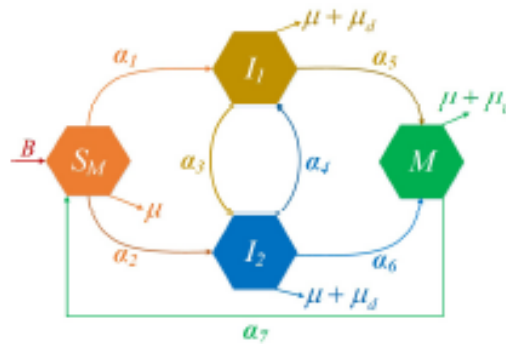


Figure 1: Mathematical model

Using parameters, the mathematical model is formulated. The system of non-linear differential equations is as follows:

$$(2.1) \quad \begin{aligned} \frac{dS_M}{dt} &= B - \alpha_1 S_M I_1 - \alpha_2 S_M I_2 + \alpha_7 M - \mu S_M \\ \frac{dI_1}{dt} &= \alpha_1 S_M I_1 - \alpha_3 I_1 + \alpha_4 I_2 - \alpha_5 I_1 - (\mu + \mu_d) I_1 \\ \frac{dI_2}{dt} &= \alpha_2 S_M I_2 + \alpha_3 I_1 - \alpha_4 I_2 - \alpha_6 I_2 - (\mu + \mu_d) I_2 \\ \frac{dM}{dt} &= \alpha_5 I_1 + \alpha_6 I_2 - \alpha_7 M - (\mu + \mu_d) M \end{aligned}$$

where, $S_M + I_1 + I_2 + M \leq N$. Also, $S_M > 0; I_1, I_2, M \geq 0$.

Adding the above differential equations of system (1.1), we have

$$\frac{d}{dt} (S_M + I_1 + I_2 + M) = B - \mu (S_M + I_1 + I_2 + M) - \mu_d (I_1 + I_2 + M) \geq 0$$

which implies that $\lim_{t \rightarrow \infty} \sup (S_M + I_1 + I_2 + M) \leq \frac{B}{\mu}$.

Therefore, the feasible region of the model is

$$\Lambda = \left\{ (S_M, I_1, I_2, M) \in R^4 : S_M + I_1 + I_2 + M \leq \frac{B}{\mu} \right\}.$$

Now, the disease-free equilibrium point of the model is $Y_0 = \left(\frac{B}{\mu}, 0, 0, 0 \right)$ and endemic point is $Y^* = (S_M^*, I_1^*, I_2^*, M^*)$

where,

$$\begin{aligned} S_M^* &= r, \\ I_1^* &= \frac{(B - \mu r) (\alpha_7 + \mu + \mu_d) (\alpha_1 (\alpha_4 + \alpha_6 - \alpha_2 r + \mu + \mu_d) - \alpha_2 \alpha_4)}{r \alpha_1 \alpha_2 (\alpha_6 - \alpha_5) (\mu + \mu_d)} \\ &\quad \left\{ \begin{aligned} &+ (\alpha_1 - \alpha_2) (\mu_d^2 (\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + 3\mu + \mu_d) \\ &+ \alpha_4 ((\mu + \mu_d) (\alpha_5 + \alpha_7 + \mu) + \mu \mu_d)) - \alpha_2 \mu_d \mu^2 \\ &+ \mu (\alpha_1 \alpha_3 - \alpha_2 (\alpha_3 + \alpha_5 + \mu)) (\alpha_6 + \alpha_7 + \mu) \\ &+ \mu_d (\alpha_1 \alpha_3 - \alpha_2 (\alpha_3 + \alpha_5 + 2\mu)) (\alpha_6 + \alpha_7 + 2\mu) \\ &+ \alpha_1 \mu (\alpha_6 + \mu) (\alpha_5 + \alpha_7 + \mu) + \alpha_1 \mu_d (\alpha_5 (\alpha_6 + 2\mu) \\ &+ \alpha_6 (\alpha_7 + 2\mu) + \mu (2\alpha_7 + 3\mu)) \end{aligned} \right\} \end{aligned}$$

$$I_2^* = \frac{\alpha_3 (B - r\mu) (\alpha_7 + \mu + \mu_d)}{\left\{ \begin{aligned} &(\alpha_5 + \alpha_7 + \mu) (\alpha_4 (\mu + \mu_d) - r\alpha_2\mu) - r\alpha_2\mu_d (\alpha_5 + \alpha_7 + 2\mu + \mu_d) \\ &+ \mu_d^2 (\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + 3\mu + \mu_d) \\ &+ (\mu + \mu_d) (\alpha_3 (\alpha_6 + \alpha_7 + \mu) + \alpha_5 (\alpha_6 + \mu)) \\ &+ (\alpha_7 + \mu) (\alpha_6 (\mu + \mu_d) + \mu^2) \\ &+ \mu\mu_d (\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + 2\alpha_7 + 3\mu) \end{aligned} \right\}},$$

$$\begin{aligned} M^* = & - \left[(\alpha_1 - \alpha_2) \mu_d^2 (\alpha_5 (\mu_d - \alpha_2 r) + \alpha_1 \alpha_3 \alpha_6 \mu r) - \mu_d \alpha_1 \alpha_3 (\alpha_5 + \alpha_6) \right. \\ & (B \mu_d (\alpha_3 - \alpha_2) + \alpha_2 \mu r (\alpha_3 + \alpha_4)) + (\alpha_7 + 2\mu) \{ (\alpha_6 + \mu) (\alpha_1 \alpha_2 \alpha_5^2 \\ & (B - \mu r) - \alpha_5^2 \mu (\alpha_1 (\alpha_6 + \mu) - 2\alpha_2 \alpha_3)) + \mu_d \alpha_5 \alpha_6 (\alpha_1 (B \alpha_2 + \alpha_6 \mu) \\ & + \mu (2\alpha_1 \alpha_4 - \alpha_2 (2\alpha_3 + \alpha_4))) + \alpha_5 \mu^2 (3\mu_d^2 (\alpha_1 - \alpha_2) + \alpha_1 \alpha_5 (\alpha_3 + 2\alpha_4)) \\ & - \alpha_2 \alpha_6 \mu (\alpha_1 \mu_d r (\alpha_3 + \alpha_5) + \alpha_5 (\alpha_4 \alpha_5 + 2\alpha_1 \alpha_3 r)) + \mu_d B \alpha_1 \alpha_5 \\ & (\alpha_2 (\alpha_3 + \alpha_4) - \alpha_1 \alpha_6) \} + (2\alpha_7 + 3\mu) \{ -\mu_d \alpha_5 \mu (\alpha_1 - \alpha_2) (B \alpha_1 + \alpha_2 \mu r) \\ & + \alpha_5 \mu^2 (\alpha_2 (\alpha_2 \alpha_5 r - \alpha_6 (\alpha_5 + \mu_d) - (\alpha_5 \mu + 2\mu_d \alpha_3)) + 2\mu_d (\alpha_1 (\alpha_3 + \alpha_6) \\ & + \alpha_4 (\alpha_1 + \alpha_2))) \} + (\alpha_5 + \alpha_7 + \mu) \{ B \alpha_1 \alpha_5 (\alpha_2 r (\alpha_4 (\alpha_1 + \alpha_2) \\ & - \alpha_2 \alpha_3) - \alpha_4^2 (\alpha_1 - \alpha_2)) + \alpha_2 \alpha_5 \mu (\alpha_2 r (\alpha_3 (\alpha_3 + \alpha_4) + (\mu_d + \alpha_5)^2) \\ & - \mu_d \alpha_4 (\mu_d + \alpha_3)) \} + (\alpha_6 + \mu) \{ \alpha_1 \alpha_5^2 (B \alpha_1 - \alpha_5 \mu) (\alpha_2 r - 2\mu_d - 2\alpha_4) \\ & + \alpha_2 \alpha_5^2 (\alpha_3 + \alpha_5) (B \alpha_1 - \mu (\alpha_3 + \alpha_5)) - \alpha_1 \alpha_3 \alpha_6^2 \mu (\alpha_3 + \alpha_5) \} + (\alpha_7 \\ & + \mu) \{ (\alpha_6 + \mu) (\alpha_1 \alpha_5 \mu (B \alpha_2 + \mu (\alpha_6 + \mu)) - B \alpha_1^2 \alpha_5 (2\alpha_4 + \alpha_6 + \mu) \\ & - \alpha_2 \alpha_5 \mu (\alpha_3 + \mu^2)) - \alpha_1 (\alpha_4 + \alpha_6 + \mu) (\alpha_2 \alpha_5 \mu^2 r + B \alpha_1 \alpha_3 \alpha_6) \\ & + \alpha_1 r (B \alpha_2 \alpha_5 (\alpha_1 \alpha_6 + \mu (\alpha_1 - \alpha_2)) + \alpha_3 \alpha_6 \mu (\alpha_1 \mu - \alpha_2 (\alpha_3 + \alpha_4))) \\ & + \alpha_2 \mu^2 r (\alpha_2 \alpha_5 (2\alpha_3 + \alpha_4 + \mu) - \alpha_1 \alpha_3) + B \alpha_1 \alpha_2 (\alpha_4 \alpha_6 (\alpha_3 + \alpha_5) + \alpha_5 \mu \\ & (\alpha_3 + 2\alpha_4)) + \alpha_5 \mu (\alpha_1 (\alpha_3 \mu (\alpha_3 + 2\alpha_4 + 2\mu) + \alpha_4 (\mu (\alpha_4 + 2\mu) + 2\alpha_6 (\alpha_5 \\ & + \mu))) - \alpha_2 \alpha_4 (\alpha_3 (\alpha_6 + 2\mu) + \mu (\alpha_4 + \alpha_6 + 2\mu))) \} + (\alpha_5 + \alpha_7 + 2\mu) \\ & \{ \alpha_5^2 (\alpha_2 r (\alpha_2 \mu (2\alpha_3 + \alpha_4) - \alpha_1 (B \alpha_2 + \alpha_4 \mu)) + \alpha_4^2 \mu (\alpha_1 - \alpha_2)) \} \\ & + \alpha_1 \alpha_3 \alpha_6 (\alpha_6 + \alpha_7 + \mu) \{ \alpha_1 \mu r (\alpha_3 + \alpha_4 + \alpha_6) + B (\alpha_2 \mu - \alpha_3 (\alpha_1 - \alpha_2)) \} \\ & + \alpha_5 \mu (\alpha_7 + 3\mu) \{ \mu_d (2\alpha_1 \alpha_5 \mu - \alpha_2 \alpha_6 (\mu_d + 2\alpha_5)) - \alpha_2 \alpha_5^2 (\alpha_6 + \mu) \} \\ & + \alpha_1 \alpha_3 \alpha_5 (2\alpha_6 + \alpha_7 + \mu) \{ \alpha_2 r (B \alpha_1 - \mu (\alpha_3 + \alpha_4)) - B \alpha_4 (\alpha_1 - \alpha_2) \} \\ & + (\alpha_3 + \alpha_4) \{ (\alpha_1 - \alpha_2) (\mu_d \alpha_5 (B \alpha_1 \alpha_2 r + 2\mu_d^2 \mu) - \mu_d B \alpha_1 \alpha_3 \alpha_6) + \mu_d \mu \\ & (\alpha_3 r (\alpha_1^2 \alpha_6 + \alpha_2^2 \alpha_5) - \alpha_1 \alpha_3 \alpha_6^2) \} + \mu_d \alpha_5 \mu (2\alpha_5 + \alpha_7 + 2\mu) \{ \alpha_2 (\alpha_2 \mu \\ & (2\alpha_3 + \alpha_4 - \alpha_1 (B \alpha_2 + \alpha_4 \mu)) + \alpha_4^2 (\alpha_1 - \alpha_2)) \} + \alpha_5 \mu^2 (\alpha_5 + 3\alpha_7 + 5\mu) \\ & (\alpha_1 \alpha_3 \alpha_6 - \alpha_2 \alpha_4 \alpha_5) + \alpha_1 \alpha_5 (2\alpha_6 + \alpha_7 + 3\mu) (\alpha_3^2 \alpha_6 \mu - B \alpha_1 (\mu_d^2 + \alpha_3 \alpha_6)) \\ & + \alpha_2 \alpha_4 \alpha_5^2 (\alpha_5 + \alpha_6 + \alpha_7 + 3\mu) (B \alpha_1 - \mu (\alpha_3 + \alpha_5)) + \alpha_1 \alpha_5^2 (\alpha_6 + \mu)^2 \\ & (\alpha_5 \mu - B \alpha_1) + \alpha_1 \alpha_3 (\alpha_5^2 \mu (\alpha_4 + \alpha_6) (3\alpha_6 + \alpha_7 + 2\mu) - \alpha_5 \alpha_6 \mu (\alpha_4 + \alpha_6) \\ & (\alpha_6 - \alpha_7 - 2\mu) + \mu (\alpha_4 \alpha_6 \mu (\alpha_5 - \alpha_6) + 2\alpha_5 (\alpha_5 \alpha_6 \mu - \mu_d^2 \alpha_2 r))) - \mu_d \mu \end{aligned}$$

$$\begin{aligned}
& (\alpha_2\alpha_5\mu(\alpha_7 + \mu)(4\alpha_5 + 3\mu) - \mu_d\alpha_1\alpha_3\alpha_6^2 - \alpha_2\alpha_5\mu^2(5\alpha_5 + \mu)) + (\alpha_6 \\
& + \alpha_7 + 2\mu)\{\mu_d\alpha_2\alpha_5(B\alpha_1\mu_d - \alpha_5^2\mu - 2\mu_d\alpha_3\mu) + B\alpha_1(\alpha_1\alpha_5(\mu_d\alpha_2r - \alpha_3 \\
& \mu) + \alpha_2\alpha_3\alpha_6(\alpha_5 + \mu_d)) + \mu_d\alpha_5\mu(\alpha_1(\mu_d\alpha_6 - \alpha_2\alpha_5r + 2\alpha_5(\alpha_4 + \alpha_6) - B \\
& (2\alpha_4 + \alpha_6)) + \mu_d^2(\alpha_1 - \alpha_2) - \alpha_2(\alpha_3(\alpha_3 + 2\alpha_5) + \alpha_4(\alpha_3 + \alpha_5))) + \alpha_1\alpha_3 \\
& (\mu_d\alpha_1\alpha_6(\mu r - B) - \alpha_2\alpha_5\mu r(2\mu_d + \alpha_5)) + \mu_d\alpha_5(\alpha_1\mu(\mu_d + \alpha_3)(\alpha_3 + 2\alpha_4 \\
& + \alpha_5) + \alpha_2(\alpha_4 + \alpha_5)(B\alpha_1 - \mu_d))\} + (\alpha_5 + \alpha_6 + 2\mu)\{\mu_d\alpha_5(\mu(\mu_d\alpha_1 \\
& (\alpha_5 + \mu_d) + 2(\alpha_1\alpha_3 - \mu_d\alpha_5) - \alpha_2\alpha_5(2\alpha_3 + \alpha_4 + \alpha_5 + \alpha_1r)) + \alpha_1\alpha_5 \\
& (B\alpha_2 + 2\alpha_4\mu))\} + \mu_d^2\alpha_5(\alpha_4 + 2\alpha_5 + \mu)(\alpha_4 + 2\alpha_5 + \mu)(\alpha_1\alpha_2(B - \alpha_2r) \\
& + \mu(\alpha_1\alpha_4 - \alpha_2\alpha_3)) + \mu_d\alpha_1\alpha_5\mu(\alpha_3(\alpha_5 + \alpha_6)(\alpha_4 - 2\alpha_2r) + (3\alpha_7 + 4\mu) \\
& (\alpha_3\alpha_6 + \mu^2)) + \mu_d\alpha_5\{\alpha_5\mu((\alpha_6 + \mu)(2\mu_d\alpha_1 - \alpha_2\alpha_5) + \mu_d(\mu_d\alpha_1 - \alpha_2(\alpha_5 \\
& + \alpha_7 + 3\mu)) - \alpha_2(\alpha_5\mu + \alpha_4(2\alpha_7 + 7\mu))) + B\alpha_1((\alpha_4 + \alpha_5)(\alpha_2(\alpha_3 + \alpha_4) \\
& - \alpha_1(\alpha_4 + \mu_d)) - \alpha_1\alpha_4(\mu_d + \alpha_5) + \alpha_2\alpha_5(2\alpha_4 + \alpha_1r) - \alpha_1\alpha_3(\alpha_4 + 3\alpha_6 \\
& + \alpha_7 + 2\mu)) + \alpha_1\alpha_3\mu(\mu_d(\alpha_3 + 2\alpha_4 + 2\alpha_6 + \alpha_7) + \alpha_4\alpha_5 + 2(\alpha_3\alpha_6 + \alpha_5 \\
& (2\alpha_6 + \mu))) + \alpha_2\mu(\alpha_2r(\mu_d(2\alpha_3 + \alpha_4 + 2\alpha_5 + 2\mu) + \alpha_5(\alpha_5 + 4\mu)) - \alpha_4(\mu_d \\
& (\alpha_4 + 4\alpha_5 + 3\mu) + \alpha_3(2\alpha_5 + \mu)) - \mu_d(\alpha_3(\alpha_3 + \alpha_4 + 2\alpha_5) + \mu_d(3\alpha_5 + 2\mu))) \\
& + \alpha_1(\mu_d\alpha_4(B\alpha_2 + 2\alpha_5\mu) + \alpha_6\mu(\mu_d(\alpha_7 + 4\mu) + 4\alpha_5\mu))\} + \alpha_1\alpha_3\alpha_6(\alpha_1\alpha_6r \\
& (\mu_d\mu + B\alpha_2) + 2\alpha_5\mu(3\mu_d\mu + \alpha_2\alpha_5r) + B\alpha_2\alpha_5(\alpha_7 + 2\mu_d)) + \alpha_1\alpha_3\mu(\mu_d\alpha_6^2 \\
& (-2\alpha_5 + \alpha_6 + 2\mu) + B\alpha_1^2\alpha_5(2\alpha_6 - \mu)) + \alpha_2\alpha_5^2\mu(2\alpha_2\alpha_5\mu r - \mu_d\alpha_3^2)\Big] \Big/ \Big[\alpha_1 \\
& \{(\alpha_1 - \alpha_2)(\mu_d^3(\mu_d + 2\mu) - \mu_d\alpha_2r(\mu_d^2 + 3\alpha_5\mu)) + (\alpha_3 + \alpha_4)(\mu_d^2(\alpha_1 - \alpha_2) \\
& (\alpha_3 - \alpha_2r + \mu) + 2\mu_d^2(\alpha_1 - \mu_d\alpha_2)) + (\alpha_5 + \alpha_6)(2\mu_d(\alpha_3 + \alpha_5)(\alpha_1\alpha_4 - \alpha_2\mu) \\
& + \mu_d^2\alpha_1(\alpha_4 + \mu_d + 2\mu) + \mu_d^2(\alpha_1 - \alpha_2)(\alpha_3 + \alpha_4 + \alpha_5 + \mu + \mu_d) - \mu_d\alpha_2(\alpha_4 \\
& + \mu_d + (\alpha_4 + \alpha_5)(\alpha_3 + \alpha_5 + \mu_d) + \alpha_1r(\alpha_5 + \mu_d)) - \alpha_2\alpha_4\alpha_5^2) + \mu_d^2(\alpha_5 + \mu) \\
& (\alpha_2r(3\alpha_2 - \alpha_1) + \alpha_1(\alpha_4 + \mu) - \alpha_2\mu) + (\alpha_5 + \alpha_7 + \mu)((\alpha_1 - \alpha_2)(\alpha_4^2(2\mu_d + \alpha_5) \\
& - 2\mu_d\alpha_2r(\alpha_3 + \alpha_4)) + \alpha_2^2\alpha_5r(\alpha_3 + \alpha_4) + \alpha_2(3\mu_d + \alpha_5)(\alpha_2\alpha_5r - \alpha_4\mu_d) \\
& - \alpha_2\alpha_4\alpha_5(\alpha_1r + \mu_d)) + (\alpha_6 + \alpha_7 + \mu)((\alpha_3 + \alpha_5)(2\mu_d\alpha_1(\alpha_3 + \mu_d) + 2\mu_d(\alpha_1\alpha_6 \\
& - \alpha_2\alpha_3) - \alpha_2\alpha_3\alpha_6) + \alpha_3\mu_d(\mu_d(\alpha_1 - \alpha_2) - \alpha_2(\alpha_5 + \alpha_6)) + 2\alpha_1\alpha_5(\mu_d(\alpha_4 + \mu) \\
& + \alpha_3(\alpha_4 + \alpha_6 + \mu)) + \alpha_1(\alpha_3^2\alpha_6 + 2\mu_d\alpha_2\mu) - \alpha_2\alpha_5(\alpha_5(2\mu_d + \mu) + \alpha_3(\alpha_1r \\
& - \alpha_4))) + \alpha_1\mu_d(\alpha_6 + \alpha_7)(\alpha_6\mu_d - \alpha_2\alpha_5r) + \mu_d^2(\alpha_1 - \alpha_2)(\alpha_3 + \alpha_4)(\alpha_4 + \alpha_5 \\
& + \mu) + (\alpha_6 + \mu)(\alpha_5(\alpha_3 + \alpha_5)(2\alpha_1\mu_d - \alpha_2\alpha_5) + \alpha_1\alpha_5(\mu_d^2 - \alpha_2\alpha_5r + \alpha_4\alpha_5 \\
& + \alpha_5(\alpha_4 + \alpha_6 + \mu))) - (2\mu_d + \alpha_5)(\alpha_2\alpha_5\mu(2\alpha_4 + \mu) + \mu_d\alpha_2^2\alpha_7r) + \alpha_2\alpha_3\alpha_6^2 \\
& (\alpha_1r - \mu) + \mu_d\alpha_5\mu(2\alpha_1\mu - 2\alpha_2\alpha_5 - 3\mu_d) - \mu_d^2\alpha_5(2\alpha_5 + \alpha_1\mu) + \mu_d\alpha_1\mu \\
& (2\alpha_3\alpha_6 + \alpha_4(4\alpha_5 - \mu_d)) + \alpha_2^2\alpha_5^2\mu r + (\alpha_7 + \mu)((\alpha_7 + \mu)((\alpha_1 - \alpha_2)(-\alpha_2r
\end{aligned}$$

$$\begin{aligned}
& (\mu_d + \mu) + \mu_d \alpha_6 + \mu^2) + \alpha_1(\alpha_6 + \mu)(2\alpha_4 + \alpha_6) - \alpha_2(\alpha_4(\alpha_5 + \mu) + \alpha_6(\alpha_3 + \alpha_4) \\
& + \mu(\alpha_4 + \alpha_6)) + \alpha_1(\alpha_6(\mu_d + \mu) - \alpha_2 r(\alpha_3 + \alpha_6) + 2\mu_d(\alpha_3 + \alpha_4 + \mu))) + (\alpha_5 \\
& + \alpha_7 + \mu)(\alpha_4(\alpha_1 - \alpha_2)(\alpha_4 - \alpha_2 r) + \alpha_2(\alpha_2 r(\alpha_3 + \alpha_5) - 2\mu_d \alpha_4 + \alpha_5 \alpha_6)) + (\alpha_6 \\
& + \alpha_7 + \mu)(\mu_d(\mu_d(\alpha_1 - \alpha_2) - \alpha_2(2\alpha_3 + \alpha_5 + 2\mu)) + \alpha_3(\alpha_1(\alpha_3 + 2\alpha_4 + 2\alpha_6 \\
& + 2\mu) - \alpha_2(\alpha_3 + 2\alpha_4 + 2\alpha_5 + 2\mu)) - \alpha_2\mu(2\alpha_3 + \alpha_5)) + (\alpha_6 + \mu)(\alpha_5(\mu(2\alpha_1 \\
& - \alpha_2) - \alpha_2(2\alpha_3 + \alpha_5 + 2\alpha_1 r + 2\mu_d) + 2\alpha_1(2\alpha_4 + \alpha_6)) + \mu_d^2(\alpha_1 - \alpha_2) - \mu_d \alpha_2 \\
& (\alpha_3 + \alpha_5)) - \mu_d \mu(\alpha_1 - \alpha_2)(\alpha_2 r - 2\mu) + 3\mu_d(\alpha_3 + \mu)(\alpha_1 \mu_d - \alpha_2 \alpha_3) - 2\mu_d \\
& (\alpha_2 \mu_d(\alpha_3 + 2\alpha_5) - 2\alpha_1 \mu(\alpha_3 + \alpha_4 + \alpha_6)) - 3\mu_d \alpha_2 \alpha_5 \mu - \alpha_2 \mu(\alpha_5(\mu - 2\alpha_2 r) \\
& + 4\alpha_4(\mu_d + \alpha_5)) - \mu_d^2(3\alpha_2 \mu - \alpha_1 \alpha_6) + \alpha_1 \mu_d(2\alpha_4 \alpha_6 - \alpha_2 r(2\mu_d + \alpha_5 + 2\alpha_6)) \\
& + 2\mu_d^2(\alpha_1 \mu_d - \alpha_2) + \mu_d^2(\alpha_1 - \alpha_2)(\alpha_3 + \alpha_4) + \alpha_3 \mu_d(4\alpha_1(\alpha_4 + \alpha_6) - \alpha_2(\alpha_4 \\
& + \alpha_5)) + (\alpha_5 + \alpha_6)(2\mu_d \alpha_1(\alpha_4 + \alpha_6) - 2\alpha_2 \alpha_5(\alpha_4 + \mu_d))) \} \Big]
\end{aligned}$$

and

$$r = \frac{1}{2\alpha_1\alpha_2} \left\{ \begin{aligned} & \mu_d(\alpha_1 + \alpha_2) + \alpha_1(\alpha_4 + \alpha_6 + \mu) + \alpha_2(\alpha_3 + \alpha_5 + \mu) \\ & + \left(\begin{aligned} & \mu_d^2(\alpha_1 - \alpha_2)^2 + (\alpha_4 + \alpha_6 + \mu) \left(\alpha_1^2(\alpha_4 + \alpha_6 + \mu + 2\mu_d) \right)^{\frac{1}{2}} \\ & - 2\alpha_1\alpha_2(\alpha_5 + \mu) - 2\mu_d\alpha_1\alpha_2(\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + 2\mu) \\ & + \alpha_2^2(\alpha_3 + \alpha_5 + \mu + 2\mu_d) + 2\alpha_1\alpha_2\alpha_3(\alpha_4 - \alpha_6 - \mu) \end{aligned} \right) \end{aligned} \right\}$$

Now, we evaluate the basic reproduction number known as threshold R_0 using next generation matrix method [6].

$$\text{Let } X' = (S_M, I_1, I_2, M)' \text{ and } X' = \frac{dX}{dt} = F(X) - V(X)$$

where, $F(X)$ denotes the rate of arrival of new individual in the compartment and $V(X)$ denotes the rate of transmission of infection which are given by

$$F(X) = \begin{bmatrix} \alpha_1 S_M I_1 \\ \alpha_2 S_M I_2 \\ 0 \\ 0 \end{bmatrix} \text{ and } V(X) = \begin{bmatrix} \alpha_3 I_1 - \alpha_4 I_2 + \alpha_5 I_1 + (\mu + \mu_d) I_1 \\ -\alpha_3 I_1 + \alpha_4 I_2 + \alpha_6 I_2 + (\mu + \mu_d) I_2 \\ -\alpha_5 I_1 - \alpha_6 I_2 + \alpha_7 M + (\mu + \mu_d) M \\ -B + \alpha_1 S_M I_1 + \alpha_2 S_M I_2 - \alpha_7 M + \mu S_M \end{bmatrix}$$

$$\text{Now, } DF(X_0) = \begin{bmatrix} f & 0 \\ 0 & 0 \end{bmatrix} \text{ and } DV(X_0) = \begin{bmatrix} v & 0 \\ J_1 & J_2 \end{bmatrix}, \text{ where } f \text{ and } v \text{ are}$$

$$4 \times 4 \text{ matrices defined as } f = \left[\frac{\partial F_i(X_0)}{\partial X_j} \right] \text{ and } v = \left[\frac{\partial V_i(X_0)}{\partial X_j} \right]$$

Finding f and v , we get

$$f = \begin{bmatrix} \alpha_1 S_M & 0 & 0 & \alpha_1 I_1 \\ 0 & \alpha_2 S_M & 0 & \alpha_2 I_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } v = \begin{bmatrix} \alpha_3 + \alpha_5 + \mu + \mu_d & -\alpha_4 & 0 & 0 \\ -\alpha_3 & \alpha_4 + \alpha_6 + \mu + \mu_d & 0 & 0 \\ -\alpha_5 & -\alpha_6 & \alpha_7 + \mu + \mu_d & 0 \\ \alpha_1 S_M & \alpha_2 S_M & -\alpha_7 & \alpha_1 I_1 + \alpha_2 I_2 + \mu \end{bmatrix}$$

Here, v is non-singular matrix.

For this model, matrix fv^{-1} is calculated and the basic reproduction (threshold) R_0 at Y_0 is obtained as

$$R_0 = \frac{B(\alpha_1 \alpha_4 + \alpha_2 \alpha_3)}{\mu((\mu + \mu_d)^2 + (\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)(\mu + \mu_d) + \alpha_3 \alpha_6 + \alpha_5(\alpha_4 + \alpha_6))} \quad (2.2)$$

3. Stability

3.1. Local Stability

In this section, for both equilibria points local as well as global stability is discussed.

Theorem 3.1.1 (Stability at Y_0): $Y_0 = \left(\frac{B}{\mu}, 0, 0, 0\right)$ is locally asymptotically stable if

$$\mu((\alpha_4 + \alpha_6)(B\alpha_1 - \alpha_5\mu) + B\alpha_2(\alpha_3 + \alpha_5)) < (\mu(\mu_d + \mu)(\mu(\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \mu) - (\alpha_1 + \alpha_2)B) + B^2\alpha_1\alpha_2 + \mu^2\alpha_3\alpha_6).$$

Proof: The Jacobian matrix J_0 at Y_0 is given by

$$J_0 = \begin{bmatrix} -\mu & -\frac{\alpha_1 B}{\mu} & -\frac{\alpha_2 B}{\mu} & \alpha_7 \\ 0 & \frac{\alpha_1 B}{\mu} - \mu_d - \alpha_3 - \alpha_5 - \mu & \alpha_4 & 0 \\ 0 & \alpha_3 & \frac{\alpha_2 B}{\mu} - \mu_d - \alpha_4 - \alpha_6 - \mu & 0 \\ 0 & \alpha_5 & \alpha_6 & -\alpha_7 - \mu - \mu_d \end{bmatrix}$$

Jacobian J_0 has eigenvalues,

$$\begin{aligned} \lambda_1 &= -\mu < 0, \lambda_2 = -(\alpha_7 + \mu + \mu_d) < 0, \\ \lambda_3 &= \frac{1}{\mu} \left(\begin{aligned} & -\frac{\mu}{2} (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + 2(\mu + \mu_d)) + \frac{B}{2} (\alpha_1 + \alpha_2) \\ & + \frac{1}{2} \left(\begin{aligned} & B^2 (\alpha_1 - \alpha_2)^2 - 2B\mu (\alpha_1 - \alpha_2) (\alpha_3 - \alpha_4 + \alpha_5 - \alpha_6) \\ & + \mu^2 ((\alpha_3 + \alpha_4)^2 + 2(\alpha_5 - \alpha_6) (\alpha_3 - \alpha_4) + (\alpha_5 - \alpha_6)^2) \end{aligned} \right)^{\frac{1}{2}} \end{aligned} \right), \\ \lambda_4 &= \frac{1}{\mu} \left(\begin{aligned} & -\frac{\mu}{2} (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + 2(\mu + \mu_d)) + \frac{B}{2} (\alpha_1 + \alpha_2) \\ & - \frac{1}{2} \left(\begin{aligned} & B^2 (\alpha_1 - \alpha_2)^2 - 2B\mu (\alpha_1 - \alpha_2) (\alpha_3 - \alpha_4 + \alpha_5 - \alpha_6) \\ & + \mu^2 ((\alpha_3 + \alpha_4)^2 + 2(\alpha_5 - \alpha_6) (\alpha_3 - \alpha_4) + (\alpha_5 - \alpha_6)^2) \end{aligned} \right)^{\frac{1}{2}} \end{aligned} \right) \end{aligned}$$

Here λ_3 and λ_4 are negative if

$$\mu ((\alpha_4 + \alpha_6) (B\alpha_1 - \alpha_5\mu) + B\alpha_2 (\alpha_3 + \alpha_5)) < (\mu (\mu_d + \mu) (\mu (\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \mu) - (\alpha_1 + \alpha_2) B) + B^2 \alpha_1 \alpha_2 + \mu^2 \alpha_3 \alpha_6)$$

As all eigenvalues of matrix are negative, equilibrium point of system is stable.

Theorem 3.1.2 (Stability at Y^*): Endemic point Y^* is locally asymptotically stable if the following conditions hold:

- i) x_5 and $x_7 < 0$,
- ii) $\alpha_3 (x_1 + x_8) < x_3 x_4$,
- iii) $x_4 (x_7 + x_8) > \alpha_5 \alpha_7$,
- iv) $x_6 (x_5 + x_8) > \alpha_6 \alpha_7$, v) $x_2 x_6 < \alpha_4 x_1$, vi) $\alpha_5 x_7 > \alpha_3 \alpha_6$, vii) $\alpha_6 x_5 > \alpha_4 \alpha_5$

Proof: We determine the local stability behavior of endemic equilibrium point Y^* by using the Jacobian matrix J^* which is given by

$$J^* = \begin{bmatrix} x_1 & x_4 & x_6 & \alpha_7 \\ x_2 & x_5 & \alpha_4 & 0 \\ x_3 & \alpha_3 & x_7 & 0 \\ 0 & \alpha_5 & \alpha_6 & x_8 \end{bmatrix}$$

where, $x_1 = -(\alpha_1 I_1^* + \alpha_2 I_2^* + \mu)$, $x_2 = \alpha_1 I_1^*$, $x_3 = \alpha_2 I_2^*$, $x_4 = -\alpha_1 S_M^*$,
 $x_5 = \alpha_1 S_M^* - \alpha_3 - \alpha_5 - \mu - \mu_d$, $x_6 = -\alpha_1 S_M^*$,
 $x_7 = \alpha_2 S_M^* - \alpha_4 - \alpha_6 - \mu - \mu_d$, $x_8 = -(\alpha_7 + \mu + \mu_d)$

The corresponding characteristic equation of Jacobian matrix is

$$\lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0$$

where,

$$a_1 = -x_1 - x_5 - x_7 - x_8,$$

$$a_2 = x_1(x_5 + x_7 + x_8) + x_5(x_7 + x_8) + x_7x_8 - \alpha_3\alpha_4 - x_2x_4 - x_3x_6,$$

$$a_3 = \alpha_4(\alpha_3(x_1 + x_8) - x_3x_4) + x_2(x_4(x_7 + x_8) - \alpha_5\alpha_7) + x_3\left(x_6(x_5 + x_8) - \alpha_6\alpha_7\right) + \alpha_3x_8(x_2x_6 - \alpha_4x_1) + \alpha_7x_2(\alpha_5x_7 - \alpha_3\alpha_6) - x_1\left(x_5(x_7 + x_8) + x_7x_8\right) - x_5x_7x_8 - \alpha_3x_2x_6,$$

$$a_4 = \alpha_3x_8(x_2x_6 - \alpha_4x_1) + \alpha_7x_2(\alpha_5x_7 - \alpha_3\alpha_6) + \alpha_7x_3(\alpha_6x_5 - \alpha_4\alpha_5) + x_8(x_4(\alpha_4x_3 - x_2x_7) + x_5(x_1x_7 - x_3x_6))$$

Here, $x_1, x_4, x_6, x_8 < 0$ and $x_2, x_3 > 0$ is obvious.

The equilibrium point is locally asymptotically stable [15], if following conditions are hold:

- i) x_5 and $x_7 < 0$,
- ii) $\alpha_3(x_1 + x_8) < x_3x_4$,
- iii) $x_4(x_7 + x_8) > \alpha_5\alpha_7$,
- iv) $x_6(x_5 + x_8) > \alpha_6\alpha_7$, v) $x_2x_6 < \alpha_4x_1$, vi) $\alpha_5x_7 > \alpha_3\alpha_6$, vii) $\alpha_6x_5 > \alpha_4\alpha_5$

3.2. Global Stability

Now, let us discuss the global stability behavior of the equilibrium Y_0 and Y^* using Lyapunov's function.

Theorem 3.2.1 (Stability at Y_0): The disease-free equilibrium point Y_0 is globally asymptotically stable.

Proof: Consider the Lyapunov's function

$$L_1(t) = I_1(t) + I_2(t) + M(t)$$

then

$$\begin{aligned} L_1'(t) &= I_1'(t) + I_2'(t) + M'(t) \\ &= S_M(\alpha_1 I_1 + \alpha_2 I_2) - (I_1 + I_2 + M)(\mu + \mu_d) - \alpha_7 M \end{aligned}$$

we get $\frac{dL_1}{dt} < 0$ if $\alpha_1 B < \mu^2$ and $\alpha_2 B < \mu^2$

whereas, $\frac{dL_1}{dt} = 0$ only if $I_1, I_2, M = 0$.

Therefore, Y_0 is globally asymptotically stable [11].

Theorem 3.2.2 (Stability at Y^*): The endemic equilibrium point Y^* is globally asymptotically stable.

Proof: Consider the Lyapunov function,

$$L(t) = \frac{1}{2}[(S_M - S_M^*) + (I_1 - I_1^*) + (I_2 - I_2^*) + (M - M^*)]^2$$

then

$$\begin{aligned} L'(t) &= [(S_M - S_M^*) + (I_1 - I_1^*) + (I_2 - I_2^*) \\ &\quad + (M - M^*)](S_M' + I_1' + I_2' + M') \\ &= -\left[((S_M - S_M^*) + (I_1 - I_1^*) + (I_2 - I_2^*) + (M - M^*)) \right. \\ &\quad \left. (\mu((S_M - S_M^*) + (I_1 - I_1^*) + (I_2 - I_2^*) + (M - M^*)) \right. \\ &\quad \left. + \mu_d(I_1 + I_2 + M)) \right] < 0 \end{aligned}$$

Here, we denote $B = \mu(S_M^* + I_1^* + I_2^* + M^*)$.

Therefore, Y^* is globally stable.

In next section, we validate the model by numerical simulation using the parametric values given in Table 1 and system of equation (2.1).

4. Numerical Simulations

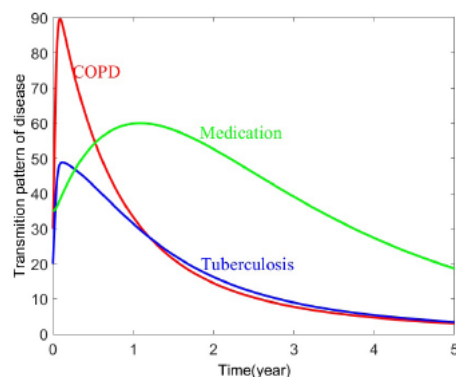


Figure 2: Transmission pattern of diseased individuals procuring medication

Figure 2 shows the transmission pattern of the disease. It can be noted that as the disease is detected, around 35 individuals starts taking precautions or medication. Some may neglect the symptoms, but after approximately a year they starts procuring medications. Also, around 29 COPD patients suffer from tuberculosis after a year.

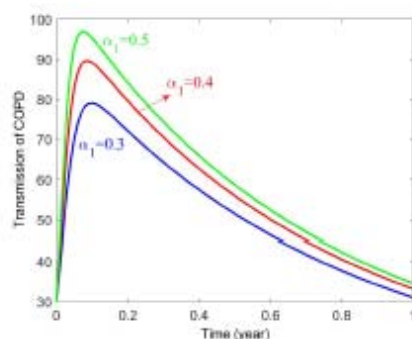


Figure 3: Effect of change in α_1 on COPD

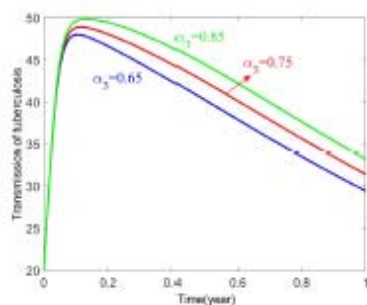


Figure 4: Effect of change in α_3 on tuberculosis

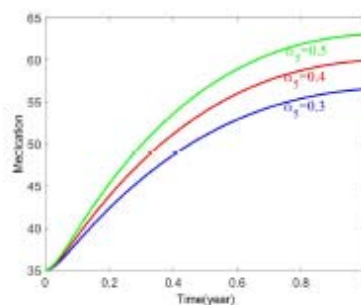


Figure 5: Impact of change of α_5 on medication

Increase in the rates corresponding to disease and medication results in change of outputs. So, the following graphs is to justify the findings. As shown in Figure 3, increase in α_1 by 20% results in increase of individual suffering from COPD by 20.79%. Also, 4.07% increase of individual suffering from tuberculosis will be observed if α_3 increases by 20% (Figure 4). Figure 5 shows that 20% increase in rate of individuals suffering from COPD (α_5) results in increase of rate of individual procuring medication from 56 individuals to 63 individuals.

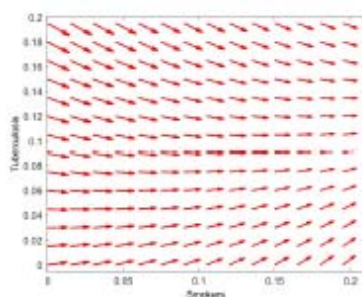


Figure 6: Intensity of tuberculosis among smokers

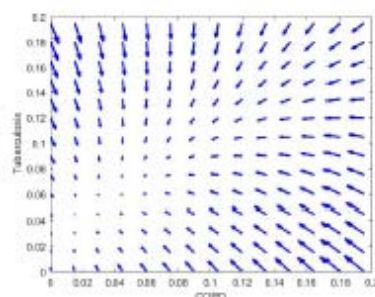


Figure 7: Directional flow of COPD and tuberculosis

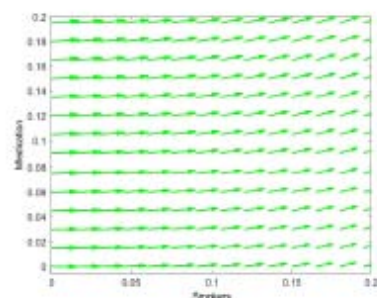


Figure 8: Intensity of medication among smokers

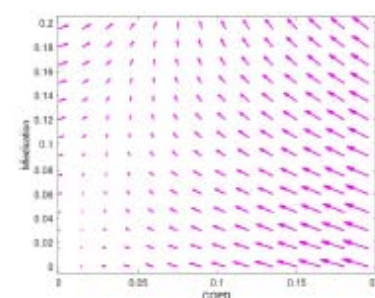


Figure 9: Intensity flow of COPD towards medication

To visualize the intensity or flow among the compartments, following quiver graphs are analyzed. Intensity of individuals suffering from tuberculosis due to smoking gradually decreases after taking proper medication (Figure 6). Intensity of individuals suffering from COPD is more as compared to tuberculosis, but it gradually decreases after taking proper medication (Figure 7). Intensity of medication increases among smokers and slowly decreases as they start getting cured or quit smoking (Figure 8). Intensity of individuals suffering from COPD decreases while taking medication (Figure 9).

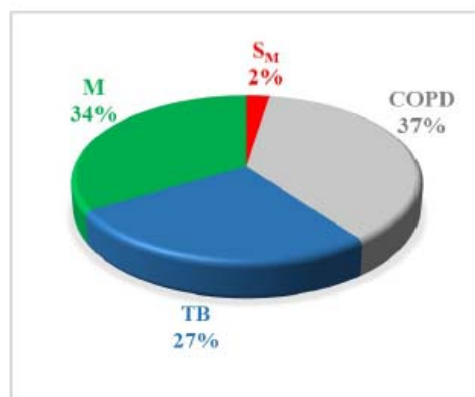


Figure 10: Percentage of all compartment

Here, among 2% smokers, 37% are affected by COPD and 27% by Tuberculosis. Out of these many patients, 34% procures medication (Figure 10).

Conclusion

To study the impact of medication on individuals suffering from COPD and tuberculosis due to smoking, a mathematical model is formulated. Equilibrium points are calculated for proposed system of nonlinear ordinary differential equations. Also, at both equilibrium points, system is locally as well as globally stable. A basic reproduction number has been evaluated which is equal to 0.7064 which connotes that approximately 71% of the individuals in the society are suffering from COPD and tuberculosis due to smoking who needs medication. Numerical simulations have been carried out to justify the results, which interprets that with the increase in the rate of affected individuals due to smoking will increase the morbidity and mortality rate all over the world. Also, quiver graph shows the intensity

or flow of individuals in their respective compartment. To keep the body healthy and protect lungs and other symptoms from getting worse, smoking cessation is the best thing one can do. This can help people to stay more active and enjoy life.

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