Proyecciones Journal of Mathematics
Vol. 38, N^o 1, pp. 13-30, March 2019.
Universidad Católica del Norte
Antofagasta - Chile

3-product cordial labeling of some snake graphs

P. Jeyanthi
Govindammal aditanar college for women, India
A. Maheswari
Kamaraj college of engineering and technology, India
and
M. Vijayalakshmi
Dr. G. U. Pope college of engineering, India
Received : March 2016. Accepted : October 2018

Abstract

Let G be a (p,q) graph. A mapping $f : V(G) \rightarrow \{0, 1, 2\}$ *is called 3-product cordial labeling if* $|v_f(i) - v_f(j)| \leq 1$ *and* $|e_f(i) - e_f(j)| \leq 1$ *for any* $i, j \in \{0, 1, 2\}$*,where* $v_f(i)$ *denotes the number of vertices labeled with i,* $e_f(i)$ *denotes the number of edges xy with* $f(x)f(y) \equiv i(mod 3)$*. A graph with 3-product cordial labeling is called 3-product cordial graph. In this paper we investigate the 3-product cordial behavior of alternate triangular snake, double alternate triangular snake and triangular snake graphs.*

Keywords *: cordial labeling, product cordial labeling, 3-product cordial labeling, 3-product cordial graph, alternate triangular snake, double alternate triangular snake, triangular snake graph.*

AMS Subject Classification (2010) *: 05C78.*

1. Introduction

Let G be a graph with p vertices and q edges. All graphs considered here are simple, finite, connected and undirected. For basic notations and terminology, we follow [3]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a complete survey of graph labeling is available in [2]. Cordial labeling is a weaker version of graceful labeling and harmonious labeling introduced by Cahit in [1]. Let f be a function from the vertices of G to $\{0,1\}$ and for each edge xy assign the label $|f(x)-f(y)|$. f is called a cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1, and the number of edges labeled 0 and the number of edges labeled 1 differ at most by 1. Let f be a function from $V(G)$ to $\{0,1\}$. For each edge uv, assign the label $f(u)f(v)$. Then f is called product cordial labeling if $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$ where $v_f(i)$ and $e_f(i)$ denotes the number of vertices and edges respectively labeled with $i(i=0,1)$. Sundaram et al. introduced the concept of EP-cordial labeling in [9]. A vertex labeling $f:V(G)\rightarrow\{-1,0,1\}$ is said to be an EP-cordial labeling if it induces the edge labeling f^* defined by $f^*(uv)=f(u)f(v)$ for each $uv\in E(G)$ and if $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(i)-e_f(j)|\leq 1$ for any $i\neq j$ $i,j\in\{-1,0,1\}$,where $v_f(x)$ and $e_f(x)$ denotes the number of vertices and edges of G having the label $x\in\{-1,0,1\}$. In [8] it is remarked that any EP-cordial labeling is 3-product cordial labeling. A mapping $f:V(G)\rightarrow\{0,1,2\}$ is called 3-product cordial labeling if $|v_f(i)-v_f(j)|\leq 1$ and $|e_f(i)-e_f(j)|\leq 1$ for any $i,j\in\{0,1,2\}$,where $v_f(i)$ denotes the number of vertices labeled with i, $e_f(i)$ denotes the number of edges xy with $f(x)f(y)\equiv i(mod3)$. A graph with 3-product cordial labeling is called 3-product cordial graph. Jeyanthi and Maheswari [4]-[7] proved that the graphs $\langle B_{n,n}:w\rangle$, $C_n\cup P_n$, $C_m\circ\overline{K_n}$ if $m\geq 3$ and $n\geq 1$, $P_m\circ\overline{K_n}$ if $m,n\geq 1$,duplicating arbitrary vertex of a cycle C_n , duplicating arbitrary edge of a cycle C_n, duplicating arbitrary vertex of a wheel W_n , middle graph of P_n , the splitting graph of P_n, total graph of P_n , $P_n[P_2]$, P_n^2, $K_{2,n}$, vertex switching of C_n, ladder L_n, triangular ladder TL_n, graph $\left\langle w_n^{(1)},w_n^{(2)}\ldots w_n^{(k)}\right\rangle$, splitting graphs $S'(K_{1,n})$, $S'(B_{n,n})$, shadow graph $D_2(B_{n,n})$, square graph $B_{n,n}^2$ and star graphs are 3-product cordial graphs. Also they proved that a complete graph K_n is a 3-product cordial graph if and only if $n\leq 2$.

In addition, they proved that if $G(p,q)$ is a 3-product cordial graph (i) $p\equiv 1(mod3)$ then $q\leq\frac{p^2-2p+7}{3}$.(ii) $p\equiv 2(mod3)$ then $q\leq\frac{p^2-p+4}{3}$ (iii)

$p \equiv 0(mod3)$ then $q \leq \frac{p^2-3p+6}{3}$ and if G_1 is a 3-product cordial graph with $3m$ vertices and $3n$ edges and G_2 is any 3-product cordial graph then $G_1 \cup G_2$ is also 3-product cordial graph.

We use the following definitions in the subsequent section.

Definition 1.1. *A triangular snake T_n is obtained from a path P_n by replacing each edge of the path by a triangle C_3.*

Definition 1.2. *An alternate triangular snake $A(T_n)$ is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i . That is every alternate edge of path is replaced by C_3. We have three types of alternate triangular snake namely (i). $A_1(T_n)$- the triangle starts from u_1 and ends with u_n, (ii). $A_2(T_n)$ - the triangle starts from u_1 and ends with u_{n-1} (or the triangle starts from u_2, ends with u_n) and (iii). $A_3(T_n)$ - if the triangle starts from u_2 and ends with u_{n-1}.*

Definition 1.3. *A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, a double alternate triangular snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to two new vertices v_i and w_i. We have three types of double alternate triangular snake namely (i). $DA_1(T_n)$ - the double triangle starts from u_1 and ends with u_n,*
(ii). $DA_2(T_n)$ - the double triangle starts from u_1
and ends with u_{n-1} (or the triangle starts from u_2 , ends with u_n) and
(iii). $DA_3(T_n)$ - the double triangle starts from u_2 and ends with u_{n-1}. For any real number n, $\lceil n \rceil$ denotes the smallest integer $\geq n$ and $\lfloor n \rfloor$ denotes the greatest integer $\leq n$.

2. Main Results

In this section we investigate the 3-product cordial behaviour of alternate triangular snake, double alternate triangular snake and triangular snake graphs.
Let $A(T_n)$ be an alternate triangular snake graph obtained from a path $u_1, u_2 \ldots, u_n$ by joining u_i and u_{i+1} (alternately) to a new vertex v_i where $1 \leq i \leq n-1$ for even n and $1 \leq i \leq n-2$ for odd n.

Therefore, $V(A(T_n)) = \{u_i, v_j : 1 \leq i \leq n, 1 \leq j \leq \lfloor \frac{n}{2} \rfloor\}$.

We note that $|V(A(T_n))| = \begin{cases} \frac{3n}{2}, & n \equiv 0(mod2) \\ \frac{3n-1}{2}, & n \equiv 1(mod2), \end{cases}$ and
$|E(A(T_n))| = \begin{cases} 2n-1, & n \equiv 0(mod2) \\ 2n-2, & n \equiv 1(mod2). \end{cases}$

Theorem 2.1. *(1). An alternate triangular snake graph $A_1(T_n)$ is a 3-product cordial graph if and only if $n \equiv 0, 1(mod3)$.*
(2). An alternate triangular snake graph $A_2(T_n)$ is a 3 - product cordial graph.
(3). An alternate triangular snake graph $A_3(T_n)$ is a 3 - product cordial graph.

Proof. (1). Define a vertex labeling $f : V(A_1(T_n)) \to \{0, 1, 2\}$ by considering the following two cases.

Case (i). $n \equiv 0(mod3)$. Take $n = 3k$.
Then $|V(A_1(T_n))| = \frac{9k}{2}$ and $|E(A_1(T_n))| = 6k - 1$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k$,
$f(u_i) = \begin{cases} 1, & if\ j \equiv 0, 1(mod4) \\ 2, & if\ j \equiv 2, 3(mod4); \end{cases}$.

For $1 \leq i \leq \frac{k}{2}, f(v_i) = 0$; For $i = \frac{k}{2} + j, 1 \leq j \leq k$,
$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if j \equiv 1(mod2). \end{cases}$

In view of the above labeling pattern we have $v_f(0) = v_f(1) = v_f(2) = \frac{3k}{2}$ and $e_f(0) = e_f(1) + 1 = e_f(2) = 2k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod3)$.

Case (ii). $n \equiv 1(mod3)$. Take $n = 3k + 1$.
Then $|V(A_1(T_n))| = \frac{9k+3}{2}$ and $|E(A_1(T_n))| = 6k + 1$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 1$,
$f(u_i) = \begin{cases} 1, & if\ j \equiv 1, 2(mod4) \\ 2, & if\ j \equiv 0, 3(mod4); \end{cases}$

For $1 \leq i \leq \frac{k+1}{2}, f(v_i) = 0$; For $i = \frac{k+1}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0 (mod 2) \\ 2, & if \ j \equiv 1 (mod 2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) = v_f(2) = \frac{3k+1}{2}$ and $e_f(0) - 1 = e_f(1) = e_f(2) = 2k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod 3)$.

Conversely assume that $n \equiv 2(mod 3)$ and take $n = 3k + 2$. Then $|V(A_1(T_n))| = 3\left(\frac{3k+2}{2}\right)$ and $|E(A_1(T_n))| = 6k + 3$. Hence we have $v_f(0) = v_f(1) = v_f(2) = \frac{3k+2}{2}$ and $e_f(0) = e_f(1) = e_f(2) = 2k + 1$. If either $f(u_i) = 0$ or $f(v_i) = 0$ for $1 \leq i \leq \frac{3k}{2} + 1$ then $e_f(0) = 3k + 2$. If $f(u_i) = 0$ for $1 \leq i \leq k + 1$ and $f(v_i) = 0$ for $1 \leq i \leq \frac{k}{2}$ then $e_f(0) = 2k + 2$. If $f(u_i) = 0$ for $1 \leq i \leq \frac{k}{2}$ and $f(v_i) = 0$ for $1 \leq i \leq k+1$ then $e_f(0) = \frac{5k}{2} + 2$. In each case we get $e_f(0) > 2k + 1$. Hence, f is not a 3-product cordial labeling when $n \equiv 2(mod 3)$.

(2). Here $|V(A_2(T_n))| = \frac{3n-1}{2}$ and $|E(A_2(T_n))| = 2n - 2$.
Define a vertex labeling $f : V(A_2(T_n)) \rightarrow \{0, 1, 2\}$by considering the following three cases.

Case (i). $n \equiv 0(mod 3)$, $n = 3k$.
Then $|V(A_2(T_n))| = \frac{9k}{2} - 1$ and $|E(A_2(T_n))| = 6k - 2$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k$,

$$f(u_i) = \begin{cases} 1, & if \ j \equiv 1, 2(mod 4) \\ 2, & if \ j \equiv 0, 3(mod 4); \end{cases}$$

For $k > 1$, $1 \leq i \leq \frac{k-1}{2}, f(v_i) = 0$;

For $i = \frac{k-1}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod 2) \\ 2, & if \ j \equiv 1(mod 2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) - 1 = v_f(2) = \frac{3k-1}{2}$ and $e_f(0) = e_f(1) + 1 = e_f(2) + 1 = 2k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod 3)$.

Case (ii). $n \equiv 1(mod 3)$, $n = 3k + 1$.

Then $|V(A_2(T_n))| = \frac{9k+2}{2}$ and $|E(A_2(T_n))| = 6k - 1$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 0,1(mod4) \\ 2, & if\ j \equiv 2,3(mod4); \end{cases}$$

For $1 \leq i \leq \frac{k}{2}, f(v_i) = 0$; For $i = \frac{k}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) - 1 = v_f(2) = \frac{3k}{2}$ and $e_f(0) = e_f(1) = e_f(2) = 2k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod3)$.

Case (iii). $n \equiv 2(mod3)$, $n = 3k + 2$.
Then $|V(A_2(T_n))| = \frac{9k+5}{2}$ and $|E(A_2(T_n))| = 6k + 2$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 2$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1,2(mod4) \\ 2, & if\ j \equiv 0,3(mod4); \end{cases}$$

For $1 \leq i \leq \frac{k+1}{2}, f(v_i) = 0$; For $i = \frac{k+1}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) = v_f(2) - 1 = \frac{3k+1}{2}$ and $e_f(0) = e_f(1) = e_f(2) + 1 = 2k + 1$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 2(mod3)$.
(3). Here $|V(A_3(T_n))| = \frac{3n-2}{2}$ and $|E(A_3(T_n))| = 2n - 3$.
Define a vertex labeling $f : V(A_3(T_n)) \to \{0, 1, 2\}$by considering the following three cases.

Case (i). $n \equiv 0(mod3)$, $n = 3k$.

Then $|V(A_3(T_n))| = \frac{9k}{2} - 1$ and $|E(A_3(T_n))| = 6k - 3$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1,2(mod4) \\ 2, & if\ j \equiv 0,3(mod4); \end{cases}$$

For $k > 2$, $1 \leq i \leq \frac{k-2}{2}, f(v_i) = 0$;

For $i = \frac{k-2}{2} + j, 1 \leq j \leq k$,
$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = \frac{3k}{2}$ and $e_f(0) = e_f(1) = e_f(2) = 2k - 1$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod3)$.

Case (ii). $n \equiv 1(mod3)$, $n = 3k + 1$.
Then $|V(A_3(T_n))| = \frac{9k+1}{2}$ and $|E(A_3(T_n))| = 6k - 1$.
For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 1$,
$$f(u_i) = \begin{cases} 1, & if\ j \equiv 0, 1(mod4) \\ 2, & if\ j \equiv 2, 3(mod4); \end{cases}$$

For $k > 1$, $1 \leq i \leq \frac{k-1}{2}, f(v_i) = 0$;

For $i = \frac{k-1}{2} + j, 1 \leq j \leq k$,
$$f(v_i) = \begin{cases} 1, & if j \equiv 1(mod2) \\ 2, & if\ j \equiv 0(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = \frac{3k+1}{2}$ and $e_f(0) + 1 = e_f(1) = e_f(2) = 2k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod3)$.

Case (iii). $n \equiv 2(mod3)$, $n = 3k + 2$.

Then $|V(A_3(T_n))| = \frac{9k+4}{2}$ and $|E(A_3(T_n))| = 6k + 1$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 1$,
$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1, 2(mod4) \\ 2, & if\ j \equiv 0, 3(mod4); \end{cases}$$
$f(u_n) = 2$;

For $1 \leq i \leq \frac{k}{2}, f(v_i) = 0$; For $i = \frac{k}{2} + j, 1 \leq j \leq k$,

$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = \frac{3k}{2} + 1$ and $e_f(0) = e_f(1) = e_f(2) - 1 = 2k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 2(mod3)$.

□

An example of 3-product cordial labeling of $A_1(T_{10})$ is shown in Figure 1.

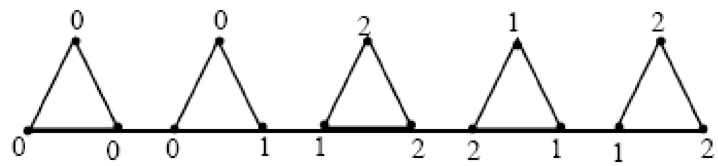

Figure 1

An example of 3-product cordial labeling of $A_2(T_9)$ is shown in Figure 2.

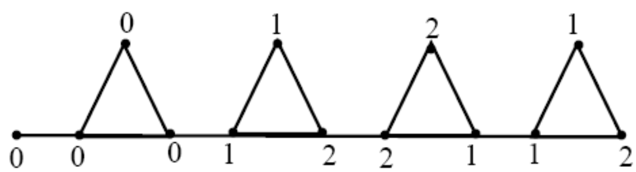

Figure 2

Theorem 2.2. *(1). A double alternate triangular snake graph $DA_1(T_n)$ is a 3-product cordial graph.*
(2). A double alternate triangular snake graph $DA_2(T_n)$ is a 3 - product cordial graph if and only if $n \equiv 0, 1(mod3)$.
(3). A double alternate triangular snake graph $DA_3(T_n)$ is a 3 - product cordial graph.

Proof. (1). Here $|V(DA_1(T_n))| = 2n$ and $|E(DA_1(T_n))| = 3n - 1$.
Define a vertex labeling $f : V(DA_1(T_n)) \rightarrow \{0, 1, 2\}$ by considering the

following two cases.

Case (i). $n \equiv 0(mod 3)$. Take $n = 3k$.
Then $|V(DA_1(T_n))| = 6k$ and $|E(DA_1(T_n))| = 9k - 1$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1, 2(mod 4) \\ 2, & if\ j \equiv 0, 3(mod 4); \end{cases}$$

For $1 \leq i \leq \frac{k}{2}, f(v_i) = f(w_i) = 0$;

For $i = \frac{k}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod 2) \\ 2, & if j \equiv 1(mod 2); \end{cases}$$

and

$$f(w_i) \begin{cases} = & 1, & if j \equiv 1(mod 2) \\ & 2, & if j \equiv 0(mod 2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) = v_f(2) = 2k$ and $e_f(0) = e_f(1) = e_f(2) + 1 = 3k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod 3)$.

Case (ii). $n \equiv 1(mod 3)$. Take $n = 3k + 1$.

Then $|V(DA_1(T_n))| = 6k + 2$ and $|E(DA_1(T_n))| = 9k + 2$.

For $k > 1, 1 \leq i \leq k - 1, f(u_i) = 0$; For $i = k - 1 + j, 1 \leq j \leq 2k + 2$;

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1, 2(mod 4) \\ 2, & if\ j \equiv 0, 3(mod 4); \end{cases}$$

For $1 \leq i \leq \frac{k+1}{2}, f(v_i) = f(w_i) = 0$;

For $i = \frac{k+1}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod 2) \\ 2, & if\ j \equiv 1(mod 2); \end{cases}$$

and

$$f(w_i) = \begin{cases} 1, & if j \equiv 1(mod 2) \\ 2, & if\ j \equiv 0(mod 2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = 2k + 1$ and $e_f(0) = e_f(1) = e_f(2) + 1 = 3k + 1$. Thus we have

$|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod3)$.

Case (iii). $n \equiv 2(mod3)$. Take $n = 3k + 2$.

Then $|V(DA_1(T_n))| = 6k + 4$ and $|E(DA_1(T_n))| = 9k + 5$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 2$,
$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1, 2(mod4) \\ 2, & if\ j \equiv 0, 3(mod4); \end{cases}$$
For $1 \leq i \leq \frac{k}{2}, f(v_i) = 0$; For $i = \frac{k}{2} + j, 1 \leq j \leq k + 1$,
$$f(v_i) = \begin{cases} 1, & ifj \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2); \end{cases}$$
For $1 \leq i \leq \frac{k}{2} + 1, f(w_i) = 0$; For $i = \frac{k}{2} + 1 + j, 1 \leq j \leq k$,
$$f(w_i) = \begin{cases} 1, & ifj \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$
In view of the above labeling pattern we have $v_f(0) = v_f(1) - 1 = v_f(2) = 2k + 1$ and $e_f(0) = e_f(1) + 1 = e_f(2) = 3k + 2$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 2(mod3)$.
(2). Suppose that $n \equiv 2(mod3)$, $n = 3k + 2$.

Hence $|V(DA_2(T_n))| = 6k + 3$ and $|E(DA_2(T_n))| = 9k + 3$.

Assume that f is a 3-product cordial labeling. Hence we have $v_f(0) = v_f(1) = v_f(2) = 2k + 1$ and $e_f(0) = e_f(1) = e_f(2) = 3k + 1$. If $f(u_i) = 0$ for $1 \leq i \leq 2k + 1$ then $e_f(0) = 6k + 3$.
If $f(v_i) = 0$ for $1 \leq i \leq k+1$ and $f(w_i) = 0$ for $1 \leq i \leq k$ then $e_f(0) = 4k+2$. If $f(u_i) = 0$ for $1 \leq i \leq k, f(v_i) = 0$ for $1 \leq i \leq \frac{k+1}{2}$ and $f(w_i) = 0$ for $1 \leq i \leq \frac{k+1}{2}$ then $e_f(0) = 3k + 2$. In either case we get a contradiction. Hence, f is not a 3-product cordial labeling if $n \equiv 2(mod3)$.

Define a vertex labeling $f : V(DA_2(T_n)) \to \{0, 1, 2\}$ by considering the following two cases.

Case (i). $n \equiv 0(mod3)$. Take $n = 3k$.

Then $|V(DA_2(T_n))| = 6k-1$ and $|E(DA_2(T_n))| = 9k-3$. For $k \succ 1, 1 \leq i \leq k-1, f(u_i) = 0$; For $i = k-1+j, 1 \leq j \leq 2k+1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 2,3(mod4) \\ 2, & if\ j \equiv 0,1(mod4); \end{cases}$$

For $k \succ 1$, $1 \leq i \leq \frac{k-1}{2}, f(v_i) = 0$; For $i = \frac{k-1}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if j \equiv 1(mod2); \end{cases}$$

and

$$f(w_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if j \equiv 1(mod2); \end{cases}$$

For $1 \leq i \leq \frac{k+1}{2}, f(w_i) = 0$; For $i = \frac{k+1}{2} + j, 1 \leq j \leq k-1$,

$$f(w_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = 2k$ and $e_f(0) = e_f(1) = e_f(2) = 3k-1$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod3)$.

Case (ii). $n \equiv 1(mod3)$. Take $n = 3k+1$.
Then $|V(DA_2(T_n))| = 6k+1$ and $|E(DA_2(T_n))| = 9k$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k+j, 1 \leq j \leq 2k+1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1,2(mod4) \\ 2, & if\ j \equiv 0,3(mod4); \end{cases}$$

For $1 \leq i \leq \frac{k}{2}, f(v_i) = f(w_i) = 0$;

For $i = \frac{k}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 1(mod2) \\ 2, & if\ j \equiv 0(mod2); \end{cases}$$

and

$$f(w_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) + 1 = 2k+1$ and $e_f(0) = e_f(1) = e_f(2) = 3k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod3)$.

(3). Here $|V(DA_3(T_n))| = 2n-2$ and $|E(DA_3(T_n))| = 3n-5$.
Define a vertex labeling $f : V(DA_3(T_n)) \rightarrow \{0,1,2\}$ by considering the following three cases.

Case (i). $n \equiv 0(mod3)$. Take $n = 3k$.

Then $|V(DA_3(T_n))| = 6k-2$ and $|E(DA_3(T_n))| = 9k-5$. For $1 \leq i \leq k-1, f(u_i) = 0$; For $i = k-1+j, 1 \leq j \leq 2k+1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 2,3(mod4) \\ 2, & if\ j \equiv 0,1(mod4); \end{cases}$$

For $1 \leq i \leq \frac{k}{2}, f(v_i) = f(w_i) = 0$;

For $i = \frac{k}{2} + j, 1 \leq j \leq k-1$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2); \end{cases}$$

and

$$f(w_i) = \begin{cases} 1, & if j \equiv 1(mod2) \\ 2, & if\ j \equiv 0(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0)+1 = v_f(1)+1 = v_f(2) = 2k$ and $e_f(0)-1 = e_f(1) = e_f(2) = 3k-2$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod3)$.

Case (ii). $n \equiv 1(mod3)$. Take $n = 3k+1$.
Then $|V(DA_3(T_n))| = 6k$ and $|E(DA_3(T_n))| = 9k-2$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k+j, 1 \leq j \leq 2k+1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 2,3(mod4) \\ 2, & if\ j \equiv 0,1(mod4); \end{cases}$$

For $k > 1, 1 \leq i \leq \frac{k-1}{2}, f(v_i) = 0$;

For $i = \frac{k-1}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2); \end{cases}$$

For $1 \leq i \leq \frac{k+1}{2}, f(w_i) = 0$; For $i = \frac{k+1}{2} + j$, $1 \leq j \leq k-1$,

$$f(w_i) = \begin{cases} 1, & if j \equiv 1(mod2) \\ 2, & if\ j \equiv 0(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) = v_f(2) = 2k$ and $e_f(0) = e_f(1)+1 = e_f(2)+1 = 3k$. Thus we have $|v_f(i)-v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod3)$.

Case (iii). $n \equiv 2(mod3)$. Take $n = 3k + 2$.

Then $|V(DA_3(T_n))| = 6k + 2$ and $|E(DA_3(T_n))| = 9k + 1$.

For $1 \leq i \leq k, f(u_i) = 0$; For $i = k + j, 1 \leq j \leq 2k + 2$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 2, 3(mod4) \\ 2, & if\ j \equiv 0, 1(mod4); \end{cases}$$

For $1 \leq i \leq \frac{k}{2}, f(v_i) = f(w_i) = 0$;

For $i = \frac{k}{2} + j, 1 \leq j \leq k$,

$$f(v_i) = \begin{cases} 1, & if j \equiv 1(mod2) \\ 2, & if\ j \equiv 0(mod2); \end{cases}$$

and

$$f(w_i) = \begin{cases} 1, & if j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = 2k + 1$ and $e_f(0) = e_f(1) = e_f(2) - 1 = 3k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 2(mod3)$.
□

An example of 3-product cordial labeling of $DA_3(T_{10})$ is shown in Figure 3.

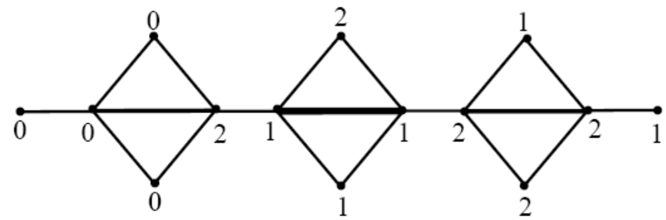

Figure 3

An example of 3-product cordial labeling of $DA_2(T_7)$ is shown in Figure 4.

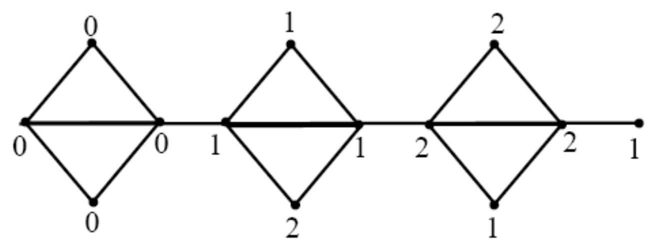

Figure 4

Theorem 2.3. *A triangular snake T_n admits 3-product cordial labeling if (i). $n \equiv 0(mod 3)$. (ii). $n \equiv 1(mod 3)$, n is odd. Also T_n is not a 3-product cordial graph if $n \equiv 2(mod 3)$.*

Proof. Let P_n be the path $u_1, u_2, \dots u_n$. Let $V(T_n) = V(P_n) \cup \{v_i/1 \leq i \leq n-1\}$ and $E(T_n) = E(P_n) \cup \{u_i v_i, v_i u_{i+1}/1 \leq i \leq n-1\}$. In this graph $|V(T_n)| = 2n-1$ and $|E(T_n)| = 3n-3$.
Define a vertex labeling $f : V((T_n)) \to 0, 1, 2$ by considering the following cases.

Case (i). $n \equiv 0(mod 3)$. Take $n = 3k$.
Then $|V(T_n)| = 6k-1$ and $|E(T_n)| = 9k-3$.

For $1 \leq i \leq k, f(u_i) = 0$; For $1 \leq i \leq k-1$, $f(v_i) = 0$;

For $i = k + j$, $1 \leq j \leq 2k - 1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1,2(mod4) \\ 2, & if\ j \equiv 0,3(mod4); \end{cases}$$

and

$f(u_{3k}) = 2$;

For $i = k - 1 + j$, $1 \leq j \leq 2k$,

$$f(v_i) = \begin{cases} 1, & if\ j \equiv 0(mod2),\ k\ is\ even \\ 2, & if\ j \equiv 1(mod2),\ k\ is\ even; \end{cases}$$

and

$$f(v_i) = \begin{cases} 1, & if\ j \equiv 1(mod2),\ k\ is\ odd \\ 2, & if\ j \equiv 0(mod2),\ k\ is\ odd. \end{cases}$$

In view of the above labeling pattern we have $v_f(0) + 1 = v_f(1) = v_f(2) = 2k$ and $e_f(0) = e_f(1) = e_f(2) = 3k - 1$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 0(mod3)$.

Case (ii). $n \equiv 1(mod3)$, Take $n = 3k + 1$, k is even.
Then $|V(T_n)| = 6k + 1$ and $|E(T_n)| = 9k$.

For $1 \leq i \leq k, f(u_i) = f(v_i) = 0$;

For $i = k + j$, $1 \leq j \leq 2k + 1$,

$$f(u_i) = \begin{cases} 1, & if\ j \equiv 1,2(mod4) \\ 2, & if\ j \equiv 0,3(mod4); \end{cases}$$

For $i = k + j$, $1 \leq j \leq 2k$,

$$f(v_i) = \begin{cases} 1, & if\ j \equiv 0(mod2) \\ 2, & if\ j \equiv 1(mod2). \end{cases}$$

In view of the above labeling pattern we have $v_f(0) = v_f(1) - 1 = v_f(2) = 2k$ and $e_f(0) = e_f(1) = e_f(2) = 3k$. Thus we have $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $i, j = 0, 1, 2$. Hence, f is a 3-product cordial labeling when $n \equiv 1(mod3)$ if k is even.

Case (iii). $n \equiv 2(mod3)$. Take $n = 3k + 2$.
Then $|V(T_n)| = 6k + 3$ and $|E(T_n)| = 9k + 3$. Assume that f is a 3-product cordial labeling. We have $v_f(0) = v_f(1) = v_f(2) = 2k + 1$ and $e_f(0) = e_f(1) = e_f(2) = 3k + 1$. If we assign $f(u_i) = 0$ for $1 \leq i \leq k + 1$ and $f(v_i) = 0$ for $1 \leq i \leq k$ then $e_f(0) = 3k + 2$.

If we assign $f(v_i) = 0$ for $1 \leq i \leq k$ and $f(v_i) = 0$ for $1 \leq i \leq k+1$ then $e_f(0) = 3k+2$.

In either case we get a contradiction. Hence, f is not a 3-product cordial labeling if $n \equiv 2(mod3)$. □

An example of 3-product cordial labeling of T_7 is shown in Figure 5.

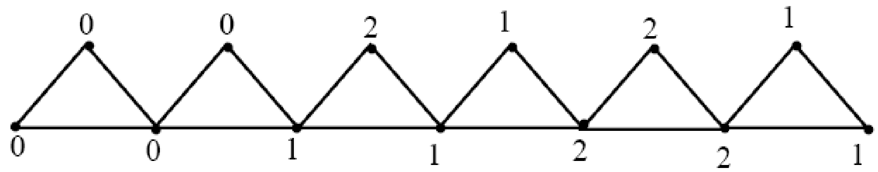

Figure 5

An example of 3-product cordial labeling of T_9 is shown in Figure 6.

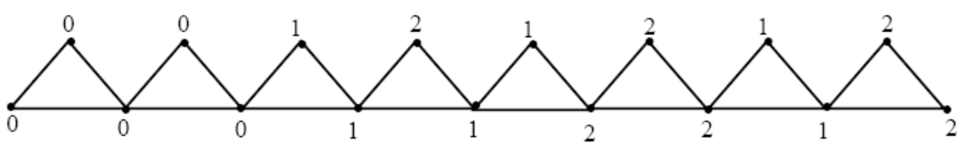

Figure 6

References

[1] I. Cahit, Cordial Graphs :A weaker version of graceful and harmonious graphs, Ars Combinatoria, Vol. 23, pp. 201-207, (1987).

[2] Joseph A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, (2017),# DS6.

[3] F. Harary, Graph theory, Addison Wesley, Massachusetts, (1972).

[4] P. Jeyanthi and A. Maheswari, 3-Product cordial labeling, SUT Journal of Mathematics, 48, pp. 231-140, (2012).

[5] P. Jeyanthi and A. Maheswari, 3-Product cordial labeling of some graphs, International Journal on Mathematical Combinatorics, Vol. 1, pp. 96-105, (2012).

[6] P. Jeyanthi and A. Maheswari, 3-Product cordial labeling of star graphs, Southeast Asian Bulletin of Mathematics, Vol. 39 (3), pp. 429-437, (2015).

[7] P. Jeyanthi and A. Maheswari, Some results on 3-Product cordial labeling, Utilitas Math., Vol. 99, pp. 215-229, (2016).

[8] R. Ponraj, M. Sivakumar and M. Sundaram, k-Product Cordial Labeling of Graphs, Int. J. Contemp. Math. Sciences, Vol. 7 (15), pp. 733-742, (2012).

[9] M. Sundaram, R. Ponraj and S. Somasundaram, EP-cordial labelings of graphs, Varahmihir Journal of Mathematical Sciences,Vol. 7 (1), pp. 183-194, (2007).

P. Jeyanthi
Research Centre
Department of Mathematics
Govindammal Aditanar College for Women
Tiruchendur 628215, Tamilnadu
India
e-mail : jeyajeyanthi@rediffmail.com

A.Maheswari
Department of Mathematics
Kamaraj College of Engineering and Technology
Virudhunagar,
India
e-mail : bala_nithin@yahoo.co.in

and

M. Vijayalakshmi
Department of Mathematics
Dr. G. U. Pope College of Engineering
Sawyerpuram, Thoothukudi District
Tamil Nadu,
India
e-mail: viji_mac@rediffmail.com