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# Multi-item multi-objective fixed charged solid transportation problem with type-2 fuzzy variables

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## Abstract:

*A multi-item multi-objective fixed charged solid transportation problema with guidelines e.g. unit transportation penalty, amounts, requirements, and conveyances as type-2 triangular fuzzy variables with conditions on few items and conveyances is formulated here. A chance constrained programming model applying generalized credibility measure for the objective function as well as the constraints is formed with the critical value based reductions of corresponding type-2 fuzzy guidelines for this particular problem. The model is then converted into the equivalent crisp deterministic form. The optimal compromise solutions are obtained by fuzzy programming technique. An example is contributed to highlight the model and is then solved by applying Generalized Reduced Gradient (GRG) technique (applying LINGO 16). The sensitivity analysis of the model is also given to illustrate the model.*

**Keywords:** Solid transportation problem; Fixed charge transportation problem; Multi-item multi-objective transportation problem; Chance constrained programming; Type-2 fuzzy set; Type-2 triangular fuzzy variable.

**MSC (2010):** 90B06, 90C70.

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## 1. Introduction

The Solid Transportation Problem (STP) is an exclusive form of linear programming model where we deal with condition of sources, stations, and carriages. The classical Transportation Problem (TP) is an exclusive form of STP if only one type of carriage is taken under consideration. TP is associated with additional costs along with shipping cost. These fixed penalties might be due to road taxes, toll charges etc. In this case it is called Fixed Charged Solid Transportation Problem (FCSTP). During the transportation movement due to complex situation, a few important criterions in the STP are always treated as uncertain variables to fit the realistic positions. There are cases to form a transportation plan for the later months; the amount of quantity at every origin, the requirement at every station, and the carriage quantity are frequently necessary to be determined by experienced knowledge or probability statistics as a result of no definite data. It is much better to explore this issue by applying fuzzy or stochastic optimization models. It is difficult to predict the exact transportation cost for a certain time period. Fuzzy set theory is the one of the popular approaches to deal with uncertainty. Type-2 fuzzy sets were proposed by Zadeh [20] as a development of typical fuzzy sets [19]. Type-2 fuzzy sets have membership functions as type-1 fuzzy sets. The advantage of type-2 fuzzy sets is that they are helpful in some cases where it is uncertain to find the definite membership functions for fuzzy sets.

The model of shipping multiple components from multiple sources to multiple destinations over a few carriages is known as multi-item STP. A situation that may arise while transporting multiple components from the source where not all brands of components can be shipped over all brands of carriages because of quality of components (e.g. liquid, breakable, etc.). Multi-item Fixed Charged Solid Transportation Problem (MIFCSTP) with condition on carriages is a model of shipping goods to a few destinations over a particular carriage with additional fixed charge for that particular route. Multi-item multi-objective solid transportation models are models that are used to find optimal solutions of multiple objective functions of shipping multiple components from multiple sources to multiple destinations over a few carriages.

The motivation behind this paper is to study solid transportation example with type-2 fuzzy parameters. Pandian et al. [16] found out a new method to solve the STP. Li et al. [10] discussed a neural network method to express bi-criteria STP, and Li et al. [11] also studied multi-objective

STP with fuzzy numbers and used improved genetic algorithm to solve it. Kundu et al. [8] investigated two models namely fixed charged transportation problem with type-2 fuzzy cost parameters and the same model with costs, supplies and demands as type-2 fuzzy variables. Maiti et al. [9] has solved one model with multi-item STP with restriction on conveyances with the type-2 fuzzy variables. Kundu et. al. [7] investigated the multi-objective STP under various uncertain environments. Amrit et al. [2] investigated multi stage STP under budget with Gaussian type-2 fuzzy parameters. Anushree et al. [3] discussed STP under type-2 trapezoidal fuzzy environment. Dhiman et al. [4]-[6] investigated a new method for the solution of the multi-item multi-objective fixed charged solid shipment model with type-2 fuzzy variables.

The paper has 5 Sections: Section 2 where some basic preliminaries relating to the notions of reductions of type-2 fuzzy variables are discussed and in Section 3 where CV-based reduction methods for type-2 fuzzy variables are discussed. We have formulated a multi-item multi-objective fixed charged solid transportation model with conditions on a few items and carriages in the sense that a few specific items are restricted to be shipped over a few particular carriages in Section 4. We have taken transportation variables e.g. unit transportation penalty, fixed costs, amounts, requirements, and carriage quantities as type-2 triangular fuzzy variables. The model is investigated by developing a chance constrained programming model applying the CV based reduction in Section 5. Finally the model is solved numerically in Section 6 applying fuzzy programming technique and LINGO 16 solver.

## 2. Preliminaries

**Definition 2.1.** [1] *A triplet* $(\eta', p', Pos)$ *is termed as a possibility space, where* $\eta'$ *is non-empty set of points,* $p'$ *is power set of* $\eta'$ *and* $Pos : \eta' \mapsto [0, 1]$ *is a mapping, called possibility measure explained as*

1. $Pos(\emptyset) = 0$ *and* $Pos(\eta') = 1$.
2. *For any* $\{A_j | j \in J\} \subset \eta'$, $Pos(\bigcup A_j) = \sup_j Pos(A_j)$.

**Definition 2.2.** [15] *The possibility measure (Pos) of a fuzzy event* $\{\tilde{\tau}' \in C'\}$, $C' \subset R$ *is explained as* $Pos\{\tilde{\tau}' \in C'\} = \sup_{y \in C'} \mu_{\tilde{\tau}'}(y)$, *where* $\mu_{\tilde{\tau}'}(y)$ *is explained as a possibility distribution of* $\tilde{\tau}'$.

**Definition 2.3.** [12] *The necessity (Nec) and credibility measure (Cr) of a normalized fuzzy variable* ($\sup_{y'\in\mathbf{R}} \mu_{\tilde{\tau}'}(y') = 1$) *is explained as follows:*

1. $Nec\{\tilde{\tau}' \in C'\} = 1 - Pos\{\tilde{\tau}' \in C'^c\} = 1 - \sup_{y'\in C'^c} \mu_{\tilde{\tau}'}(y')$.
2. $Cr\{\tilde{\tau}' \in C'\} = (Pos\{\tilde{\tau}' \in C'\} + Nec\{\tilde{\tau}' \in C'\})/2$.

**Definition 2.4.** [12] *The generalised credibility measure of a fuzzy variable is explained as* $\tilde{C}r\{\tilde{\tau}' \in C'\} = (\sup_{y'\in\mathbf{R}} \mu_{\tilde{\tau}'}(y') + \sup_{y'\in C'} \mu_{\tilde{\tau}'}(y') - \sup_{y'\in C'^c} \mu_{\tilde{\tau}'}(y'))/2$.

**Definition 2.5.** [13] *For a possibility space* $(\eta', p', Pos)$, *a Regular Fuzzy Variable (RFV)* $\tilde{\tau}'$ *is explained as a mapping from* $\eta'$ *to* $[0,1]$ *in the sense that for every* $s' \in [0,1]$, $\{\delta' \in \eta' |\, \mu_{\tilde{\tau}'}(\delta') \leq s'\} \in p'$.

**Definition 2.6.** [13] *If* $(\eta', p', Pos)$ *is a fuzzy possibility space then a Type-2 Fuzzy Variable (T2 FV)* $\tilde{\tau}$ *is expressed as* $\eta' \mapsto \mathbf{R}$ *such that for any* $t \in \mathbf{R}$ *the set* $\{\delta' \in \eta' |\, \mu_{\tilde{\tau}'}(\delta') \leq s'\} \in p'$.

**Definition 2.7.** [20] *A type-2 fuzzy set* $\tilde{B}$ *explained on the universe of discourse* $Y$ *is described by a membership function* $\tilde{\mu}_{\tilde{B}} : Y \mapsto F([0,1])$ *and is expressed by the following set notation :* $\tilde{B} = \{\Big(y, \tilde{\mu}_{\tilde{B}}(y)\Big) : y \in Y\}$.

**Example 2.1.** [13] *A type-2 triangular fuzzy variable* $\tilde{\tau}'$ *is expressed by* $(r'_1, r'_2, r'_3; \theta'_l, \theta'_r)$, *where* $r'_1, r'_2, r'_3 \in \mathbf{R}$ *and* $\theta'_l, \theta'_r$ *are two criterion defining the grade of ambiguity that* $\tilde{\tau}'$ *takes a value* $x'$ *and the secondary possibility distribution function* $\tilde{\mu}_{\tilde{\tau}'}(x')$ *of* $\tilde{\tau}'$ *is defined as*

$$\tilde{\mu}_{\tilde{\tau}'}(x') = \begin{cases} (\frac{x'-r'_1}{r'_2-r'_1} - \theta'_l \frac{x'-r'_1}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1} + \theta'_r \frac{x'-r'_1}{r'_2-r'_1}), if x' \in [r'_1, \frac{r'_1+r'_2}{2}]; \\ (\frac{x'-r'_1}{r'_2-r'_1} - \theta'_l \frac{r'_2-x'}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1}, \frac{x'-r'_1}{r'_2-r'_1} + \theta'_r \frac{r'_2-x'}{r'_2-r'_1}), if x' \in (\frac{r'_1+r'_2}{2}, r'_2]; \\ (\frac{r'_3-x'}{r'_3-r'_2} - \theta'_l \frac{x'-r'_2}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2} + \theta'_r \frac{x'-r'_2}{r'_3-r'_2}), if x' \in (r'_2, \frac{r'_2+r'_3}{2}]; \\ (\frac{r'_3-x'}{r'_3-r'_2} - \theta'_l \frac{r'_3-x'}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2}, \frac{r'_3-x'}{r'_3-r'_2} + \theta'_r \frac{r'_3-x'}{r'_3-r'_2}), if x' \in (\frac{r'_2+r'_3}{2}, r'_3]. \end{cases}$$

**Example 2.2.** *The secondary possibility distribution of* $\tilde{\tau}' = (1, 2, 3; 0.5, 1)$ *is given by*

$$\tilde{\mu}_{\tilde{\tau}'}(x') = \begin{cases} (0.5x' - 0.5), (x' - 1), (1.5x' - 1.5)), if x' \in [1, 1.5]; \\ ((1.5x' - 2), (x' - 1), 0.5x'), if x' \in (1.5, 2]; \\ ((4 - 1.5x'), (3 - x'), (2 - 0.5x')), if x' \in (2, 2.5]; \\ ((4.5 - 0.5x'), (3 - x'), (4.5 - 1.5x')), if x' \in (2.5, 3]. \end{cases}$$

### 2.1. Critical Values for RFVs

The different kinds of Critical Values (CVs) [17] of a RFV $\tilde{\tau}'$ is defined below.

(i) the optimistic CV of $\tilde{\tau}'$, denoted by $CV^*[\tilde{\tau}']$, is defined as

$$CV^*[\tilde{\tau}'] = \sup_{\delta' \in [0,1]} [\delta' \wedge Pos\{\tilde{\tau}' \geq \delta'\}]$$

(ii) the pessimistic CV of $\tilde{\tau}'$, denoted by $CV_*[\tilde{\tau}']$, is defined as

$$CV_*[\tilde{\tau}'] = \sup_{\delta' \in [0,1]} [\delta' \wedge Nec\{\tilde{\tau}' \geq \delta'\}]$$

(iii) the CV of $\tilde{\tau}'$, denoted by $CV[\tilde{\tau}']$, is defined as

$$CV[\tilde{\tau}'] = \sup_{\delta' \in [0,1]} [\delta' \wedge Cr\{\tilde{\tau}' \geq \delta'\}].$$

## 3. CV-based reduction method for T2 FVs

CV-based reduction approach was introduced by Qin et al. [17] which reduces a T2 FV to a type-1 fuzzy variable. Let $\tilde{\tau}'$ be a type-2 fuzzy variable with secondary membership function $\tilde{\nu}_{\tilde{\tau}'}(y)$. The method is to propose the CVs as a defining value for RFV $\tilde{\nu}_{\tilde{\tau}'}(y)$, i.e. $CV^*[\tilde{\nu}_{\tilde{\tau}'}(y)], CV_*[\tilde{\nu}_{\tilde{\tau}'}(y)]$ or $CV[\tilde{\nu}_{\tilde{\tau}'}(y)]$. Then these are accordingly called optimistic CV reduction, pessimistic CV reduction and CV reduction method.

**Theorem 3.1.** [17] *Suppose that $\tilde{\tau}' = (s_1', s_2', s_3'; \eta_l', \eta_r')$ be a type-2 triangular fuzzy variable. The following results are given below:*

*(a) The reduction of $\tilde{\tau}'$ to $\tau_1'$ applying the optimistic CV reduction method has the following possibility distribution*

$$\mu_{\tau_1'}(x') = \begin{cases} \frac{(1+\eta_r')(x'-s_1')}{s_2'-s_1'+\eta_r'(x'-s_1')}, \ if x' \in [s_1', \frac{s_1'+s_2'}{2}]; \\ \frac{(1-\eta_r')x'+\eta_r's_2'-s_1'}{s_2'-s_1'+\eta_r'(s_2'-x')}, \ if x' \in (\frac{s_1'+s_2'}{2}, s_2']; \\ \frac{(-1+\eta_r')x'-\eta_r's_2'+s_3'}{s_3'-s_2'+\eta_r'(x'-s_2')}, \ if x' \in (s_2', \frac{s_2'+s_3'}{2}]; \\ \frac{(1+\eta_r')(s_3'-x')}{s_3'-s_2'+\eta_r'(s_3'-x')}, \ if x' \in (\frac{s_2'+s_3'}{2}, s_3']. \end{cases}$$

*(b) The reduction of $\tilde{\tau}'$ to $\tau_2'$ applying the pessimistic CV reduction method has the following possibility distribution*

$$\mu_{\tau_2'}(x') = \begin{cases} \frac{(x'-s_1')}{s_2'-s_1'+\eta_l'(x'-s_1')}, \ if x' \in [s_1', \frac{s_1'+s_2'}{2}]; \\ \frac{x'-s_1'}{s_2'-s_1'+\eta_l'(s_2'-x')}, \ if x' \in (\frac{s_1'+s_2'}{2}, s_2']; \\ \frac{(s_3'-x')}{s_3'-s_2'+\eta_l'(x'-s_2')}, \ if x' \in (s_2', \frac{s_2'+s_3'}{2}]; \\ \frac{(s_3'-x')}{s_3'-s_2'+\eta_l'(s_3'-x')}, \ if x' \in (\frac{s_2'+s_3'}{2}, s_3']. \end{cases}$$

*(c) The reduction of $\tilde{\tau}'$ to $\tau_3'$ applying the CV reduction method has the*

*following possibility distribution*

$$\mu_{\tau_3'}(x') = \begin{cases} \frac{(1+\eta_r')(x'-s_1')}{s_2'-s_1'+2\eta_r'(x'-s_1')}, \; if x' \in [s_1', \frac{s_1'+s_2'}{2}]; \\ \frac{(1-\eta_l')x'+\eta_l's_2'-s_1'}{s_2'-s_1'+2\eta_l'(s_2'-x')}, \; if x' \in (\frac{s_1'+s_2'}{2}, s_2']; \\ \frac{(-1+\eta_l')x'-\eta_l's_2'+s_3'}{s_3'-s_2'+2\eta_l'(x'-s_2')}, \; if x' \in (s_2', \frac{s_2'+s_3'}{2}]; \\ \frac{(1+\eta_r')(s_3'-x')}{s_3'-s_2'+2\eta_r'(s_3'-x')}, \; if x' \in (\frac{s_2'+s_3'}{2}, s_3']. \end{cases}$$

**Theorem 3.2.** [17] *Let $\xi_i$ be the reduction of the type-2 fuzzy variable $\tilde{\xi}_i = (r_1^i, r_2^i, r_3^i; \theta_{l,i}, \theta_{r,i})$ obtained by the CV reduction method for $i = 1, 2, ..., n$. Let $\xi_1, \xi_2, ...., \xi_n$ are freely independent, and $k_i \geq 0$ for $i = 1, 2, ..., n$.*
*(i) Assumed the generalized credibility level $\alpha \in (0, 0.5]$, if $\alpha \in (0, 0.25]$, then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ is identical to $\sum_{i=1}^{n} \frac{(1-2\alpha+(1-4\alpha)\theta_{r,i})k_ir_1^i+2\alpha k_ir_2^i}{1+(1-4\alpha)\theta_{r,i}} \leq t$, and if $\alpha \in (0.25, 0.5]$, then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ is identical to $\sum_{i=1}^{n} \frac{(1-2\alpha)k_ir_1^i+(2\alpha+(4\alpha-1)\theta_{l,i})k_ir_2^i}{1+(4\alpha-1)\theta_{l,i}} \leq t$.*
*(ii) Assumed the generalized credibility level $\alpha \in (0.5, 1]$, if $\alpha \in (0.5, 0.75]$, then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ is identical to $\sum_{i=1}^{n} \frac{(2\alpha-1)k_ir_3^i+(2(1-\alpha)+(3-4\alpha)\theta_{l,i})k_ir_2^i}{1+(3-4\alpha)\theta_{l,i}} \leq t$,*
*and if $\alpha \in (0.75, 1]$, then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \leq t\} \geq \alpha$ is identical to $\sum_{i=1}^{n} \frac{(2\alpha-1+(4\alpha-3)\theta_{r,i})k_ir_3^i+2(1-\alpha)k_ir_2^i}{1+(4\alpha-3)\theta_{r,i}} \leq t$.*

**Corollary 3.1.** *The identical expression of $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \geq t\} \geq \alpha$ are easily obtained from the above theorem because*

$$\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \geq t\} \geq \alpha \Rightarrow \tilde{C}r\{\sum_{i=1}^{n} -k_i\xi_i \leq -t\} \geq \alpha$$
$$\Rightarrow \tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i' \leq t'\} \geq \alpha$$

*where $\xi_i' = -\xi_i$ is the CV reduction of $-\tilde{\xi}_i = (-r_3^i, -r_2^i, -r_1^i; \theta_{r,i}, \theta_{l,i})$ and $t' = -t$.*
*Assumed the generalized credibility level $\alpha \in (0, 0.5]$, if $\alpha \in (0, 0.25]$ from (i) of the above theorem, then $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i \geq t\} \geq \alpha$ i.e. $\tilde{C}r\{\sum_{i=1}^{n} k_i\xi_i' \leq t'\} \geq \alpha$ is identical to*
$\sum_{i=1}^{n} \frac{(1-2\alpha+(1-4\alpha)\theta_{l,i})k_i(-r_3^i)+2\alpha k_i(-r_2^i)}{1+(1-4\alpha)\theta_{l,i}} \leq t' = -t,$

$\Rightarrow \sum\limits_{i=1}^{n} \frac{(1-2\alpha+(1-4\alpha)\theta_{l,i})k_i r_3^i + 2\alpha k_i r_2^i}{1+(1-4\alpha)\theta_{l,i}} \geq t,$

*and if* $\alpha \in (0.25, 0.5]$, *then* $\tilde{Cr}\{\sum\limits_{i=1}^{n} k_i\xi_i \geq t\} \geq \alpha$ *is identical to*

$\sum\limits_{i=1}^{n} \frac{(1-2\alpha)k_i(-r_3^i)+(2\alpha+(4\alpha-1)\theta_{r,i})k_i(-r_2^i)}{1+(4\alpha-1)\theta_{r,i}} \leq -t$

$\Rightarrow \sum\limits_{i=1}^{n} \frac{(1-2\alpha)k_i r_3^i+(2\alpha+(4\alpha-1)\theta_{r,i})k_i r_2^i}{1+(4\alpha-1)\theta_{r,i}} \geq t.$

*The other values of* $\alpha$ *are similarly derived from other identical expressions.*

## 4. Model: Multi-item multi-objective fixed charged solid transportation problem with condition on conveyances

Suppose that $k\,(k = 1, 2, ..., K)$ different modes of carriages are necessary to transport $l$ components from $m$ sources $O_i\,(i = 1, 2, ..., m)$ to $n$ stations $D_j\,(j = 1, 2, ..., n)$ and also $(t = 1, 2, ...., R)$ objectives are to be minimized. In addition to that there are a few conditions on a few particular components and carriages so that a few components can not be shipped over a few carriages. Suppose that $I_k$ be the set of components which can be shipped over carriages $k\,(k = 1, 2, .., K)$. We use character $p\,(= 1, 2, ..., l)$ to stand for the items.

The fixed charged solid transportation problem is linked with two categories of penalties, unit transportation penalty for shipping unit amount from origin $i$ to station $j$ and a fixed cost for the direction $(i, j)$. We develop a multi-item multi-objective fixed charged solid transportation model with $m$ sources, $n$ stations, $k$ carriages, direct penalty and fixed penalty criterion as T2 FVs as follows:

$$\text{Min}\, Z_t = \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}(c_{ijk}^{tp}x_{ijk}^{p}) + e_{ijk}^{tp}y_{ijk}^{p},\ t = 1, 2, 3, ......R$$

$$\text{subject to} \sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \leq a_i^p, i = 1, 2, ..., m; p = 1, 2, ..., l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \geq b_j^p, j = 1, 2, ..., n; p = 1, 2, ..., l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p}x_{ijk}^{p} \leq f_k, k = 1, 2, ..., K, \tag{4.1}$$

$$x_{ijk}^{p} \geq 0,\ \forall i, j, k, p,$$

where $d^p_{ijk}$ is defined as

$$d^p_{ijk} = \begin{cases} 1, if\, p \in I_K \,\forall\, i,j,k,p; \\ 0, \text{ otherwise,} \end{cases}$$

and

$$y^p_{ijk} = \begin{cases} 1, if\, x > 0; \\ 0, \text{ otherwise.} \end{cases}$$

Here, $x^p_{ijk}$is the decision variable representing the measure of $p$-th component shipped from source $i$ to station $j$, $e^{tp}_{ijk}$ is the type-2 fuzzy fixed cost linked with direction $(i,j)$ for the objective $Z_t$. The unit transportation cost $c^{tp}_{ijk}$(from $i$-th origin to $j$-th station by $k$-th carriage for $p$-th thing) for the objective $Z_t$, full supply of $p$-th component $a^p_i$ at $i$-th source, full demand of $p$-th component $b^p_j$ at $j$-th station and full quantity $f_k$ of $k$-th carriage are all type-2 fuzzy variables.

## 5. Solution Procedure (Chance Constrained programming using generalized credibility)

Let $c^{tp\prime}_{ijk}, e^{tp\prime}_{ijk}, a^{p\prime}_i, b^{p\prime}_j$and $f_{k\prime}$ be the reduced fuzzy variables from type-2 fuzzy variables $c^{tp}_{ijk}, e^{tp}_{ijk}, a^p_i, b^p_j$ and $f_k$ respctively based on CV-based reduction method. We develop a chance-constrained programming with these reduced fuzzy variables to solve the above problem. The uncertain constraints are granted to be opposed such that constraints must be fulfilled at a few chance level in chance-constrained programming. Yang et al. [18], Liu et al. [14], Kundu et al. [9] advanced chance-constrained programming with fuzzy guidelines using credibility measure. The natural credibility measure can not be used if the reduced fuzzy guidelines $c^{tp\prime}_{ijk}$, $e^{tp\prime}_{ijk}$, $a^{p\prime}_i$, $b^{p\prime}_j$and $f_{k\prime}$ are not normalized. The succeeding chance-constrained programming example is developed for the raised model (4.1) using generalized credibility.

$\text{Min}_x(\text{Min}\tilde{f}_t)$

subject to

$$Cr\{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d^p_{ijk}c^{tp\prime}_{ijk}x^p_{ijk} + e^{tp\prime}_{ijk}y^p_{ijk} \le \tilde{f}_t\} \ge \alpha, t = 1,2,..,R$$

$$Cr\{\sum_{j=1}^{n}\sum_{k=1}^{K} d^p_{ijk}x^p_{ijk} \le a^{p'}_i\} \ge \alpha^p_i, i = 1,2,..,m; p = 1,2,..,l,$$

$$Cr\{\sum_{i=1}^{m}\sum_{k=1}^{K} d^p_{ijk}x^p_{ijk} \ge b^{p'}_j\} \ge \beta^p_j, j = 1,2,..,n; p = 1,2,..,l,$$

$$Cr\{\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p}x_{ijk}^{p} \le f_{k'}\} \ge \gamma_k, k=1,2,..,K, \tag{5.1}$$

$$\mathrm{x}_{ijk}^{p} \ge 0,\ d_{ijk}^{p} = \begin{cases} 1, \text{if}\, p \in I_K\, \forall\, i,j,k,p; \\ 0, \text{ otherwise,} \end{cases},\ y_{ijk}^{p} = \begin{cases} 1, \text{if}\, x_{ijk}^{p} > 0; \\ 0, \text{ otherwise.} \end{cases}$$

where Min$\tilde{f}_t$ expresses the lowest likely crisp form that the objective function attains with generalized credibility slightly $\alpha\,(0 < \alpha \le 1)$. Especially, $\alpha$ specifies that we are working to reduce the $\alpha$ critical value of the objective function. $\alpha_i^p, \beta_j^p$ and $\gamma_k\,(0 < \alpha_i^p, \beta_j^p, \gamma_k \le 1)$ are fixed generalized credibility levels of fulfilment of the particular constraints.

### 5.1. Crisp Equivalence

Let $c_{ijk}^{tp}, e_{ijk}^{tp}, a_i^p, b_j^p$ and $f_k$ are all jointly independent type-2 triangular fuzzy variables defined by $c_{ijk}^{tp} = (c_{ijk}^{tp1}, c_{ijk}^{tp2}, c_{ijk}^{tp3}; \theta_{l,ijk}^{tp}, \theta_{r,ijk}^{tp})$, $e_{ijk}^{tp} = (e_{ijk}^{tp1}, e_{ijk}^{tp2}, e_{ijk}^{tp3}; \theta_{l,ijk}^{\prime tp}, \theta_{r,ijk}^{\prime tp})$, $a_i^p = (a_i^{p1}, a_i^{p2}, a_i^{p3}; \theta_{l,i}^p, \theta_{r,i}^p)$, $b_j^p = (b_j^{p1}, b_j^{p2}, b_j^{p3}; \theta_{l,j}^p, \theta_{r,j}^p)$, and$f_k = (f_k^1, f_k^2, f_k^3; \theta_{l,k}, \theta_{r,k})$. The chance constrained model formulation (5.1) is passed into the next crisp identical parametric programming models from Theorem 3.2 and its corollary 3.1:

Event A$:0 < \alpha \le 0.25$ : The identical parametric programming for example (5.1) is

$$\text{Min} \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}[\frac{(1-2\alpha+(1-4\alpha)\theta_{r,ijk}^{tp})c_{ijk}^{tp1}x_{ijk}^{p}+2\alpha c_{ijk}^{tp2}x_{ijk}^{p}}{1+(1-4\alpha)\theta_{r,ijk}^{tp}}]$$
$$+\frac{(1-2\alpha+(1-4\alpha)\theta_{r,ijk}^{\prime tp})e_{ijk}^{tp1}y_{ijk}^{p}+2\alpha e_{ijk}^{tp2}y_{ijk}^{p}}{1+(1-4\alpha)\theta_{r,ijk}^{\prime tp}}$$

subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \quad \le F_{a_i^p}, i=1,2,...,m; p=1,2,...,l,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \quad \ge F_{b_j^p}, j=1,2,...,n; p=1,2,...,l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p}x_{ijk}^{p} \le F_{f_k}, k=1,2,...,K, \tag{5.2}$$

$$x_{ijk}^{p} \ge 0, d_{ijk}^{p} = \begin{cases} 1, \text{if}\, p \in I_K\, \forall\, i,j,k,p; \\ 0, \text{ otherwise,} \end{cases}, y_{ijk}^{p} = \begin{cases} 1, \text{if}\, x_{ijk}^{p} > 0; \\ 0, \text{ otherwise.} \end{cases}$$

where $F_{a_i^p}, F_{b_j^p}, F_{f_k}$ is defined by (5.6)-(5.8) appropriately.
Event B : $0.25 < \alpha \leq 0.5$ : The identical parametric programming for example (5.1) is

$$\text{Min} \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}\Big[\frac{(1-2\alpha)c_{ijk}^{tp1}x_{ijk}^{p}+(2\alpha+(4\alpha-1)\theta_{l,ijk}^{tp})c_{ijk}^{tp2}x_{ijk}^{p}}{1+(4\alpha-1)\theta_{l,ijk}^{tp}}\Big]$$
$$+\frac{(1-2\alpha)e_{ijk}^{tp1}y_{ijk}^{p}+(2\alpha+(4\alpha-1)\theta_{l,ijk}^{\prime tp})e_{ijk}^{tp2}y_{ijk}^{p}}{1+(4\alpha-1)\theta_{l,ijk}^{\prime tp}}$$

$$\text{subject to} \sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \leq F_{a_i^p}, i=1,2,...,m; p=1,2,...,l,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \geq F_{b_j^p}, j=1,2,...,n; p=1,2,...,l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p}x_{ijk}^{p} \leq F_{f_k}, k=1,2,...,K, \tag{5.3}$$

$$x_{ijk}^{p} \geq 0, d_{ijk}^{p} = \begin{cases} 1, \text{if}\, p \in I_K\ \forall\, i,j,k,p; \\ 0, \text{ otherwise,}\end{cases} , y_{ijk}^{p} = \begin{cases} 1, \text{if}\, x_{ijk}^{p} > 0; \\ 0, \text{ otherwise.}\end{cases}$$

Event C: $0.5 < \alpha \leq 0.75$ : The identical parametric problem for the example (5.1) is

$$\text{Min} \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}\frac{(2\alpha-1)c_{ijk}^{tp3}x_{ijk}^{p}+(2(1-\alpha)+(3-4\alpha)\theta_{l,ijk}^{tp})c_{ijk}^{tp2}x_{ijk}^{p}}{1+(3-4\alpha)\theta_{l,ijk}^{tp}}$$
$$+\frac{(2\alpha-1)e_{ijk}^{tp3}y_{ijk}^{p}+(2-2\alpha+(3-4\alpha)\theta_{l,ijk}^{\prime tp})e_{ijk}^{tp2}y_{ijk}^{p}}{1+(3-4\alpha)\theta_{l,ijk}^{\prime tp}}$$

$$\text{subject to} \sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \leq F_{a_i^p}, i=1,2,...,m; p=1,2,...,l,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \geq F_{b_j^p}, j=1,2,...,n; p=1,2,...,l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p}x_{ijk}^{p} \leq F_{f_k}, k=1,2,...,K, \tag{5.4}$$

$$x_{ijk}^{p} \geq 0, d_{ijk}^{p} = \begin{cases} 1, \text{if}\, p \in I_K\ \forall\, i,j,k,p; \\ 0, \text{ otherwise,}\end{cases} , y_{ijk}^{p} = \begin{cases} 1, \text{if}\, x_{ijk}^{p} > 0; \\ 0, \text{ otherwise.}\end{cases}$$

Event D: $0.75 < \alpha \leq 1$ : The identical parametric problem for the example (5.1) is

$$\text{Min} \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}[\frac{(2\alpha-1+(4\alpha-3)\theta_{r,ijk}^{tp})c_{ijk}^{tp3}x_{ijk}^{p}+2(1-\alpha)c_{ijk}^{tp2}x_{ijk}^{p}}{1+(4\alpha-3)\theta_{r,ijk}^{tp}}]$$
$$+\frac{(2\alpha-1+(4\alpha-3)\theta'^{tp}_{r,ijk})e_{ijk}^{tp3}y_{ijk}^{p}+2(1-\alpha)e_{ijk}^{tp2}y_{ijk}^{p}}{1+(4\alpha-3)\theta'^{tp}_{r,ijk}}$$

subjet to

$$\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \leq F_{a_i^p}, i=1,2,...,m; p=1,2,...,l,$$
$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p}x_{ijk}^{p} \geq F_{b_j^p}, j=1,2,...,n; p=1,2,...,l,$$
$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p}x_{ijk}^{p} \leq F_{f_k}, k=1,2,...,K, \tag{5.5}$$

$$x_{ijk}^{p} \geq 0, d_{ijk}^{p} = \begin{cases} 1, \text{if}\, p \in I_K\; \forall\, i,j,k,p; \\ 0, \text{ otherwise}, \end{cases}, y_{ijk}^{p} = \begin{cases} 1, \text{if}\, x_{ijk}^{p} > 0; \\ 0, \text{ otherwise}. \end{cases}$$

where,

$$F_{a_i^p} = \begin{cases} \frac{(1-2\alpha_i^p+(1-4\alpha_i^p)\theta_{l,i}^p)a_i^{p3}+2\alpha_i^p a_i^{p2}}{1+(1-4\alpha_i^p)\theta_{l,i}^p}, & \text{if } 0<\alpha_i^p\leq 0.25; \\ \frac{(1-2\alpha_i^p)a_i^{p3}+(2\alpha_i^p+(4\alpha_i^p-1)\theta_{r,i}^p)a_i^{p2}}{1+(4\alpha_i^p-1)\theta_{r,i}^p}, & \text{if } .25<\alpha_i^p\leq .5; \\ \frac{(2\alpha_i^p-1)a_i^{p1}+(2(1-\alpha_i^p)+(3-4\alpha_i^p)\theta_{r,i}^p)a_i^{p2}}{1+(3-4\alpha_i^p)\theta_{r,i}^p}, & \text{if } 0.5<\alpha_i^p\leq 0.75; \\ \frac{(2\alpha_i^p-1+(4\alpha_i^p-3)\theta_{l,i}^p)a_i^{p1}+2(1-\alpha_i^p)a_i^{p2}}{1+(4\alpha_i^p-3)\theta_{l,i}^p}, & \text{if } 0.75<\alpha_i^p\leq 1. \end{cases} \tag{5.6}$$

$$F_{b_j^p} = \begin{cases} \frac{(1-2\beta_j^p+(1-4\beta_j^p)\theta_{r,j}^p)b_j^{p1}+2\beta_j^p b_j^{p2}}{1+(1-4\beta_j^p)\theta_{r,j}^p}, & \text{if } 0<\beta_j^p\leq 0.25; \\ \frac{(1-2\beta_j^p)b_j^{p1}+(2\beta_j^p+(4\beta_j^p-1)\theta_{l,j}^p)b_j^{p2}}{1+(4\beta_j^p-1)\theta_{l,j}^p}, & \text{if } .25<\beta_j^p\leq .5; \\ \frac{(2\beta_j^p-1)b_j^{p3}+(2(1-\beta_j^p)+(3-4\beta_j^p)\theta_{l,j}^p)b_j^{p2}}{1+(3-4\beta_j^p)\theta_{l,j}^p}, & \text{if } 0.5<\beta_j^p\leq 0.75; \\ \frac{(2\beta_j^p-1+(4\beta_j^p-3)\theta_{r,j}^p)b_j^{p3}+2(1-\beta_j^p)b_j^{p2}}{1+(4\beta_j^p-3)\theta_{r,j}^p}, & \text{if } 0.75<\beta_j^p\leq 1. \end{cases} \tag{5.7}$$

$$F_{f_k} = \begin{cases} \frac{(1-2\gamma_k+(1-4\gamma_k)\theta_{l,k})f_k^3+2\gamma_k f_k^2}{1+(1-4\gamma_k)\theta_{l,k}}, & \text{if } 0<\gamma_k\leq 0.25; \\ \frac{(1-2\gamma_k)f_k^3+(2\gamma_k+(4\gamma_k-1)\theta_{r,k})f_k^2}{1+(4\gamma_k-1)\theta_{r,k}}, & \text{if } 0.25<\gamma_k\leq 0.5; \\ \frac{(2\gamma_k-1)f_k^1+(2(1-\gamma_k)+(3-4\gamma_k)\theta_{r,k})f_k^2}{1+(3-4\gamma_k)\theta_{r,k}}, & \text{if } 0.5<\gamma_k\leq 0.75; \\ \frac{(2\gamma_k-1+(4\gamma_k-3)\theta_{l,k})f_k^1+2(1-\gamma_k)f_k^2}{1+(4\gamma_k-3)\theta_{l,k}}, & \text{if } 0.75<\gamma_k\leq 1. \end{cases} \tag{5.8}$$

### 5.2. Fuzzy programming technique

Zimmerman [21] presented that fuzzy linear programming technique constantly provides powerful solutions and a best possible optimal solution for multiple objective problems. The stages to solve the multi-objective problems by means of fuzzy programming method are given below:

Stage 1: The multi-objective model is solved as a one objective problem applying, every time, single objective $\bar{Z}_t(t = 1, 2, ..., R)$ to obtain the best possible solution $X^{t*} = x^p_{ijk}$ of $R$ distinct single objective model.

Stage 2: The values of all the $R$ objective functions at all these $R$ best possible solutions $X^{t*}(t = 1, 2, ......, R)$ are calculated and the upper and lower bound for individual objective is specified by
$U_t = \text{Max}\{\bar{Z}_t(X^{1*}), \bar{Z}_t(X^{2*}), ......., \bar{Z}_t(X^{t*})\}$ and $L_t = \bar{Z}_t(X^{t*})$, $t = 1, 2, ....., R$ appropriately.

Stage 3: An introductory fuzzy example is assumed as

Find $x$
subject to $Z_t(x) \leq L_t$, $t = 1, 2, ......, R$
and the constraints of (4.1).

where $x = x^p_{ijk}$, $i = 1, 2, ...., m$; $j = 1, 2, ...., n; k = 1, 2, ...., K$;
$p = 1, 2, ..., l$.

Stage 4: The linear membership function $\mu_t(\bar{Z}_t)$ identical to $t^{th}$ objective is calculated as

$$\mu_t(\bar{Z}_t) = \begin{cases} 1, \text{if } \bar{Z}_t \leq L_t; \\ \frac{U_t - \bar{Z}_t}{U_t - L_t}, \text{if } L_t < \bar{Z}_t < U_t; \\ 0, \text{if } \bar{Z}_t \geq U_t, \forall t. \end{cases}$$

Stage 5: The fuzzy linear programming model is formulated applying max-min operator as
Max $\delta$
such that

$$\delta \leq \mu_t(\bar{Z}_t) = \frac{U_t - \bar{Z}_t}{U_t - L_t}, \forall t \tag{5.9}$$

and the constraints of (4.1)

$\delta \geq 0$ and $\delta = \min_t\{\mu_t(\bar{Z}_t)\}$.

Stage 6: The reduced model is worked out by a linear optimization method and the best possible optimal solutions are achieved.

## 6. Numerical Model

The projected problem is illustrated numerically in this section. The proposed methodology is solved numerically by taking one example of the model. Consider the model with objective functions ($t = 1, 2$), sources ($i = 1, 2, 3$), destinations ($j = 1, 2, 3$), conveyance ($k = 1, 2, 3, 4$) and items ($p = 1, 2, 3$). Suppose that $I_1 = \{1, 2\}$, $I_2 = \{1, 2, 3\}$, $I_3 = \{3\}$, $I_4 = \{1, 2, 3\}$. The transportation penalties and fixed penalties for this model are given in Tables 2-13 (Appendix). The amounts, requirements and conveyance capacities are the consecutive type-2 fuzzy data:
$a_1^1 = (21, 24, 27; 0.5, 1), a_1^2 = (26, 28, 30; 0.5, 0.9), a_1^3 = (24, 26, 29; 0.6, 1)$, $a_2^1 = (26, 28, 32; 0.6, 0.9), a_2^2 = (20, 24, 27; 0.6, 0.9), a_2^3 = (22, 24, 26; 0.5, 1)$, $a_3^1 = (27, 28, 29; 0.7, 1), a_3^2 = (32, 35, 36; 0.8, 1), a_3^3 = (23, 25, 29; 0.5, 1)$, $b_1^1 = (9, 12, 14; 0.8, 0.9), b_2^1 = (15, 16, 17; 0.5, 0.6), b_3^1 = (15, 18, 20; 0.5, 0.6)$, $b_1^2 = (11, 13, 15; 0.5, 0.7), b_2^2 = (12, 13, 15; 0.9, 0.5), b_3^2 = (11, 14, 16; 0.7, 1)$, $b_1^3 = (10, 12, 15; 0.4, 0.6), b_2^3 = (9, 11, 12; 0.3, 0.5), b_3^3 = (11, 15, 17; 0.4, 0.5)$, $e_1 = (34, 36, 37; 0.5, 0.7), e_2 = (46, 49, 50; 0.6, 1), e_3 = (28, 30, 33; 0.7, 1)$, $e_4 = (40, 43, 45; 0.5, 0.7)$.
The chance constrained programming model for this model as (5.1) is formulated here. The fixed general credibility levels for objective function and constraints are reserved as $\alpha = 0.7$, $\alpha_i^p = 0.7$, $\beta_j^p = 0.7$, $\gamma_k = 0.7$, $t = 1, 2$, $p = 1, 2, 3$, $i = 1, 2, 3$, $j = 1, 2, 3$, $k = 1, 2, 3, 4$. The corresponding deterministic form of the model using (5.4) is given below:

$$\text{Min} \sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^p \frac{0.4c_{ijk}^{tp3}x_{ijk}^p + (0.6+0.2\theta_{l,ijk}^{tp})c_{ijk}^{tp2}x_{ijk}^p}{1+0.2\theta_{l,ijk}^{tp}} + \frac{0.4e_{ijk}^{tp3}y_{ijk}^p + (0.6+0.2\theta_{l,ijk}^{\prime tp})e_{ijk}^{tp2}y_{ijk}^p}{1+0.2\theta_{l,ijk}^{\prime tp}}$$

$$\text{(6.1)} \quad \text{subject to} \sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^p x_{ijk}^p \leq F_{a_i^p}, i = 1, 2, ..., m; p = 1, 2, .., l,$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} d_{ijk}^{p} x_{ijk}^{p} \geq F_{b_j^p}, j = 1, 2, ..., n; p = 1, 2, ..., l,$$

$$\sum_{p=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ijk}^{p} x_{ijk}^{p} \leq F_{f_k}, k = 1, 2, ..., K$$

$$x_{ijk}^{p} \geq 0,\ d_{ijk}^{p} = \begin{cases} 1, \text{if } p \in I_K \, \forall\, i, j, k, p; \\ 0, \text{ otherwise,} \end{cases},\ y_{ijk}^{p} = \begin{cases} 1, \text{if } x_{ijk}^{p} > 0; \\ 0, \text{ otherwise.} \end{cases}$$

where $F_{a_i^p}, F_{b_j^p}, F_{f_k}$ are calculated from (5.6), (5.7) and (5.8) as follows:
$F_{a_1^1} = 23$, $F_{a_1^2} = 27.32$, $F_{a_1^3} = 25.33$, $F_{a_2^1} = 27.32$, $F_{a_2^2} = 22.64$, $F_{a_2^3} = 34$, $F_{a_3^1} = 27.67$, $F_{a_3^2} = 23.33$, $F_{a_3^3} = 24.33$, $F_{b_1^1} = 12.69$, $F_{b_1^2} = 16.36$, $F_{b_1^3} = 18.73$, $F_{b_2^1} = 13.72$, $F_{b_2^2} = 13.67$, $F_{b_2^3} = 14.70$, $F_{b_3^1} = 13.11$, $F_{b_3^2} = 11.38$, $F_{b_3^3} = 15.74$, $F_{f_1} = 35.30$, $F_{f_2} = 48$, $F_{f_3} = 29.33$, $F_{f_4} = 41.94$.
The optimal solution of the first objective function (6.1) are as follows:
$x_{111}^1 = 4.27$, $x_{131}^1 = 18.73$, $x_{211}^1 = 8.42$, $x_{321}^1 = 3.88$, $x_{322}^1 = 12.48$, $x_{122}^2 = 8.4$, $x_{114}^2 = 13.72$, $x_{134}^2 = 5.2$, $x_{224}^2 = 5.27$, $x_{234}^2 = 9.5$, $x_{132}^3 = 15.74$, $x_{322}^3 = 11.38$, $x_{213}^3 = 4.86$, $x_{114}^3 = 8.25$, and the minimum first transportation cost (first objective value) is 505.8202.
The running time and full solver repetitions concerning this solution are 0.05 sec and 14 appropriately.
The optimal solution of the second objective function (6.1) are as follows:
$x_{111}^1 = 4.27$, $x_{131}^1 = 18.73$, $x_{211}^1 = 8.42$, $x_{322}^1 = 16.36$, $x_{122}^2 = 13.67$, $x_{114}^2 = 13.65$, $x_{214}^2 = 0.07$, $x_{234}^2 = 14.7$, $x_{332}^3 = 15.74$, $x_{213}^3 = 10.97$, $x_{114}^3 = 2.14$, $x_{124}^3 = 11.38$, and the minimum second transportation cost (second objective value) is 617.2989.
The running time and full solver repetitions concerning this solution are 0.05 sec and 14 appropriately.
Here, $L_1 = 505.8202$, $U_1 = 525.2601$, and $L_2 = 617.2989$, $U_2 = 656.6142$ and then applying fuzzy linear programming technique the compromise optimal solution of both the objective functions (5.9) are as follows:
$x_{111}^1 = 4.27$, $x_{131}^1 = 18.73$, $x_{211}^1 = 8.42$, $x_{321}^1 = 3.88$, $x_{322}^1 = 12.48$, $x_{122}^2 = 13.67$, $x_{114}^2 = 13.65$, $x_{214}^2 = 0.07$, $x_{234}^2 = 14.7$, $x_{132}^3 = 12.00588$, $x_{322}^3 = 6.11$, $x_{332}^3 = 3.734125$, $x_{213}^3 = 4.86$, $x_{114}^3 = 8.25$, $x_{124}^3 = 5.27$, $\delta = 0.3203944$ and the minimum first and second transportation cost (first and second objective value) is 519.0317 and 644.0178.
The running time and full solver repetitions concerning this solution are 0.05 sec and 22 appropriately.

## 7. Sensitivity Analysis

A sensitivity analysis of the model is given to view the effectiveness and reasonably accuracy of the crisp equivalent form and solution methods of the given problem. The changes of the transportation amount and transportation cost for different generalized credibility levels for the origin, station and carriage constraints of the problem are given in table 7.1.

Table 7.1: Sensitivity Analysis

| $\alpha$ | $\alpha_i^p$ | $\beta_j^p$ | $\gamma_k$ | Transportation Amount | Transportation Cost |
|---|---|---|---|---|---|
| 0.65 | 0.7 | 0.7 | 0.7 | 130.120002 | $Z_1 = 498.0995, Z_2 = 617.4242$ |
| 0.7 | 0.7 | 0.7 | 0.7 | 130.100005 | $Z_1 = 519.0317, Z_2 = 644.0178$ |
| 0.75 | 0.7 | 0.7 | 0.7 | 130.12 | $Z_1 = 543.93, Z_2 = 678.075$ |
| 0.7 | 0.5 | 0.7 | 0.7 | 130.10004 | $Z_1 = 523.111, Z_2 = 647.5476$ |
| 0.7 | 0.9 | 0.7 | 0.7 | 130.10004 | $Z_1 = 520.8784, Z_2 = 645.8103$ |
| 0.7 | 0.7 | 0.5 | 0.7 | 113.65001 | $Z_1 = 447.2653, Z_2 = 547.6762$ |
| 0.7 | 0.7 | 0.9 | 0.7 | 138.61 | $Z_1 = 552.7917, Z_2 = 678.9297$ |
| 0.7 | 0.7 | 0.7 | 0.5 | 130.12 | $Z_1 = 517.6526, Z_2 = 641.5595$ |
| 0.7 | 0.7 | 0.7 | 0.9 | 130.100098 | $Z_1 = 522.0067, Z_2 = 648.4144$ |

It is viewed from the table 7.1 that for fixed $\alpha, \alpha_i^p, \gamma_k$ and different $\beta_j^p$, the transportation cost, transportation amount increases/decreases with the increased/decreased value of the crisp amount of the demands. The cause of the increase of transportation cost is due to the increase/decrease of the defuzzified amount of the demands. The transportation amount almost remains same for different $\alpha, \alpha_i^p, \gamma_k$ and fixed $\beta_j^p$ .

## Conclusion

A multi-item multi-objective fixed charged solid transportation problem with type-2 triangular fuzzy variables have been developed and worked out for the first time ever here. A chance constrained programming is formed for the model and solved using fuzzy programming technique. The method can be used in other decision-making problem in distinct fields with type-2 fuzzy guidelines. The price discounts, transportation time constraints etc. can be used in different types of transportation problems to extend the presented model.

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