Proyecciones Journal of Mathematics
Vol. 37, N^o 4, pp. 637-681, December 2018.
Universidad Católica del Norte
Antofagasta - Chile

Fuzzy soft attribute correlation coefficient and application to data of human trafficking

Santanu Acharjee
Debraj Roy College, India
Diganta Jyoti Sarma
Central Institute of Technology, India
Robert A. Hanneman
University of California-Riverside, U.S.A.
John N. Mordeson
Creighton University, U.S.A.
and
Davender S. Malik
Creighton University, U.S.A.
Received : January 2018. Accepted : April 2018

Dedicated to Late Prof. Lotfi A. Zadeh

Abstract

In this paper, we introduce fuzzy soft attribute correlation coefficient and apply it to find the correlation between vulnerability and government response of various countries related to human trafficking based on six regions with the help of data from "The Global Slavery Index 2016". Comparison of fuzzy soft attribute correlation coefficients is done with the conventional analysis of sociology by calculating Pearson's zero-order product-moment correlations. Along with these, some fundamental concepts of mathematical statistics are developed with respect to fuzzy soft set.

2010 AMS Subject Classifications : *03E72, 03E99, 62A86, 62A99, 91D99.*

Keywords : *Fuzzy soft set; correlation coefficient; α-cut; soft set; vulnerability; human trafficking.*

1. Introduction

A mathematical theory is based on various abstract thoughts. One has full freedom to establish certain environments by neglecting many facts. For example in physics, we often neglect the frictional effect of air on a free falling body, but this fact is fully impossible in real life. Similarly, other branches like medical science, economics, engineering, social sciences, etc. are full of uncertainties. Molodtsov [12] introduced the concept of soft set theory in the year 1999 and investigated various applications in game theory, smoothness of functions, operation research, Perron integration, probability theory, the theory of measurement, etc.

Later Maji et al. [10] defined various operations on soft sets to study some of the fundamental properties. Pei and Miao [14], Chen [3] pointed out errors in some of the results of Maji et al. [10] and introduced some new notions and properties. At present, investigations of different properties and applications of soft set theory have attracted many researchers from various backgrounds. Since then many applications of soft set theory can be found in other branches of science and social science. Fuzzy soft set was introduced by Maji et al. [9] as a hybrid structure of soft set with fuzzy set. Fuzzy aspects of sociological researches can be found in [8,6].

Trafficking in persons has been defined as the recruitment, transportation, transfer, harboring or receipt of persons by means of threat or use of force or other forms of coercion, of abduction, of fraud, of deception, of the abuse of power or of a position of vulnerability or the giving or receiving of payments or benefits to achieve the consent of a person having control over another person, for the purpose of exploitation [6]. It is one of the most heinous crimes of our times.

Accurate data concerning the flow of trafficking in persons is impossible to obtain due to the nature of the problem. The goal of the trafficker is to be undetected and to deceive. Besides the scale of the trafficking, there are many other reasons for the scarcity of data. Among these are the victims's reluctance to report crimes or testify for fear of reprisals, disincentives, both structural and legal, for law enforcement to act against traffickers, a lack of harmony among existing data sources, and an unwillingness of some countries and agencies to share data. Thus, concepts from the mathematics of uncertainty provide a valuable way to study the problems of human trafficking.

The data on vulnerability and government response that appears in our application is taken from [4]. "The Global Slavery Index 2016" is an annual study of worldwide slavery conditions by country published by the Walk Free Foundation. In 2016, the study estimated a total of 45.8 million people to be in some form of modern slavery in 167 countries. The report includes three data points for each country, national estimates of the prevalence of modern slavery, vulnerability measures, and assessment of the strength of government response. The index pioneered the use of random-sampled nationally-representative surveys to estimate prevalence. This included commissioning seven such surveys in 2014 and a further 19 surveys through Gallop World Poll in 2015.

The goal of this paper is to develop a method to compare the relationship of two fuzzy soft sets by using techniques from fuzzy soft set theory. We do this by developing a notion of fuzzy soft attribute correlation coefficient. This correlation coefficient is constructed by using the attributes of a fuzzy soft set and α-cuts with respect to utility. We apply our results to determine the relationship between vulnerability and government response averages of countries making up a particular region. The regions are determined from combining certain regions defined in [15]. In [15], the size of the flow of human trafficking between countries is given linguistically. By using the regions in [15], this allows for the development of future researches involving the concepts from both [4] and [15]. One may refer to [1,13] for the recent research related to the mathematics of uncertainties and human trafficking.

In Section 3, we discuss the construction of a fuzzy soft representation with respect to attributes and level cuts along with foundational notions of fuzzy soft statistics. In Section 4, we introduce fuzzy soft attribute correlation coefficients in terms of level sets. We show that The Americas and Europe have the highest correlation levels in this case. In Section 7, we give the fuzzy soft attribute correlation coefficient of the complement of vulnerability and government response in terms of level sets. The importance of the findings here is due to the fact that numbers in [4] representing the vulnerability of countries are high if the vulnerability is high and the numbers representing a country's government response are high if the response is high. One would be particularly interested in knowing the relationship between low vulnerability ratings and high government response ratings. We show that The Americas and Europe have the highest correlation levels in this case also. In fact, the correlation levels for the complement of vulnerability and government response are higher than those for vulnerability and government response.

2. Preliminaries

Following definitions are due to Çağman et al. [2].

Definition 2.1. [2] A soft set F_A on the universe U is defined by the set of ordered pairs $F_A = \{(x, f_A(x) : x \in E, f_A(x) \in P(U)\}$, where $f_A : E \to P(U)$ such that $f_A(x) = \emptyset$ if $x \notin A$.

Here, f_A is called an *approximate function* of the soft set F_A. The value of $f_A(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. We will denote the set of all soft sets over U as $S(U)$.

Definition 2.2. [2] Let $F_A \in S(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called a *soft empty set*, denoted by $F_\emptyset$.

$f_A(x) = \emptyset$ means there is no element in U related to the parameter $x \in E$. Therefore, we do not display such elements in the soft sets, as it is meaningless to consider such parameters.

Definition 2.3. [2] Let $F_A \in S(U)$. If $f_A(x) = U$ for all $x \in A$, then F_A is called an *A-universal soft set*, denoted by $F_{\widetilde{A}}$.

If $A = E$, then the A-universal soft set is denoted by $F_{\widetilde{E}}$.

Definition 2.4. [2] Let $F_A, F_B \in S(U)$. Then F_A is a *soft subset* of F_B, denoted by $F_A \widetilde{\subseteq} F_B$, if $f_A(x) \subseteq f_B(x)$ for all $x \in E$.

Definition 2.5. [2] Let $F_A, F_B \in S(U)$. Then F_A and F_B are soft equal, denoted by $F_A{=}F_B$, if and only if $f_A(x){=}f_B(x)$ for all $x \in E$.

Definition 2.6. [2] Let $F_A, F_B \in S(U)$. Then, the *soft union* $F_A \widetilde{\cup} F_B$, *the* *soft intersection* $F_A \widetilde{\cap} F_B$ and the *soft difference* $F_A \widetilde{\setminus} F_B$ of F_A and F_B are defined by the approximation functions

$f_{A\widetilde{\cup}B}(x) = f_A(x) \cup f_B(x)$, $f_{A\widetilde{\cap}B}(x) = f_A(x) \cap f_B(x)$ and $f_{A\widetilde{\setminus}B}(x) = f_A(x) \setminus f_B(x)$ respectively and the soft complement $F_A^{\widetilde{c}}$ of F_A is defined by the approximate function, $f_A^{\widetilde{c}}(x) = f_A^c(x)$, where $f_A^c(x)$ is the compliment of the set $f_A(x)$; that is $f_A^c(x) = U \setminus f_A(x)$ for all $x \in E$.

It is easy to see that $(F_A^{\widetilde{c}})^{\widetilde{c}} = F_A$ and $F_\emptyset^{\widetilde{c}} = F_{\widetilde{E}}$.

Example 2.1. [2] Let us consider a universe $U = \{a, b, c\}$ and $E = \{e_1, e_2, e_3, e_4\}$. Let $A = \{e_1, e_2, e_3\}$. We define a soft set $(F, A) = \{(e_1, \{a, b\}), (e_2, \{a, c\}), (e_3, \{a, b, c\})\}$.

Then, the representation of (F, A) in tabular form is shown in Table 1:

	$F(e_1)$	$F(e_2)$	$F(e_3)$
a	1	1	1
b	1	0	1
c	0	1	1

Table 1

Definition 2.7. [9] Let U be a universe and let A be a set of parameters. The pair (F, A) is called a fuzzy soft set over U, where $F : A \to [0, 1]^U$.

$[0, 1]^U$ denotes the set of all fuzzy sets of U.

Example 2.2. [11] Suppose that a fuzzy soft set (F, E) describes attractiveness of the shirts which the authors are going to wear. $U=$ the set of all shirts under consideration $= \{x_1, x_2, x_3, x_4, x_5\}$. Let I^U be the collection of all fuzzy subsets of U. Also, let $E = \{colourful, bright, cheap, warm\} = \{e_1, e_2, e_3, e_4\}$.

Let $F(e_1) = \{\frac{x_1}{0.5}, \frac{x_2}{0.9}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0}\}$; $F(e_2) = \{\frac{x_1}{1.0}, \frac{x_2}{0.8}, \frac{x_3}{0.7}, \frac{x_4}{0}, \frac{x_5}{0}\}$; $F(e_3) = \{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}, \frac{x_4}{0.6}, \frac{x_5}{0}\}$; $F(e_4) = \{\frac{x_1}{0}, \frac{x_2}{1}, \frac{x_3}{0}, \frac{x_4}{0}, \frac{x_5}{0.3}\}$

So, the fuzzy soft set (F, E) is a family $\{F(e_i) : i = 1, 2, 3, 4\}$ of I^U.

Definition 2.8. [11] For two fuzzy soft sets (F, A) and (G, B) over a common universe U, we say that (F, A) is a fuzzy soft subset of (G, B); denoted by $(F, A)\widetilde{\subseteq}(G, B)$ if (i) $A \subseteq B$ (ii) $F(e) \subseteq G(e)$.

Definition 2.9. [11] Union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) denoted by $(F, A)\widetilde{\cup}(G, B) = (H, C)$; where

$$H(e) = F(e) \text{ if } e \in A \setminus B;\ G(e) \text{ if } e \in B \setminus A;\ F(e) \cup G(e) \text{ if } e \in A \cap B.$$

Definition 2.10. [11] For two fuzzy soft sets (F, A) and (G, B) over a common universe U are said to be fuzzy soft equal if (F, A) is a fuzzy soft subset of (G, B) and (G, B) is a fuzzy soft subset of (F, A).

Definition 2.11. [11] A fuzzy soft set (F, A) over U is said to be null fuzzy soft set denoted by ϕ, if $\forall e \in A; F(e)=$ null fuzzy set of U.

Definition 2.12. [11] A fuzzy soft set (F, A) over U is said to be absolute fuzzy soft set denoted by $\widetilde{A}$, if $\forall e \in A, F(e) = U$.

Definition 2.13. [17] Restricted intersection of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H, C) denoted by $(F, A)\widetilde{\cap}(G, B) = (H, C)$; where $C = A \cap B \neq \emptyset$ and $\forall e \in C; H(e) = F(e) \cap G(e)$.

Proposition 2.1. [16] Every fuzzy set can be considered as soft set.

Following definition is due to Zadeh [17] whenever A, B are two fuzzy sets.

Definition 2.14. [17] A is contained in B (or, equivalently, A is a subset of B, or A is smaller than or equal to B) if and only if $f_A \leq f_B$. In symbols $A \subseteq B \Leftrightarrow f_A \leq f_B$.

3. Some fundamental statistical notions of fuzzy soft set with respect to attributes, utility and α-cut

In this section, we introduce fuzzy soft mean, α-cut fuzzy soft standard deviation, etc. with examples and proved some theorems. Throughout this paper, we shall denote $I = \{1, 2, 3, ..., n\}$ and Δ is an index set.

3.1. Fuzzy soft mean, α-cut fuzzy soft standard deviation, etc.

Definition 3.1. If (F, A) be a fuzzy soft set over a universe U; where $F(e_i)$ is a fuzzy set for the attribute $e_i \in A$, then fuzzy soft mean of (F, A) is denoted by $\widetilde{F_A} = \{(A, F(A))\}$ and $F(A)$ is defined as follows:

$F(A) = \{\frac{x_1}{min\{\alpha_1^i\}}, \frac{x_2}{min\{\alpha_2^i\}}, \frac{x_3}{min\{\alpha_3^i\}}, ..., \frac{x_k}{min\{\alpha_k^i\}}, ...\}$. Here, α_k^i is the membership value of x_k associated with attributes e_i, where $k \in \Delta$ and $i \in I$.

Here, we consider *minimum* in $F(A)$ as we want to find the minimum of degrees of membership of x_k $\forall i \in I, k \in \Delta$. It is due to the fact that $min\{\alpha_k^i\} \leq \alpha_k^i$ $\forall$ $e_i \in A$. Since, $\forall$ $e_i \in A$, $F(A) \subseteq F(e_i)$ in the sense of fuzzy set, thus we have used the notion $\widetilde{F_A} = \{(A, F(A))\}$, rather than writing as $\widetilde{F_A} = \{(e_i, F(A)) | e_i \in A\}$.

If $0 \leq \alpha \leq 1$, then $\alpha\overrightarrow{F_A}$ is defined as $\alpha\overrightarrow{F_A} = ({\alpha_1}', {\alpha_2}', {\alpha_3}', ..., {\alpha_n}', ...)$, where ${\alpha_k}' = 1$ if $min\{{\alpha_k}^i\} \geq \alpha$, otherwise 0 for $i \in I, k \in \Delta$

Example 3.1. Let us consider a fuzzy soft set $(F, A) = \{(e_1, \{\frac{x_1}{0.9}, \frac{x_2}{0.8}, \frac{x_3}{0.3}\}),$ $(e_2, \{\frac{x_1}{0.4}, \frac{x_2}{0.6}, \frac{x_3}{0.4}\}), (e_3, \{\frac{x_1}{0.3}, \frac{x_2}{0.6}, \frac{x_3}{0.2}\})\}$. Then, $\widetilde{F_A} = \{(A, \{\frac{x_1}{0.3}, \frac{x_2}{0.6}, \frac{x_3}{0.2}\})\}$, where $A = \{e_1, e_2, e_3\}$ and $U = \{x_1, x_2, x_3\}$.

Thus, $0.3\overrightarrow{F_A} = (1, 1, 0)$, $0.4\overrightarrow{F_A} = (0, 1, 0)$, $0.5\overrightarrow{F_A} = (0, 1, 0)$, $0.6\overrightarrow{F_A} = (0, 1, 0)$, $0.7\overrightarrow{F_A} = (0, 0, 0)$ and so on.

Definition 3.2. Let us consider a fuzzy soft set (F, A) where $e_i \in A$. Then, $\alpha\overrightarrow{F(e_i)} = ({\beta_1}', {\beta_2}', {\beta_3}', ..., {\beta_n}', ...)$ where ${\beta_j}' = 1$ if $F_j(e_i)(x_j) \geq \alpha$, otherwise 0. Here, $F_j(e_i)(x_j)$ denotes the membership value of x_j in the j^{th} place of $F(e_i)$.

Example 3.2. Let us consider Example 3.1, where $0.3\overrightarrow{F(e_1)} = (1, 1, 1)$, $0.6\overrightarrow{F(e_1)} = (1, 1, 0)$, $0.6\overrightarrow{F(e_2)} = (0, 1, 0)$ and so on.

Definition 3.3. Let (F, A) be a fuzzy soft set then scale of (F, A) is defined as $h = \max\ \{F(e_i)(x_j) : e_i \in A, x_j \in U, i \in I, j \in \Delta\}$.

Definition 3.4. Let us consider a universe U and a set of attributes E, where $A \subseteq E$ with $\mid A \mid = n$. If (F, A) be a fuzzy soft set over U, then α-cut fuzzy soft standard deviation of (F, A) is denoted by $\sigma(\alpha\overrightarrow{(F, A)})$ and it is defined by $\sqrt{\frac{1}{n}\sum\limits_{i\in I} \|\ \alpha\overrightarrow{F(e_i)} - \alpha\overrightarrow{F_A}\ \|^2}$, where $\|\ \alpha\overrightarrow{F(e_i)} - \alpha\overrightarrow{F_A}\ \|^2 = \langle\alpha\overrightarrow{F(e_i)} - \alpha\overrightarrow{F_A}, \alpha\overrightarrow{F(e_i)} - \alpha\overrightarrow{F_A}\rangle$.

Example 3.3. Let $(F, A) = \{(e_1, \{\frac{x_1}{0.9}, \frac{x_2}{0.8}, \frac{x_3}{0.5}\}), (e_2, \{\frac{x_1}{0.4}, \frac{x_2}{0.6}, \frac{x_3}{0.6}\}),$ $(e_3, \{\frac{x_1}{0.3}, \frac{x_2}{0.6}, \frac{x_3}{0.2}\})\}$. Then, $\widetilde{F_A} = \{(A, \{\frac{x_1}{0.3}, \frac{x_2}{0.6}, \frac{x_3}{0.2}\})\}$, where $A = \{e_1, e_2, e_3\}$ and $U = \{x_1, x_2, x_3\}$.

Now, $0.3\overrightarrow{F_A} = (1, 1, 0)$, $0.3\overrightarrow{F(e_1)} = (1, 1, 1)$, $0.3\overrightarrow{F(e_2)} = (1, 1, 1)$, and $0.3\overrightarrow{F(e_3)} = (1, 1, 0)$. Then, $\sigma(0.3\overrightarrow{(F, A)}) = \sqrt{\frac{2}{3}}$.

3.2. Concept of utility wise representation of fuzzy soft set

Consider a fuzzy soft set (F, A) over a universe U and $\mathbf{R}$ is the set of real numbers. We define an α-cut level utility function $\mu_\alpha : U \to \mathbf{R}$ as $x \succeq y \iff \mu_\alpha(x) \geq \mu_\alpha(y)$ for $x, y \in U$ and so on with fundamental notions of utility representations of utility theory. Here, the notion $x \succeq y$ indicates that x is strictly preferred and indifferent to y. Readers can find these notions in literature of utility theory and microeconomics.

Without any presumption of fuzzy α-cut, it must be mentioned that here we indicate α-cut, which is based on utility. Thus, it is easy to find that our α-cut doesn't generate a set with elements from x_k's as usually we do in fuzzy α-cut. Readers should not be confused with this notion.

If $F_j(e_i)(x_j) \geq \alpha$ for $x_j \in U$ and $e_i \in A$, then utility wise representation of x_j in $F(e_i)$ is shown in Table 2.

	$F(e_i)$	$F(e_i)^c$
x_j^{α}	$\mu(x_j)$	1-$\mu(x_j)$

Table 2

If $F_j(e_i)(x_j) < \alpha$, then we assume $\mu(x_j) = 0$ for the particular case beyond any presumption of $\mu(x_j)$. Here, x_j^{α} denotes $x_j \in U$ with α-cut level utility.

Example 3.4. Let us consider Example 3.3. We define a 0.3-cut level utility function $\mu_{0.3} : U \to \mathbf{R}$ as $\mu_{0.3}(x_1) = 5$, $\mu_{0.3}(x_2) = -2$ and $\mu_{0.3}(x_3) = 3$. Thus, utility wise representation of (F, A) at 0.3-cut level utility is shown in Table 3.

	$F(e_1)$	$F(e_2)$	$F(e_2)$
$x_1^{0.3}$	5	5	5
$x_2^{0.3}$	-2	-2	-2
$x_3^{0.3}$	3	3	0

Table 3

Then, we call $(5, -2, 3)$ as the origin of (F, A) at 0.3-cut level utility and this concept can be extended for any fuzzy soft set.

It is important to note that the origin of a fuzzy soft set is not fixed as the concept of classical mathematics or statistics. It is based on one's choice and α-cut level of utility.

3.3. Generating process of a new fuzzy soft set from the old one with respect to α-cut and utility

(i) **α-cut generated fuzzy soft set**

Consider a fuzzy soft set (F, A) whose α-cut level representation is denoted by $\alpha\overrightarrow{(F, A)}$ over a universe U with elements $x_i, i \in \Delta$ and $e_j \in A, j \in I$.

Then, we can generate a new fuzzy soft set $(G_F, A)_\alpha$ with α-cut level of representation $\alpha\overrightarrow{(G_F, A)}$ if $(\alpha_1, \alpha_2, ..., \alpha_i, ...)+\alpha\overrightarrow{(F, A)}$ exists, where

$(\alpha_1, \alpha_2, ..., \alpha_i, ...) \neq (0, 0, 0, ..., 0, ...)$, $\alpha_i \in \mathbf{R}$ and $i \in \Delta$.

Now, we discuss the generating process. We define an α-cut level utility function $\mu_\alpha : U \to \mathbf{R}$ such that if $\alpha_i + F_i(e_j) \geq \mu_\alpha(x_i)$, $i \in \Delta, j \in I$; then $x_i \in G_i(e_j)$ with the membership value minimum of { membership value of x_i in $F(e_j)$, α}, otherwise, $x_i \notin G_i(e_j)$.

Here, $(\alpha_1, \alpha_2, ..., \alpha_i, ...) + \alpha\overrightarrow{(F, A)} = \{(\alpha_1, \alpha_2, ..., \alpha_i, ...) + \alpha\overrightarrow{F(e_1)}, (\alpha_1, \alpha_2, ..., \alpha_i, ..) + \alpha\overrightarrow{F(e_2)}, ..., (\alpha_1, \alpha_2, ..., \alpha_i, ...) + \alpha\overrightarrow{F(e_n)}\}$, where $i \in \Delta, j \in I$.

The new fuzzy soft set $(G_F, A)_\alpha$ with representation $\alpha\overrightarrow{(G_F, A)} = (\alpha_1, \alpha_2, ..., \alpha_i, ...) + \alpha\overrightarrow{(F, A)}$ is called an *α-cut generated fuzzy soft set* of (F, A). The fuzzy soft mean of $(G_F, A)_\alpha$ is denoted by $\widetilde{G_A}$ with α-cut level of representation $\alpha\overrightarrow{G_A}$.

(ii) **α-cut generated fuzzy soft set with scale**

Let the scale of a fuzzy soft set (F, A) be h. Then, similarly as discussed above we can generate a new *α-cut generated fuzzy soft set* $(G_F, A)^h_\alpha$ with representation $\alpha\overrightarrow{(G_F, A)_h} = \frac{\alpha\overrightarrow{(G_F, A)}}{h}$.

The fuzzy soft mean of $(G_F, A)^h_\alpha$ is denoted by $\widetilde{G_A}^h$ with α-cut level representation of fuzzy soft mean of $(G_F, A)^h_\alpha$ as $\alpha\overrightarrow{G^h_A}$.

Example 3.5. Let us consider a fuzzy soft set $(F, A) = \{(e_1, \{\frac{x_1}{0.4}, \frac{x_2}{0.8}, \frac{x_3}{0.5}\}), (e_2, \{\frac{x_1}{0.6}, \frac{x_2}{0.2}, \frac{x_3}{0.2}\})\}$. We define a 0.3-cut level of utility function $\mu_{0.3} : U \to \mathbf{R}$ as $\mu_{0.3}(x_1) = 5$, $\mu_{0.3}(x_2) = -2$ and $\mu_{0.3}(x_3) = 3$.

Let $(\alpha_1, \alpha_2, \alpha_3) = (1, 4, 5)$, then $(1, 4, 5) + 0.3\overrightarrow{(F, A)} = \{(1, 4, 5) + (1, 1, 1), (1, 4, 5) + (1, 0, 0)\} = \{(2, 5, 6), (2, 4, 5)\}$.

Then, we have $(G_F, A)_{0.3} = \{(e_1, \{\frac{x_1}{0}, \frac{x_2}{0.3}, \frac{x_3}{0.3}\}), (e_2, \{\frac{x_1}{0}, \frac{x_2}{0.2}, \frac{x_3}{0.2}\})\}$.

Theorem 3.1. Let (F, A) be a fuzzy soft set over U and $(G_F, A)_\alpha$ be an α-cut generated fuzzy soft set of (F, A), then $\sigma(\alpha\overrightarrow{(F, A)}) = \sigma(\alpha\overrightarrow{(G_F, A)})$

Proof: Let $(\alpha_1, \alpha_2, \alpha_3, ..., \alpha_i, ...) \neq (0, 0, 0, ..., 0, ...)$, where $\alpha_i \in \mathbf{R}$, $i \in \Delta$.

We define an α-cut level utility function $\mu_\alpha : U \to \mathbf{R}$ as $\mu_\alpha(x_i) = \beta_i$ such that $x_i \in G_i(e_j)$ if $\alpha_i + F_i(e_j) \geq \beta_i$, where $i \in \Delta, j \in I$. Here, $x_i \in G_i(e_j)$ indicates that x_i is in i^{th} place of $G(e_j)$ with membership value.

Then, $\alpha\overrightarrow{G_A}= (min\{\alpha_1+\alpha F_1(e_1), \alpha_1+\alpha F_1(e_2), ..., \alpha_1+\alpha F_1(e_n)\}, min\{\alpha_2+\alpha F_2(e_1), \alpha_2+\alpha F_2(e_2), ..., \alpha_2+\alpha F_2(e_n)\}, ..., min\{\alpha_i+\alpha F_i(e_1), \alpha_i+\alpha F_i(e_2), ..., \alpha_i+\alpha F_i(e_n)\}, ...)$, where $\alpha F_i(e_n)$ indicates the i^{th}-coordinate of $\alpha\overrightarrow{F(e_n)}$ with respect to x_i.

$= (\alpha_1+\alpha F_1(e_{j_1}), \alpha_2+\alpha F_2(e_{j_2}), \alpha_3+\alpha F_3(e_{j_3}), ..., \alpha_i+\alpha F_i(e_{j_i}), ...)$ (say), where each $j_i \in I$.

Now, $\alpha\overrightarrow{G(e_j)}-\alpha\overrightarrow{G_A} = (\alpha_1+\alpha F_1(e_j), \alpha_2+\alpha F_2(e_j), ..., \alpha_i+\alpha F_i(e_j),...)-(\alpha_1+\alpha F_1(e_{j_1}), \alpha_2 + \alpha F_2(e_{j_2}), \alpha_3 + \alpha F_3(e_{j_3}), ..., \alpha_i + \alpha F_i(e_{j_i}), ...)$.

$= (\alpha F_1(e_j) - \alpha F_1(e_{j_1}), \alpha F_2(e_j) - \alpha F_2(e_{j_2}), ..., \alpha F_i(e_j) - \alpha F_i(e_{j_i}), ...)$

$= \alpha\overrightarrow{F(e_j)} - \alpha\overrightarrow{F_A}$

Now, $|| \alpha\overrightarrow{G(e_j)} - \alpha\overrightarrow{G_A} ||^2 = \langle\alpha\overrightarrow{G(e_j)} - \alpha\overrightarrow{G_A}, \alpha\overrightarrow{G(e_j)} - \alpha\overrightarrow{G_A}\rangle$.

$= \langle\alpha\overrightarrow{F(e_j)} - \alpha\overrightarrow{F_A}, \alpha\overrightarrow{F(e_j)} - \alpha\overrightarrow{F_A}\rangle$

$= || \alpha\overrightarrow{F(e_j)} - \alpha\overrightarrow{F_A} ||^2$

Thus, $\sigma(\alpha\overrightarrow{(F, A)}) = \sigma(\alpha\overrightarrow{(G_F, A)})$

Corollary 3.1. Let (F, A) be a fuzzy soft set over U and $(G_F, A)_\alpha$, $(H_F, A)_\alpha$ are two distinct α-cut generated fuzzy soft sets of (F, A), then $\sigma(\alpha\overrightarrow{(F, A)}) = \sigma(\alpha\overrightarrow{(G_F, A)}) = \sigma(\alpha\overrightarrow{(H_F, A)})$.

Thus, it can be said that α-cut fuzzy soft standard deviation of an α-cut generated fuzzy soft of any fuzzy soft set is independent of the choice of $(\alpha_1, \alpha_2, ..., \alpha_i, ...)$ and utility.

Example 3.6. Let us consider Example 3.5.

Here, $0.3\overrightarrow{F_A} = (1,0,0)$, $0.3\overrightarrow{F(e_1)} = (1,1,1)$, $0.3\overrightarrow{F(e_2)} = (1,0,0)$, then $\sigma(0.3\overrightarrow{(F,A)}) = 1$.

Again, $0.3\overrightarrow{G_A} = (0,0,0)$, $0.3\overrightarrow{G(e_1)} = (0,1,1)$, $0.3\overrightarrow{F(e_2)} = (0,0,0)$, then $\sigma(0.3\overrightarrow{(G_F,A)}) = 1$.

The proof of the following theorem can be obtained by following the steps of Theorem 3.1 and using the concept of section "α-cut generated fuzzy soft set with scale".

Theorem 3.2. Let (F,A) be a fuzzy soft set over U and $(G_F,A)^h_\alpha$ be an α-cut generated fuzzy soft set of (F,A), then $\sigma(\alpha\overrightarrow{(G_F,A)}_h) = \frac{1}{h}\sigma(\alpha\overrightarrow{(F,A)})$.

3.4. Fuzzy soft coefficient of variation

Definition 3.5. Let us consider a fuzzy soft set (F,A) whose α-cut level representation is denoted by $\alpha\overrightarrow{(F,A)}$ over a universe U. Then, *α-cut level fuzzy soft coefficient of variation* is denoted by $\alpha FSCV\overrightarrow{(F,A)}$ and it is defined as $\alpha FSCV\overrightarrow{(F,A)} = \{\frac{\sigma(\alpha\overrightarrow{(F,A)})}{||\alpha\overrightarrow{F_A}||^2}\} \times 100$.

Theorem 3.3. Let (F,A) be a fuzzy soft set over U, then $\alpha FSCV\overrightarrow{(G_F,A)}$ $=\{\frac{\sigma(\alpha\overrightarrow{(F,A)})}{\theta}\} \times 100$, where $\theta = ||\ \beta\ ||^2 + 2\langle\beta, \alpha\overrightarrow{F_A}\rangle + ||\ \alpha\overrightarrow{F_A}\ ||^2$ and $\beta = (\alpha_1, \alpha_2, \alpha_3, ..., \alpha_i, ...)$.

Proof. From the definition, we know that $\alpha\overrightarrow{(G_F,A)} = \beta + \alpha\overrightarrow{(F,A)}$, where $\beta = (\alpha_1, \alpha_2, \alpha_3, ..., \alpha_i, ...)$. Thus, it is easy to find that $\alpha\overrightarrow{G_A} = \beta + \alpha\overrightarrow{F_A}$.

Thus, $||\ \alpha\overrightarrow{G_A}\ ||^2 = ||\ \beta\ ||^2 + 2\langle\beta, \alpha\overrightarrow{F_A}\rangle + ||\ \alpha\overrightarrow{F_A}\ ||^2$ ($=\theta$, say); since in our case $\langle\beta, \alpha\overrightarrow{F_A}\rangle = \langle\alpha\overrightarrow{F_A}, \beta\rangle$.

Again from Theorem 3.1., we have $\sigma(\alpha\overrightarrow{(F,A)}) = \sigma(\alpha\overrightarrow{(G_F,A)})$. Hence, Definition 3.5. yields the result.

Theorem 3.4. Let (F, A) be a fuzzy soft set over U, then $\alpha FSCV\overrightarrow{(G_F, A)_h}$ $= h \times \alpha FSCV\overrightarrow{(G_F, A)}$.

Example 3.7. Let us consider a fuzzy soft set
$(F, A) = \{(e_1, \{\frac{x_1}{0.8}, \frac{x_2}{0.7}, \frac{x_3}{0.8}\}), (e_2, \{\frac{x_1}{0.6}, \frac{x_2}{0.7}, \frac{x_3}{0.2}\})\}$.

We define a 0.3-cut level utility function $\mu_{0.3} : U \to \mathbf{R}$ as $\mu_{0.3}(x_1) = 5$, $\mu_{0.3}(x_2) = -2$ and $\mu_{0.3}(x_3) = 3$.

Let $(\alpha_1, \alpha_2, \alpha_3) = (1, 4, 5)$, then $(1, 4, 5)+0.3\overrightarrow{(F, A)} = \{(2, 5, 6), (2, 5, 5)\}$. Then $0.3\overrightarrow{(G, A)} = \{(2, 5, 6), (2, 5, 5)\}$. So, $0.3\overrightarrow{G_A} = (2, 5, 5)$. Here, h= 0.8

$\sigma(0.3\overrightarrow{(G_F, A)}) = \frac{1}{\sqrt{2}}$.

Again, $0.3\overrightarrow{G_A^{0.8}} = (\frac{2}{0.8}, \frac{5}{0.8}, \frac{5}{0.8})$ and hence, $\sigma(0.3\overrightarrow{(G_F, A)_{0.8}}) = \frac{1}{0.8} \times \frac{1}{\sqrt{2}}$.

Thus, calculating we have $0.3FSCV\overrightarrow{(G_F, A)_{0.8}} = 0.8 \times 0.3FSCV\overrightarrow{(G_F, A)}$.

Remark 3.1. Scaling on an α-cut generated fuzzy soft set may not generate distinct α-cut generated fuzzy soft set.

The above remark can be verified from the following example.

Example 3.8. Let us consider Example 3.5., where,
$(F, A) =\{(e_1, \{\frac{x_1}{0.4}, \frac{x_2}{0.8}, \frac{x_3}{0.5}\}), (e_2, \{\frac{x_1}{0.6}, \frac{x_2}{0.2}, \frac{x_3}{0.2}\})\}$.
Then, we have $(G_F, A)_{0.3} = \{(e_1, \{\frac{x_1}{0}, \frac{x_2}{0.3}, \frac{x_3}{0.3}\}), (e_2, \{\frac{x_1}{0}, \frac{x_2}{0.2}, \frac{x_3}{0.2}\})$.

Now, $\frac{(1,4,5)+0.3\overrightarrow{(F,A)}}{0.8} = \{(2.5, 6.25, 7.5), (2.5, 6.25, 6.25)\}$.

Then, for (2.5, 6.25, 7.5), membership value of $x_1 = 0$, membership value of $x_2 = 0.3$, membership value of $x_3 = 0.3$.

Again, for (2.5, 6.25, 6.25), membership value of $x_1 = 0$, membership value of $x_2 = 0.2$, membership value of $x_3 = 0.2$.

Thus, $(G_F, A)^{0.8}_{0.3} = \{(e_1, \{\frac{x_1}{0}, \frac{x_2}{0.3}, \frac{x_3}{0.3}\}), (e_2, \{\frac{x_1}{0}, \frac{x_2}{0.2}, \frac{x_3}{0.2}\})$.

This example shows that utility matters more than scaling to generate fuzzy soft set with α-cut .

3.5. Fuzzy soft covariance with α-cut

Definition 3.6.
(i) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two fuzzy soft sets, where $A, B \subseteq E, \mid A \mid = n > \mid B \mid = m$. We extend B to $C = B \cup \{f_{m+1}, f_{m+2}, ..., f_n\}$ such that $G_i(f_k)(x_i) = 0$ $\forall k \in \{m+1, m+2, ..., n\}$. Then, *$\alpha$-cut level fuzzy soft covariance of (F, A) and (G, B)* is denoted by $\alpha FSCov(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|>|B|})$ and it is defined as $\alpha FSCov(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|>|B|}) = \frac{1}{n}\ \{\|\ \Delta_1\ \|^2 + \|\ \Delta_2\ \|^2 + ... + \|\ \Delta_n\ \|^2\}$, where $\Delta_j = (min\{\alpha F_1(e_j), \alpha G_1(f_j)\}, min\{\alpha F_2(e_j), \alpha G_2(f_j)\}, ..., min\{\alpha F_i(e_j), \alpha G_i(f_j)\}, ...)$, and $e_j \in A, f_j \in C$, $i \in \Delta, j \in I$.
The attributes $f_{m+1}, f_{m+2}, ..., f_n$ with $G_i(f_k)(x_j) = 0\ \forall k \in \{m+1, m+2, ..., n\}, i \in \Delta$ are called *fuzzy soft statistical dummy attributes* for B relative to A.
(ii) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two fuzzy soft sets, where $A, B \subseteq E, \mid A \mid = n = \mid B \mid$. Then, *$\alpha$-cut level fuzzy soft covariance of (F, A) and (G, B)* is denoted by $\alpha FSCov(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|=|B|})$ and it is defined as
$\alpha FSCov(\overrightarrow{(F, A)}, \overrightarrow{(G, B)}_{|A|=|B|}) = \frac{1}{n}\ \{\|\ \Delta_1\ \|^2 + \|\ \Delta_2\ \|^2 + ... + \|\ \Delta_n\ \|^2\}$, where
$\Delta_j = (min\{\alpha F_1(e_j), \alpha G_1(f_j)\}, min\{\alpha F_2(e_j), \alpha G_2(f_j)\}, ...,$
$min\{\alpha F_i(e_j), \alpha G_i(f_j)\}, ...)$, and $e_j \in A, f_j \in B$, $i \in \Delta, j \in I$.
The above definition can be redefined if $A = B$. In this case
$e_j = f_j\ \forall j \in I$.

Definition 3.7. (i) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two fuzzy soft sets, where $A, B \subseteq E, \mid A \mid = n > \mid B \mid = m$. We extend B to $C = B \cup \{f_{m+1}, f_{m+2}, ..., f_n\}$ such that $G_i(f_k)(x_i) = 0\ \forall k \in \{m+1, m+2, ..., n\}$. Then, (F, A) and (G, B) are said to be *ϵ_α-approximation independent fuzzy soft sets* if $\Delta_j = (min\{\alpha F_1(e_j), \alpha G_1(f_j)\}, min\{\alpha F_2(e_j), \alpha G_2(f_j)\}, ...,$
$min\{\alpha F_i(e_j), \alpha G_i(f_j)\}, ...) = 0$, where $0 = (0, 0, ..., 0, ...)$, $e_j \in A$ and $f_j \in C$, $i \in \Delta$, $\forall j \in I$.

(ii) Let us consider a universe U with the set of attributes E. Let (F, A) and (G, B) be two fuzzy soft sets, where $A, B \subseteq E, \mid A \mid = n = \mid B \mid$. Then,

(F,A) and (G,B) are said to be ϵ_α-*approximation independent fuzzy soft sets* if $\Delta_j = (min\{\alpha F_1(e_j), \alpha G_1(f_j)\}, min\{\alpha F_2(e_j), \alpha G_2(f_j)\}, ...,$ $min\{\alpha F_i(e_j), \alpha G_i(f_j)\}, ...) = 0$, where $0 = (0,0,...,0,...)$, $e_j \in A$ and $f_j \in B$,$i \in \Delta$, $\forall j \in I$.

Theorem 3.5. (i) Let us consider a universe U with the set of attributes E. Let (F,A) and (G,B) be two fuzzy soft sets, where $A, B \subseteq E, \mid A \mid = n > \mid B \mid = m$. Then, $\alpha FSCov(\overrightarrow{(F,A)}, \overrightarrow{(G,B)}_{|A|>|B|}) = 0 \Leftrightarrow (F,A)$ and (G,B) are ϵ_α-approximation independent fuzzy soft sets.

(ii) Let us consider a universe U with the set of attributes E. Let (F,A) and (G,B) be two fuzzy soft sets, where $A, B \subseteq E, \mid A \mid = n = \mid B \mid$. Then, $\alpha FSCov(\overrightarrow{(F,A)}, \overrightarrow{(G,B)}_{|A|=|B|}) = 0 \Leftrightarrow (F,A)$ and (G,B) are ϵ_α-approximation independent fuzzy soft sets.

Proof: (i) $\alpha FSCov(\overrightarrow{(F,A)}, \overrightarrow{(G,B)}_{|A|>|B|}) = 0$
$\Leftrightarrow \frac{1}{n} \{|| \Delta_1 ||^2 + || \Delta_2 ||^2 + ... + || \Delta_n ||^2\} = 0$
$\Leftrightarrow || \Delta_j ||^2 = 0 \ \forall j \in I$
$\Leftrightarrow || \Delta_j || = 0 \ \forall j \in I$
$\Leftrightarrow \Delta_j = (0,0,...,0,...) \ \forall j \in I$
$\Leftrightarrow (F,A)$ and (G,B) are ϵ_α-approximation independent fuzzy soft sets.

(ii) Proof can be done following the above steps.

4. Fuzzy soft attribute correlation coefficient and α-cut

In Definition 3.2, we defined $\alpha\overrightarrow{F(e_i)}$. Now, we consider the following example.

Example 4.1. Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ be a universe and $E = \{e_1, e_2, e_3, e_4\}$ be a set of attributes. We consider $A = \{e_1, e_2, e_3\}$ and define a fuzzy soft set as

$(F,A) = \{(e_1, \{\frac{x_1}{0.1}, \frac{x_2}{0}, \frac{x_3}{0.5}, \frac{x_4}{0.9}, \frac{x_5}{0}\}), (e_2, \{\frac{x_1}{0}, \frac{x_2}{0.8}, \frac{x_3}{0.4}, \frac{x_4}{0.4}, \frac{x_5}{0}\}),$
$(e_3, \{\frac{x_1}{0.2}, \frac{x_2}{0}, \frac{x_3}{0.8}, \frac{x_4}{0.9}, \frac{x_5}{0}\})\}$; then $0.1\overrightarrow{F(e_1)} = (1,0,1,1,0)$,$0.3\overrightarrow{F(e_1)}$ $= (0,0,1,1,0)$,$0.5\overrightarrow{F(e_3)} = (0,0,1,1,0)$, etc.

Definition 4.1. Let (F, A) be a fuzzy soft set with at least two attributes e_1 and e_2, then fuzzy soft attribute correlation coefficient of $\alpha\overrightarrow{F(e_1)}$ and $\alpha\overrightarrow{F(e_2)}$ at α-cut level is defined as follows:

$$FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{F(e_2)}) = \frac{\sum_{i\in I} \Delta_i}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||}; \text{ where}$$

Δ_i= min $\{\alpha F_i(e_1), \alpha F_i(e_2)\}$, $||\alpha\overrightarrow{F(e_j)}|| = \sqrt{\langle \alpha\overrightarrow{F(e_j)}, \alpha\overrightarrow{F(e_j)}\rangle} = \sqrt{\sum_{i\in I} (\alpha F_i(e_j))^2}$, $||\alpha\overrightarrow{F(e_1)}|| \neq 0$ and $||\alpha\overrightarrow{F(e_2)}|| \neq 0$.

If $||\alpha\overrightarrow{F(e_1)}|| = 0$ or $||\alpha\overrightarrow{F(e_2)}|| = 0$, then $FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{F(e_2)})$ is not possible. In this case, we shall use notation "∞" without considering any matter of Δ_i.

Example 4.2. Let us consider a universe $U = \{x_1, x_2, x_3, x_4\}$ and a set of attributes $E = \{e_1, e_2, e_3, e_4\}$. Let $A = \{e_1, e_2, e_3\}$. We define a fuzzy soft set (F, A) =$\{(e_1, \{\frac{x_1}{0.1}, \frac{x_2}{0.7}, \frac{x_3}{0.2}, \frac{x_4}{0}\}), (e_2, \{\frac{x_1}{0.2}, \frac{x_2}{0}, \frac{x_3}{0.4}, \frac{x_4}{0.5}\}),$
$(e_3, \{\frac{x_1}{0}, \frac{x_2}{0.3}, \frac{x_3}{0.5}, \frac{x_4}{0}\})\}$. Consider $\alpha = 0.1$, then $0.1\overrightarrow{F(e_1)} = (1, 1, 1, 0)$ and $0.1\overrightarrow{F(e_2)} = (1, 0, 1, 1)$. Then Δ_1 = min $\{1, 1\} = 1$, Δ_2 = min $\{1, 0\} = 0$, Δ_3 =min $\{1, 1\} = 1$ and Δ_4 = min $\{0, 1\} = 0$. Then,
$FSACC(0.1\overrightarrow{F(e_1)}, 0.1\overrightarrow{F(e_2)}) = 0.67$.

Theorem 4.1. If (F, A) be any fuzzy soft set with at least two attributes e_1 and e_2 over a universe U, then $0 \leq FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{F(e_2)}) \leq 1$.

Proof. We know,
$\sum_{i\in I} \Delta_i = \sum_{i\in I}$ min $\{\alpha F_i(e_1), \alpha F_i(e_2)\} \leq \sum_{i\in I} (\alpha F_i(e_1).\alpha F_i(e_2))$
$\leq \sqrt{\sum_{i\in I} (\alpha F_i(e_1))^2}.\sqrt{\sum_{i\in I} (\alpha F_i(e_2))^2} = ||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||$.

So, $\frac{\sum_{i\in I} \Delta_i}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}||} \leq 1$, which implies $FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{F(e_2)}) \leq 1$.

Also, $\sum_{i\in I} \Delta_i \geq 0$ and $||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{F(e_2)}|| > 0$ which implies
$0 \leq FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{F(e_2)})$.

Hence, $0 \leq FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha F(e_2)}) \leq 1$.

Theorem 4.2. Let (F, A) be a fuzzy soft set with at least two attributes e_1 and e_2, then $FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha F(e_2)}) = FSACC(\overrightarrow{\alpha F(e_2)}, \overrightarrow{\alpha F(e_1)})$.

Proof. Proof can be obtained from Definition 4.1.

Theorem 4.3. If $F(e_1), G(e_1)$ and $H(e_1)$ are fuzzy sets of fuzzy soft sets $(F, A), (G, A)$ and (H, A) respectively over U such that $F(e_1) \subseteq G(e_1) \subseteq H(e_1)$ and $e_1 \in A$, then;

(i) $FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha H(e_1)}) \leq FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha G(e_1)})$

(ii) $FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha H(e_1)}) \leq FSACC(\overrightarrow{\alpha G(e_1)}, \overrightarrow{\alpha H(e_1)})$

(iii) $FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha G(e_1)}) \leq FSACC(\overrightarrow{\alpha H(e_1)}, \overrightarrow{\alpha G(e_1)})$ if $(||\overrightarrow{\alpha F(e_1)}||.||\overrightarrow{\alpha H(e_1)}|| - ||\overrightarrow{\alpha G(e_1)}||^2) \leq 0$

Proof. (i) Since $F(e_1) \subseteq G(e_1) \subseteq H(e_1)$, so $F_j(e_1)(x_j) \leq G_j(e_1)(x_j) \leq H_j(e_1)(x_j)$. Thus, $\Delta_i = \min\{\alpha F_i(e_1), \alpha H_i(e_1)\} = \alpha F_i(e_1)$
and $\Delta_i' = \min\{\alpha F_i(e_1), \alpha G_i(e_1)\} = \alpha F_i(e_1)$. Thus, $\Delta_i = \Delta_i'$.

Now, $FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha H(e_1)}) - FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha G(e_1)})$

$$= \frac{\sum_{i\in I} \Delta_i}{||\overrightarrow{\alpha F(e_1)}||.||\overrightarrow{\alpha H(e_1)}||} - \frac{\sum_{i\in I} \Delta_i'}{||\overrightarrow{\alpha F(e_1)}||.||\overrightarrow{\alpha G(e_1)}||}$$
$$= \frac{\sum_{i\in I} \Delta_i}{||\overrightarrow{\alpha F(e_1)}||}.\{\frac{1}{||\overrightarrow{\alpha H(e_1)}||} - \frac{1}{||\overrightarrow{\alpha G(e_1)}||}\} \text{ ......(3.1)}$$

Since, $G(e_1) \subseteq H(e_1)$ and $G(e_1)$, $H(e_1)$ are fuzzy sets, so $\alpha G_i(e_1) \leq \alpha H_i(e_1) \Rightarrow \sum_{i\in I} \{\alpha G_i(e_1)\}^2 \leq \sum_{i\in I} \{\alpha H_i(e_1)\}^2$
$\Rightarrow ||\overrightarrow{\alpha G(e_1)}|| \leq ||\overrightarrow{\alpha H(e_1)}|| \Rightarrow \{\frac{1}{||\overrightarrow{\alpha H(e_1)}||} - \frac{1}{||\overrightarrow{\alpha G(e_1)}||}\} \leq 0$ ... (3.2)

So, (3.1) and (3.2.) implies
$FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha H(e_1)}) \leq FSACC(\overrightarrow{\alpha F(e_1)}, \overrightarrow{\alpha G(e_1)})$.

(ii) Since $F(e_1) \subseteq G(e_1) \subseteq H(e_1)$, so
$F_j(e_1)(x_j) \leq G_j(e_1)(x_j) \leq H_j(e_1)(x_j)$. Thus, $\Delta_i = \min\{\alpha F_i(e_1), \alpha H_i(e_1)\} = \alpha F_i(e_1)$ and $\Delta_i' = \min\{\alpha G_i(e_1), \alpha H_i(e_1)\} = \alpha G_i(e_1)$.

Now, $FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{H(e_1)}) - FSACC(\alpha\overrightarrow{G(e_1)}, \alpha\overrightarrow{H(e_1)})$

$$= \frac{\sum\limits_{i\in I}\Delta_i}{||\alpha\overrightarrow{F(e_1)}||.||\alpha\overrightarrow{H(e_1)}||} - \frac{\sum\limits_{i\in I}\Delta_i'}{||\alpha\overrightarrow{G(e_1)}||.||\alpha\overrightarrow{H(e_1)}||}.$$

$$= \frac{1}{||\alpha\overrightarrow{H(e_1)}||}.\{\frac{\sum\limits_{i\in I}\{\alpha F_i(e_1)\}^2}{||\alpha\overrightarrow{F(e_1)}||} - \frac{\sum\limits_{i\in I}\{\alpha G_i(e_1)\}^2}{||\alpha\overrightarrow{G(e_1)}||}\}$$

$$= \frac{1}{||\alpha\overrightarrow{H(e_1)}||}.\ \{\frac{||\alpha\overrightarrow{F(e_1)}||^2}{||\alpha\overrightarrow{F(e_1)}||} - \frac{||\alpha\overrightarrow{G(e_1)}||^2}{||\alpha\overrightarrow{G(e_1)}||}\}$$

$$= \frac{1}{||\alpha\overrightarrow{H(e_1)}||}.\ \{||\alpha\overrightarrow{F(e_1)}|| - ||\alpha\overrightarrow{G(e_1)}||\} \leq 0$$

Hence, $FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{H(e_1)}) \leq FSACC(\alpha\overrightarrow{G(e_1)}, \alpha\overrightarrow{H(e_1)})$.

(iii) Proof can be done by following the above steps.

Now, we state the following theorem without proof.

Theorem 4.4. $FSACC(\alpha\overrightarrow{F(e_1)}, \alpha\overrightarrow{F(e_2)}) = 1 \Leftrightarrow \alpha\overrightarrow{F(e_1)} = \alpha\overrightarrow{F(e_2)}$

5. Application to human trafficking

In this section, we do analysis of fuzzy soft attribute correlation coefficient ($FSACC$) between mean of vulnerability and mean of government responses of the following regions: Asia Pacific, Europe, Russia and Eurasia, Sub-Saharan Africa, The Middle East and North Africa, and The Americas. For the purpose of region-wise data related to vulnerability and government responses, we consider region-wise data of "The Global Slavery Index 2016" [4].

According to "The Global Slavery index 2016" [4], twenty four vulnerability variables are clustered in following four dimensions.

Dimension 1: Civil and political protections: Confidence in judicial system, political stability, weapons access, discrimination (sexuality), displaced persons, global slavery index government response values, political rights measure.

Dimension 2: Social, health and economic rights: Financial inclusion (borrowed any money), financial inclusion (received wages), cell phone subscription, safety net, undernourishment, tuberculosis, water improved access.

Dimension 3: Personal security: Financial inclusion (availability of emergency funds), violent crime, women's physical security, GINI coefficient, discrimination (intellectual disability), discrimination (immigrants), discrimination(minorities).

Dimension 4: Refugee populations and conflict: Impact of terrorism, internal conflict, refugees resident.

Now, we denote two sets of parameters P and Q related to vulnerability and government responses respectively.

Let $P = \{e_1, e_2, e_3, e_4\}$, $Q = \{f_1, f_2, f_3, f_4, f_5\}$.

We consider the following notions for the attributes of vulnerability and government responses.

$e_1 =$ *Civil and political protection*;

$e_2 =$ *Social, health, and economic rights*;

$e_3 =$ *Personal security*;

$e_4 =$ *Refugees and conflict*;

$f_1 =$ *Survivors supported*;

$f_2 =$ *Criminal justice*;

$f_3 =$ *Coordination and accountability*;

f_4 = *Addressing risk*;

f_5 = *Government and business.*

Let $A = \{g_1, g_2, g_3\}$, where g_1 = *mean of vulnerability*, g_2 = *mean of government responses* and g_3 = *100 - mean of vulnerability.*

Throughout this article, $FSACC_\alpha(Asia\ Pacific)$ means $FSACC(\alpha\overrightarrow{F(g_1)}, \alpha\overrightarrow{F(g_2)})$ for Asia Pacific and so on for other five regions. Similarly, we shall denote $FSACC_\alpha^C(Asia\ Pacific)$ means $FSACC(\alpha\overrightarrow{F(g_2)}, \alpha\overrightarrow{F(g_3)})$ for Asia Pacific and so on for other five regions.

Let X be the set of 167 countries, which are considered in "The Global Slavery Index 2016" [4] and U_i, $i \in \{1, 2, 3, 4, 5, 6\}$ be the regions Asia Pacific, Russia and Eurasia, The Middle East and North Africa, The Americas, Europe, and Sub-Saharan Africa respectively. We define fuzzy soft sets $F_{U_i} : A \to [0, 1]^{U_i}$ $\forall$ $i \in \{1, 2, 3, 4, 5, 6\}$ with the following membership values.

fuzzy membership (α)	conditions
0	if mean value of $x_{ij} \leq 0$
0.1	if $0 <$ mean value of $x_{ij} \leq 10$
0.2	if $10 <$ mean value of $x_{ij} \leq 20$
0.3	if $20 <$ mean value of $x_{ij} \leq 30$
0.4	if $30 <$ mean value of $x_{ij} \leq 40$
0.5	if $40 <$ mean value of $x_{ij} \leq 50$
0.6	if $50 <$ mean value of $x_{ij} \leq 60$
0.7	if $60 <$ mean value of $x_{ij} \leq 70$
0.8	if $70 <$ mean value of $x_{ij} \leq 80$
0.9	if $80 <$ mean value of $x_{ij} \leq 90$
1	if $90 <$ mean value of $x_{ij} \leq 100$

Table 4

Here, x_{ij} indicates the country which is in the i^{th} place with alphabetic order in the region U_j of "The Global Slavery Index 2016" [4], for $j \in \{1, 2, 3, 4, 5, 6\}$.

(i) **Asia Pacific**

We shall use following abbreviations:

HSC = Hong Kong, SAR China

PNG = Papua New Guinea

According to "The Global Slavery Index 2016"([4], p. 55); government responses of Afghanistan were unknown for f_1, f_2, f_3, f_4 and f_5, hence we shall consider 0 for Afghanistan. It is because of either government of Afghanistan was unable to take action against human trafficking in Afghanistan or government was not interested to take part in worldwide research survey related to human trafficking. Moreover, many internal causes may be present including terrorism threat, etc.

Similar representations will be considered for similar cases.

Name	e_1	e_2	e_3	e_4	Mean	FMV	f_1	f_2	f_3	f_4	f_5	Mean	FMV
Afghanistan*	83.00	47.39	53.31	84.55	67.06	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Australia	15.14	19.85	17.45	35.49	21.98	0.30	64.44	81.85	56.25	69.05	25.00	59.32	0.60
Bangladesh	46.78	46.04	33.63	50.02	44.12	0.50	39.44	60.37	68.75	59.52	0.00	45.62	0.50
Brunei	60.78	30.99	99.99	63.93	63.92	0.70	7.41	35.74	12.50	30.95	0.00	17.32	0.20
Cambodia	53.68	42.96	57.40	12.00	41.51	0.50	25.19	33.33	37.50	64.29	0.00	32.06	0.40
China	55.12	26.90	43.84	52.78	44.66	0.50	35.56	23.70	31.25	52.38	0.00	28.58	0.30
HSC	42.28	17.55	21.44	35.65	29.23	0.30	5.93	5.19	0.00	30.95	0.00	8.41	0.10
India	37.07	36.68	43.88	87.78	51.35	0.60	44.07	45.00	43.75	45.24	0.00	35.61	0.40
Indonesia	39.15	43.35	50.38	36.01	42.22	0.50	37.59	40.56	50.00	54.76	0.00	36.58	0.40
Japan	25.23	19.09	22.16	19.13	21.40	0.30	42.59	19.44	18.75	45.24	0.00	25.20	0.30
Laos	56.64	34.01	53.98	1.19	36.45	0.40	28.70	26.48	31.25	50.00	0.00	27.29	0.30
Malaysia	34.90	32.43	46.39	40.33	38.51	0.40	36.48	51.48	31.25	35.71	0.00	30.98	0.40
Mongolia	39.22	36.74	40.87	3.54	30.09	0.40	27.78	31.67	31.25	47.62	0.00	27.66	0.30
Myanmar	57.81	50.11	50.53	66.99	56.36	0.60	41.11	8.89	50.00	50.00	0.00	30.00	0.30
Nepal	42.30	43.22	34.74	41.21	40.37	0.50	42.78	38.15	75.00	61.90	0.00	43.57	0.50
New Zealand	13.31	22.24	16.09	21.51	18.29	0.20	53.70	47.96	43.75	88.10	0.00	46.70	0.50
North Korea	71.20	48.27	62.88	1.00	45.84	0.50	0.00	−13.89	0.00	−7.14	0.00	−4.21	0.00
Pakistan	58.40	41.98	52.70	96.79	62.47	0.70	28.52	37.04	25.00	76.19	0.00	33.35	0.40
PNG	50.12	62.85	99.99	23.10	59.01	0.60	6.48	23.70	25.00	14.29	0.00	13.89	0.20
Philippines	44.76	39.62	52.34	53.95	47.67	0.50	46.48	62.78	50.00	78.57	0.00	47.57	0.50
Singapore	29.85	22.11	20.58	1.00	18.38	0.20	36.11	22.41	0.00	42.86	0.00	20.28	0.30
South Korea	38.20	34.64	28.98	17.32	29.79	0.30	35.93	31.85	12.50	33.33	0.00	22.72	0.30
Sri Lanka	47.01	35.12	31.82	31.08	36.26	0.40	25.93	38.52	37.50	83.33	0.00	37.06	0.40
Taiwan	34.91	33.38	22.34	1.76	23.10	0.30	50.56	23.62	43.75	42.86	0.00	32.16	0.40
Thailand	49.23	28.62	48.97	63.33	47.54	0.50	35.19	35.93	56.25	61.90	0.00	37.85	0.40
Timor-Leste	38.88	48.07	68.55	1.00	39.13	0.40	25.93	25.93	0.00	40.48	0.00	18.47	0.20
Vietnam	51.19	29.94	35.22	1.00	29.34	0.30	45.19	34.07	62.50	66.67	0.00	41.69	0.50

Table 5

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(Asia)	0.9623	0.9428	0.8292	0.6131	0.3273	0	∞	∞	∞	∞

Table 6

(ii) **Russia and Eurasia**

Name	e_1	e_2	e_3	e_4	Mean	FMV	f_1	f_2	f_3	f_4	f_5	Mean	FMV
Armenia	53.66	27.38	42.63	24.90	37.14	0.40	48.33	49.81	56.25	50.00	0.00	40.88	0.50
Azerbaijan	68.90	28.23	28.17	23.05	37.09	0.40	28.33	60.37	18.75	57.14	0.00	32.92	0.40
Belarus	58.41	20.48	35.88	23.80	34.64	0.40	35.37	26.11	43.75	52.38	0.00	31.52	0.40
Georgia	51.00	28.43	35.95	24.42	34.95	0.40	67.59	58.70	56.25	59.52	0.00	48.41	0.50
Kazakhstan	59.05	22.08	28.19	22.12	32.86	0.40	38.33	44.07	18.75	35.71	0.00	27.37	0.30
Kyrgyzstan	54.16	28.33	36.37	21.88	35.18	0.40	27.96	35.74	37.50	50.00	0.00	30.24	0.40
Moldova	47.52	28.20	38.04	14.41	32.04	0.40	55.56	57.59	50.00	64.29	0.00	45.49	0.50
Russia	57.21	18.47	40.66	57.47	43.45	0.50	21.48	28.33	12.50	61.90	0.00	24.84	0.30
Tajikistan	62.85	37.62	41.68	27.53	42.42	0.50	41.99	34.44	25.00	54.76	0.00	31.24	0.40
Turkmenistan	68.14	28.65	43.34	9.22	37.34	0.40	14.81	35.74	12.50	54.76	0.00	23.56	0.30
Ukraine	61.97	21.39	35.80	43.41	40.64	0.50	62.04	47.78	12.50	61.90	0.00	36.84	0.40
Uzbekistan	74.62	28.35	32.09	12.14	36.80	0.40	24.26	23.70	18.75	52.38	0.00	23.82	0.30

Table 7

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(Russia and Eurasia)	1	1	1	0.8165	0	∞	∞	∞	∞	∞

Table 8

(iii) **The Middle East and North Africa**

Throughout this paper, we shall use "MENA" as an abbreviation to "The Middle East and North Africa"

Name	e_1	e_2	e_3	e_4	Mean	FMV	f_1	f_2	f_3	f_4	f_5	Mean	FMV
Algeria	51.03	28.10	47.02	51.46	44.40	0.50	28.52	24.07	25.00	42.86	0.00	24.09	0.30
Bahrain	54.41	33.14	41.45	31.26	40.06	0.50	36.67	36.67	25.00	35.71	0.00	26.81	0.30
Egypt	51.25	27.90	62.96	54.85	49.24	0.50	35.74	32.04	50.00	52.38	0.00	34.03	0.40
Iran	70.71	32.99	48.60	51.44	50.93	0.60	0.00	7.41	0.00	16.67	0.00	4.82	0.10
Iraq*	71.22	44.91	58.04	81.13	63.82	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Israel	33.66	23.67	38.28	51.85	36.87	0.40	49.81	47.96	50.00	61.90	0.00	41.93	0.50
Jordan	48.39	27.97	54.16	37.73	42.06	0.50	45.00	42.22	56.25	42.86	0.00	37.27	0.40
Kuwait	59.29	27.97	41.30	15.89	36.11	0.40	14.81	33.33	25.00	45.24	0.00	23.68	0.30
Lebanon	55.39	29.32	50.98	58.00	48.42	0.50	37.59	32.04	37.50	42.86	0.00	30.00	0.30
Libya*	77.85	22.99	81.44	53.21	58.87	0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Morocco	55.83	18.58	56.08	22.18	38.17	0.40	6.48	24.63	18.75	52.38	0.00	20.45	0.30
Oman	57.65	21.03	62.88	12.60	38.54	0.40	36.11	29.26	12.50	47.62	0.00	25.10	0.30
Qatar	48.79	15.26	50.50	12.30	31.71	0.40	52.41	44.26	25.00	35.71	0.00	31.48	0.40
Saudi Arabia	64.94	30.92	37.20	28.84	40.48	0.50	28.70	34.44	25.00	38.10	0.00	25.25	0.30
Syria*	95.67	35.93	60.97	72.98	66.39	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Tunisia	40.01	25.97	42.84	28.91	34.43	0.40	36.11	22.22	18.75	61.90	0.00	27.80	0.30
UAE	41.71	22.64	30.75	18.36	28.36	0.30	63.89	36.67	56.25	57.14	0.00	42.79	0.50
Yemen*	75.01	51.30	54.67	62.28	60.82	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Table 9

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(MENA)	0.8819	0.8498	0.8498	0.4339	0	∞	∞	∞	∞	∞

Table 10

(iv) **The Americas**

We shall use following abbreviations:

$DR = Dominican\ Republic$

$TT = Trinidad\ and\ Tobago$

Name	e_1	e_2	e_3	e_4	Mean	FMV	f_1	f_2	f_3	f_4	f_5	Mean	FMV
Argentina	35.63	18.50	34.04	25.36	28.38	0.30	49.26	59.81	87.50	78.57	0.00	55.03	0.60
Barbados	39.70	15.41	67.00	1.00	30.78	0.40	39.44	29.26	50.00	38.10	0.00	31.36	0.40
Bolivia	49.71	32.08	38.69	16.57	34.26	0.40	17.59	36.67	50.00	57.14	0.00	32.28	0.40
Brazil	37.98	20.46	45.88	30.74	33.77	0.40	45.56	34.44	87.50	78.57	37.50	56.71	0.60
Canada	17.59	23.64	15.48	36.11	23.21	0.30	57.41	68.52	62.50	64.29	0.00	50.54	0.60
Chile	31.23	19.73	34.26	31.37	29.15	0.30	44.63	52.59	50.00	69.05	0.00	43.25	0.50
Colombia	51.72	26.54	46.78	43.49	42.13	0.50	44.07	41.85	43.75	73.81	0.00	40.70	0.50
Costa Rica	36.35	24.40	33.82	23.97	29.63	0.30	56.48	45.56	37.50	71.43	0.00	42.19	0.50
Cuba	53.60	26.60	34.00	13.99	32.05	0.40	25.00	17.78	12.50	38.10	0.00	18.68	0.20
DR	47.97	33.84	46.38	24.33	38.13	0.40	49.26	59.07	62.50	64.29	0.00	47.02	0.50
Ecuador	43.69	29.23	37.74	29.98	35.16	0.40	37.22	62.22	25.00	71.43	0.00	39.17	0.40
El Salvador	49.88	29.58	53.68	8.78	35.48	0.40	32.96	28.89	50.00	57.14	0.00	33.80	0.40
Guatemala	48.75	34.25	57.12	20.33	40.11	0.50	37.59	51.48	56.25	64.29	0.00	41.92	0.50
Guyana	39.35	25.57	83.50	5.81	38.56	0.40	25.74	50.74	25.00	52.38	0.00	30.77	0.40
Haiti	57.68	56.20	58.13	2.60	43.65	0.50	33.52	37.22	50.00	28.57	0.00	29.86	0.30
Honduras	53.76	34.82	60.12	16.38	41.27	0.50	24.63	31.67	37.50	59.52	0.00	30.66	0.40
Jamaica	43.21	30.91	46.13	8.68	32.23	0.40	47.78	61.85	81.25	54.76	0.00	49.13	0.50
Mexico	43.03	30.36	52.84	61.85	47.02	0.50	45.00	63.15	50.00	73.81	0.00	46.39	0.50
Nicaragua	43.53	31.92	39.79	23.67	34.73	0.40	44.07	66.11	56.25	52.38	0.00	43.76	0.50
Panama	37.46	29.14	46.18	23.55	34.08	0.40	11.85	73.15	37.50	57.14	0.00	35.93	0.40
Paraguay	46.02	21.22	38.92	26.14	33.08	0.40	35.93	55.56	37.50	76.19	0.00	41.04	0.50
Peru	43.85	31.98	44.81	30.33	37.74	0.40	47.78	28.33	62.50	61.90	0.00	40.10	0.50
Suriname	36.66	12.35	67.00	38.67	38.67	0.40	20.56	19.44	31.25	40.48	0.00	22.35	0.30
TT	35.88	14.40	67.00	16.26	33.38	0.40	31.11	44.63	31.25	66.67	0.00	34.73	0.40
United States	20.42	23.51	20.96	45.10	27.50	0.30	96.30	79.63	68.75	78.57	75.00	79.65	0.80
Uruguay	26.65	20.69	28.36	13.89	22.40	0.30	40.56	43.70	43.75	78.57	0.00	41.32	0.50
Venezuela	58.82	27.75	55.42	35.94	44.48	0.50	25.00	40.93	12.50	40.48	0.00	23.78	0.30

Table 11

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(The Americas)	1	1	0.9813	0.7735	0.3162	∞	∞	∞	∞	∞

Table 12

(v) **Europe**

We shall use following abbreviations:

BH = *Bosnia and Herzegovina*

Name	e_1	e_2	e_3	e_4	Mean	FMV	f_1	f_2	f_3	f_4	f_5	Mean	FMV
Albania	52.88	29.58	42.36	18.25	35.77	0.40	73.70	46.30	43.75	61.90	0.00	45.13	0.50
Austria	21.47	18.14	14.05	33.14	21.70	0.30	57.96	59.07	68.75	76.19	0.00	52.39	0.60
Belgium	25.42	19.83	24.03	31.28	25.14	0.30	71.30	50.74	75.00	71.43	0.00	53.69	0.60
BH	58.88	25.21	32.99	26.43	35.88	0.40	57.41	43.70	31.25	69.05	0.00	40.28	0.50
Bulgaria	40.41	20.54	33.79	30.50	31.31	0.40	43.15	38.52	43.75	71.43	0.00	39.37	0.40
Croatia	36.51	25.29	36.88	16.54	28.80	0.30	69.63	70.19	56.25	76.19	0.00	54.45	0.60
Cyprus	27.60	23.25	28.95	30.90	27.67	0.30	54.26	74.44	18.75	61.90	0.00	41.87	0.50
Czech Republic	27.22	19.38	31.55	27.78	26.48	0.30	54.81	50.74	56.25	66.67	0.00	45.69	0.50
Denmark	15.68	18.53	11.06	23.92	17.30	0.20	59.81	68.52	50.00	69.05	0.00	49.48	0.50
Estonia	35.94	16.87	39.31	11.14	25.82	0.30	30.19	23.33	31.25	80.95	0.00	33.14	0.40
Finland	22.07	19.69	19.08	22.66	20.88	0.30	52.78	62.04	56.25	66.67	0.00	47.55	0.50
France	26.13	20.27	18.01	44.77	27.30	0.30	52.59	76.85	43.75	69.05	0.00	48.45	0.50
Germany	23.61	19.97	20.81	40.76	26.29	0.30	61.67	64.81	43.75	78.57	0.00	49.76	0.50
Greece	37.64	22.74	38.10	38.77	34.31	0.40	53.89	36.85	18.75	38.10	0.00	29.52	0.30
Hungary	23.66	20.69	35.56	23.30	25.80	0.30	59.81	41.85	68.75	76.19	0.00	49.32	0.50
Iceland	24.88	12.22	20.43	15.03	18.14	0.20	45.37	52.22	37.50	45.24	0.00	36.07	0.40
Ireland	19.07	22.62	20.21	33.13	23.76	0.30	69.63	71.30	18.75	61.90	0.00	44.32	0.50
Italy	36.39	21.50	33.62	38.56	32.52	0.40	42.59	65.93	37.50	69.05	0.00	43.01	0.50
Kosovo	55.11	39.25	42.13	15.94	38.11	0.40	48.15	59.92	43.75	47.62	0.00	39.89	0.40
Latvia	41.95	20.25	33.22	12.95	27.09	0.30	58.89	50.19	43.75	71.43	0.00	44.85	0.50
Lithuania	35.09	20.58	34.10	16.94	26.68	0.30	59.26	54.81	25.00	73.81	0.00	42.58	0.50
Luxembourg	22.99	18.64	9.76	49.43	25.20	0.30	33.15	31.67	68.75	26.19	0.00	31.95	0.40
Macedonia	49.81	24.38	44.06	24.11	35.59	0.40	70.37	60.19	62.50	42.86	0.00	47.18	0.50
Montenegro	41.68	22.15	38.08	23.45	31.34	0.40	69.63	60.56	56.25	59.52	0.00	49.19	0.50
Netherlands	17.60	17.86	21.64	28.58	21.42	0.30	74.63	79.07	87.50	99.99	0.00	68.24	0.70
Norway	17.88	19.90	14.85	34.90	21.88	0.30	65.93	82.41	37.50	69.05	0.00	50.98	0.60
Poland	34.76	19.50	29.07	23.33	26.66	0.30	46.11	47.96	68.75	76.19	0.00	47.80	0.50
Portugal	21.50	22.62	16.06	16.89	19.27	0.20	58.52	73.52	68.75	83.33	0.00	56.82	0.60
Romania	39.26	25.86	31.83	18.74	28.92	0.30	37.59	36.85	56.25	50.00	0.00	36.14	0.40
Serbia	47.80	21.75	30.24	27.05	31.71	0.40	61.67	76.67	31.25	54.76	0.00	44.87	0.50
Slovakia	32.47	20.94	31.60	16.42	25.36	0.30	57.96	42.41	31.25	61.90	0.00	38.70	0.40
Slovenia	21.82	22.08	28.50	13.72	21.53	0.30	52.04	51.30	56.25	76.19	0.00	47.16	0.50
Spain	25.14	23.19	18.65	29.67	24.16	0.30	77.04	64.07	50.00	73.81	0.00	52.98	0.60
Sweden	21.39	19.70	10.84	38.60	22.63	0.30	72.22	62.96	75.00	73.81	0.00	56.80	0.60
Switzerland	16.93	16.60	13.69	30.98	19.55	0.20	60.19	64.81	25.00	73.81	0.00	44.76	0.50
Turkey	45.47	29.38	44.11	57.55	44.13	0.50	57.41	47.41	37.50	52.38	0.00	38.94	0.40
United Kingdom	18.45	20.37	21.83	46.50	26.79	0.30	74.63	79.07	43.75	69.05	37.50	60.80	0.70

Table 13

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(Europe)	1	1	0.9444	0.4743	0	∞	∞	∞	∞	∞

Table 14

(vi) **Sub-Saharan Africa**

We shall use following abbreviations:

CAR = *Central African Republic*

DRC = *Democratic Republic of the Congo*

RC = *Republic of the Congo*

Name	e_1	e_2	e_3	e_4	Mean	FMV	f_1	f_2	f_3	f_4	f_5	Mean	FMV
Angola	56.32	49.54	45.95	25.02	44.21	0.50	20.37	20.37	31.25	28.57	0.00	20.11	0.30
Benin	46.95	36.52	39.70	14.90	34.52	0.40	38.70	20.56	56.25	66.67	0.00	36.44	0.40
Botswana	37.25	42.82	46.38	19.21	36.41	0.40	24.81	14.81	37.50	42.86	0.00	24.00	0.30
Burkina Faso	59.55	40.77	40.73	25.98	41.76	0.50	47.41	30.56	37.50	42.86	0.00	31.67	0.40
Burundi	64.08	52.22	51.40	37.17	51.22	0.60	29.63	14.81	18.75	38.10	0.00	20.26	0.30
Cameroon	61.31	43.37	47.95	52.51	51.28	0.60	30.37	33.89	37.50	47.62	0.00	29.88	0.30
Cape Verde	33.33	20.30	55.40	36.34	36.34	0.40	15.19	15.37	25.00	30.95	0.00	17.30	0.20
CAR	83.67	48.67	85.43	62.21	70.00	0.70	14.81	17.22	12.50	7.14	0.00	10.33	0.20
Chad	70.47	49.96	47.98	40.94	52.34	0.60	26.85	12.04	31.25	52.38	0.00	24.50	0.30
DRC	78.42	56.33	56.72	82.43	68.47	0.70	7.78	11.67	31.25	26.19	0.00	15.38	0.20
Djibouti	49.13	43.61	55.25	32.42	45.10	0.50	25.00	28.89	37.50	59.52	0.00	30.18	0.40
Equatorial Guinea	56.66	40.62	46.38	1.00	36.16	0.40	0.00	18.52	0.00	23.81	0.00	8.47	0.10
Eritrea	59.44	51.05	62.88	24.82	49.55	0.50	0.00	2.96	0.00	26.19	0.00	5.83	0.10
Ethiopia	59.75	54.68	34.16	59.77	52.09	0.60	21.30	33.33	62.50	52.38	0.00	33.90	0.40
Gabon	51.58	31.51	42.38	16.97	35.61	0.40	30.00	24.26	25.00	45.24	0.00	24.90	0.30
Gambia	59.30	29.27	74.18	22.63	46.34	0.50	22.59	35.19	37.50	45.24	0.00	28.10	0.30
Ghana	51.89	38.42	47.45	28.26	41.50	0.50	22.04	30.19	25.00	45.24	0.00	24.49	0.30
Guinea	66.89	41.58	52.68	28.67	47.45	0.50	2.78	12.96	31.25	28.57	0.00	15.11	0.20
Guinea-Bissau	62.08	40.50	70.25	22.47	48.82	0.50	14.81	32.04	25.00	21.43	0.00	18.66	0.20
Ivory Coast	62.07	38.72	46.22	33.45	45.11	0.50	37.96	30.19	50.00	30.95	0.00	29.82	0.30
Kenya	54.53	52.84	46.75	72.28	56.60	0.60	21.85	27.41	6.25	42.86	0.00	19.67	0.20
Lesotho	40.33	52.26	68.23	9.58	42.60	0.50	31.48	25.74	50.00	59.52	0.00	33.35	0.40
Liberia	57.93	44.62	44.45	29.43	44.11	0.50	27.22	27.41	31.25	50.00	0.00	27.18	0.30
Madagascar	50.37	50.87	52.86	15.99	42.52	0.50	31.30	14.44	31.25	42.86	0.00	23.97	0.30
Malawi	54.63	56.74	47.78	21.06	45.05	0.50	32.22	21.11	12.50	38.10	0.00	20.79	0.30
Mali	64.04	34.08	31.69	57.41	46.80	0.50	15.19	12.04	43.75	33.33	0.00	20.86	0.30
Mauritania	65.96	40.54	49.85	30.74	46.77	0.50	25.00	32.41	12.50	40.48	0.00	22.08	0.30
Mauritius	29.24	24.49	26.58	1.00	20.33	0.30	34.44	27.41	18.75	45.24	0.00	25.17	0.30
Mozambique	39.91	48.46	54.40	35.86	44.66	0.50	53.89	50.56	12.50	47.62	0.00	32.91	0.40
Namibia	39.00	43.68	51.42	18.27	38.09	0.40	28.15	22.04	31.25	52.38	0.00	26.76	0.30
Niger	57.70	48.17	42.13	40.38	47.09	0.50	12.41	29.26	25.00	40.48	0.00	21.43	0.30
Nigeria	60.94	47.84	59.76	80.84	62.34	0.70	50.74	59.63	50.00	45.24	0.00	41.12	0.50
RC	65.17	44.69	52.43	28.94	47.81	0.50	22.22	8.89	37.50	35.71	0.00	20.86	0.30
Rwanda	55.44	47.65	46.23	42.00	47.83	0.50	30.93	45.19	31.25	54.76	0.00	32.43	0.40
Senegal	44.97	42.31	36.96	35.48	39.93	0.40	49.63	32.59	25.00	54.76	0.00	32.40	0.40
Sierra Leone	50.57	53.29	41.72	17.70	40.82	0.50	44.44	45.56	43.75	54.76	0.00	37.70	0.40
Somalia	73.03	64.82	55.97	74.46	67.07	0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00
South Africa	40.27	43.06	58.30	41.84	45.87	0.50	38.89	55.74	31.25	64.29	0.00	38.03	0.40
South Sudan	74.73	50.70	60.80	76.15	65.59	0.70	20.37	1.48	18.75	28.57	0.00	13.83	0.20
Sudan	80.64	54.12	46.18	85.04	66.50	0.70	24.07	27.41	6.25	33.33	0.00	18.21	0.20
Swaziland	57.69	53.88	67.33	15.39	48.57	0.50	36.30	22.04	37.50	42.86	0.00	27.74	0.30
Tanzania	51.66	54.67	47.66	40.46	48.61	0.50	27.04	26.48	25.00	47.62	0.00	25.23	0.30
Togo	64.78	39.82	47.78	24.09	44.12	0.50	26.48	4.63	31.25	19.05	0.00	16.28	0.20
Uganda	54.89	52.18	39.45	48.73	48.81	0.50	50.93	50.19	37.50	59.52	0.00	39.63	0.40
Zambia	45.10	58.76	50.03	24.46	44.59	0.50	33.89	29.81	43.75	38.10	0.00	29.11	0.30
Zimbabwe	60.28	52.25	48.92	26.78	47.06	0.50	15.37	20.56	12.50	42.86	0.00	18.26	0.20

Table 15

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(Sub-Saharan Africa)	0.9891	0.9668	0.8470	0.5164	0.1622	∞	∞	∞	∞	∞

Table 16

6. Analysis

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC(Asia Pacific)	0.9623	0.9428	0.8292	0.6131	0.3273	0	∞	∞	∞	∞
FSACC(Russia and Eurasia)	1	1	1	0.8165	0	∞	∞	∞	∞	∞
FSACC(MENA)	0.8819	0.8498	0.8498	0.4339	0	∞	∞	∞	∞	∞
FSACC(The Americas)	1	1	0.9813	0.7735	0.3162	∞	∞	∞	∞	∞
FSACC(Europe)	1	1	0.9444	0.4743	0	∞	∞	∞	∞	∞
FSACC(Sub-Saharan Africa)	0.9891	0.9668	0.8470	0.5164	0.1622	∞	∞	∞	∞	∞

Table 17

Thus, The Americas and Europe have the highest correlation levels in this case.

7. Complement of vulnerability and government responses

(i) Asia Pacific

Complement of vulnerability and government responses of the region "Asia Pacific" is shown in Table 18.

Name	Mean of vulnerability	100 - Mean of vulnerability	FMV	Mean of govt. responses	FMV
Afghanistan*	67.06	32.94	0.40	0.00	0.00
Australia	21.98	78.02	0.80	59.32	0.60
Bangladesh	44.12	55.88	0.60	45.62	0.50
Brunei	63.92	36.08	0.40	17.32	0.20
Cambodia	41.51	58.49	0.60	32.06	0.40
China	44.66	55.34	0.60	28.58	0.30
HSC	29.23	70.77	0.80	8.41	0.10
India	51.35	48.65	0.50	35.61	0.40
Indonesia	42.22	57.78	0.60	36.58	0.40
Japan	21.40	78.60	0.80	25.20	0.30
Laos	36.45	63.55	0.70	27.29	0.30
Malaysia	38.51	61.49	0.70	30.98	0.40
Mongolia	30.09	69.91	0.70	27.66	0.30
Myanmar	56.36	43.64	0.50	30.00	0.30
Nepal	40.37	59.63	0.60	43.57	0.50
New Zealand	18.29	81.71	0.90	46.70	0.50
North Korea	45.84	54.16	0.60	-4.21	0.00
Pakistan	62.47	37.53	0.40	33.35	0.40
PNG	59.01	40.99	0.50	13.89	0.20
Philippines	47.67	52.33	0.60	47.57	0.50
Singapore	18.38	81.62	0.90	20.28	0.30
South Korea	29.79	70.22	0.80	22.72	0.30
Sri Lanka	36.26	63.74	0.70	37.06	0.40
Taiwan	23.10	76.90	0.80	32.16	0.40
Thailand	47.54	52.46	0.60	37.85	0.40
Timor-Leste	39.13	60.88	0.70	18.47	0.20
Vietnam	29.34	70.66	0.80	41.69	0.50

Table 18

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC^C(Asia Pacific)	0.9623	0.9428	0.8819	0.7201	0.5000	0.2182	∞	∞	∞	∞

Table 19

Name	Mean of vulnerability	100 - Mean of vulnerability	FMV	Mean of govt. responses	FMV
heightArmenia	37.14	62.86	0.70	40.88	0.50
Azerbaijan	37.09	62.91	0.70	32.92	0.40
Belarus	34.64	65.36	0.70	31.52	0.40
Georgia	34.95	65.05	0.70	48.41	0.50
Kazakhstan	32.86	67.14	0.70	27.37	0.30
Kyrgyzstan	35.18	64.81	0.70	30.24	0.40
Moldova	32.04	67.96	0.70	45.49	0.50
Russia	43.45	56.55	0.60	24.84	0.30
Tajikistan	42.42	57.58	0.60	31.24	0.40
Turkmenistan	37.34	62.66	0.70	23.56	0.30
Ukraine	40.64	59.36	0.60	36.84	0.40
Uzbekistan	36.80	63.20	0.70	23.82	0.30

Table 20

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC^C(Russia and Eurasia)	1	1	1	0.8165	0.5000	∞	∞	∞	∞	∞

Table 21

(iii) **The Middle East and North Africa**

Name	Mean of vulnerability	100 - Mean of vulnerability	FMV	Mean of govt. responses	FMV
Algeria	44.40	55.60	0.60	24.09	0.30
Bahrain	40.06	59.94	0.60	26.81	0.30
Egypt	49.24	50.76	0.60	34.03	0.40
Iran	50.93	49.07	0.50	4.82	0.10
Iraq*	63.82	36.18	0.40	0.00	0.00
Israel	36.87	63.13	0.70	41.93	0.50
Jordan	42.06	57.94	0.60	37.27	0.40
Kuwait	36.11	63.89	0.70	23.68	0.30
Lebanon	48.42	51.58	0.60	30.00	0.30
Libya*	58.87	41.13	0.50	0.00	0.00
Morocco	38.17	61.83	0.70	20.45	0.30
Oman	38.54	61.46	0.70	25.10	0.30
Qatar	31.71	68.29	0.70	31.48	0.40
Saudi Arabia	40.48	59.52	0.60	25.25	0.30
Syria*	66.39	33.61	0.40	0.00	0.00
Tunisia	34.43	65.57	0.70	27.80	0.30
UAE	28.36	71.64	0.80	42.79	0.50
Yemen*	60.82	39.18	0.40	0.00	0.00

Table 22

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC^C(MENA)	0.8819	0.8498	0.8498	0.5270	0.3651	∞	∞	∞	∞	∞

Table 23

(iv) **The Americas**

We shall use following abbreviations:

DR = Dominican Republic

TT = Trinidad and Tobago

Name	Mean of vulnerability	100 - Mean of vulnerability	FMV	Mean of govt. responses	FMV
heightArgentina	28.38	71.62	0.80	55.03	0.60
Barbados	30.78	69.22	0.70	31.36	0.40
Bolivia	34.26	65.74	0.70	32.28	0.40
Brazil	33.77	66.23	0.70	56.71	0.60
Canada	23.21	76.80	0.80	50.54	0.60
Chile	29.15	70.85	0.80	43.25	0.50
Colombia	42.13	57.87	0.60	40.70	0.50
Costa Rica	29.63	70.37	0.80	42.19	0.50
Cuba	32.05	67.95	0.70	18.68	0.20
DR	38.13	61.87	0.70	47.02	0.50
Ecuador	35.16	64.84	0.70	39.17	0.40
El Salvador	35.48	64.52	0.70	33.80	0.40
Guatemala	40.11	59.89	0.60	41.92	0.50
Guyana	38.56	61.44	0.70	30.77	0.40
Haiti	43.65	56.35	0.60	29.86	0.30
Honduras	41.27	58.73	0.60	30.66	0.40
Jamaica	32.23	67.77	0.70	49.13	0.50
Mexico	47.02	52.98	0.60	46.39	0.50
Nicaragua	34.73	65.27	0.70	43.76	0.50
Panama	34.08	65.92	0.70	35.93	0.40
Paraguay	33.08	66.92	0.70	41.04	0.50
Peru	37.74	62.26	0.70	40.10	0.50
Suriname	38.67	61.33	0.70	22.35	0.30
TT	33.38	66.62	0.70	34.73	0.40
United States	27.50	72.50	0.80	79.65	0.80
Uruguay	22.40	77.60	0.80	41.32	0.50
Venezuela	44.48	55.52	0.60	23.78	0.30

Table 24

(v) **Europe**

We shall use following abbreviations:

BH = *Bosnia and Herzegovina*

Name	Mean of vulnerability	100 - Mean of vulnerability	FMV	Mean of govt. responses	FMV
Albania	35.77	64.23	0.70	45.13	0.50
Austria	21.70	78.30	0.80	52.39	0.60
Belgium	25.14	74.86	0.80	53.69	0.60
BH	35.88	64.12	0.70	40.28	0.50
Bulgaria	31.31	68.69	0.70	39.37	0.40
Croatia	28.80	71.19	0.80	54.45	0.60
Cyprus	27.67	72.33	0.80	41.87	0.50
Czech Republic	26.48	73.52	0.80	45.69	0.50
Denmark	17.30	82.70	0.90	49.48	0.50
Estonia	25.82	74.19	0.80	33.14	0.40
Finland	20.88	79.13	0.80	47.55	0.50
France	27.30	72.70	0.80	48.45	0.50
Germany	26.29	73.71	0.80	49.76	0.50
Greece	34.31	65.69	0.70	29.52	0.30
Hungary	25.80	74.20	0.80	49.32	0.50
Iceland	18.14	81.86	0.90	36.07	0.40
Ireland	23.76	76.24	0.80	44.32	0.50
Italy	32.52	67.48	0.70	43.01	0.50
Kosovo	38.11	61.89	0.70	39.89	0.40
Latvia	27.09	72.91	0.80	44.85	0.50
Lithuania	26.68	73.32	0.80	42.58	0.50
Luxembourg	25.20	74.80	0.80	31.95	0.40
Macedonia	35.59	64.41	0.70	47.18	0.50
Montenegro	31.34	68.66	0.70	49.19	0.50
Netherlands	21.42	78.58	0.80	68.24	0.70
Norway	21.88	78.12	0.80	50.98	0.60
Poland	26.66	73.34	0.80	47.80	0.50
Portugal	19.27	80.73	0.90	56.82	0.60
Romania	28.92	71.08	0.80	36.14	0.40
Serbia	31.71	68.29	0.70	44.87	0.50
Slovakia	25.36	74.64	0.80	38.70	0.40
Slovenia	21.53	78.47	0.80	47.16	0.50
Spain	24.16	75.84	0.80	52.98	0.60
Sweden	22.63	77.37	0.80	56.80	0.60
Switzerland	19.55	80.45	0.90	44.76	0.50
Turkey	44.13	55.87	0.60	38.94	0.40
United Kingdom	26.79	73.21	0.80	60.80	0.70

Table 26

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
FSACC^C(Europe)	1	1	1	0.9864	0.8699	0.4932	0.2357	∞	∞	∞

Table 27

(vi) **Sub-Saharan Africa**

We shall use following abbreviations:

CAR = *Central African Republic*

DRC = *Democratic Republic of the Congo*

RC = *Republic of the Congo*

Name	Mean of vulnerability	100 - Mean of vulnerability	FMV	Mean of govt. responses	FMV
Angola	44.21	55.79	0.60	20.11	0.30
Benin	34.52	65.48	0.70	36.44	0.40
Botswana	36.41	63.59	0.70	24.00	0.30
Burkina Faso	41.76	58.24	0.60	31.67	0.40
Burundi	51.22	48.78	0.50	20.26	0.30
Cameroon	51.28	48.72	0.50	29.88	0.30
Cape Verde	36.34	63.66	0.70	17.30	0.20
CAR	70.00	30.00	0.40	10.33	0.20
Chad	52.34	47.66	0.50	24.50	0.30
DRC	68.47	31.53	0.40	15.38	0.20
Djibouti	45.10	54.90	0.60	30.18	0.40
Equatorial Guinea	36.16	63.84	0.70	8.47	0.10
Eritrea	49.55	50.45	0.60	5.83	0.10
Ethiopia	52.09	47.91	0.50	33.90	0.40
Gabon	35.61	64.39	0.70	24.90	0.30
Gambia	46.34	53.66	0.60	28.10	0.30
Ghana	41.50	58.50	0.60	24.49	0.30
Guinea	47.45	52.55	0.60	15.11	0.20
Guinea-Bissau	48.82	51.18	0.60	18.66	0.20
Ivory Coast	45.11	54.89	0.60	29.82	0.30
Kenya	56.60	43.40	0.50	19.67	0.20
Lesotho	42.60	57.40	0.60	33.35	0.40
Liberia	44.11	55.89	0.60	27.18	0.30
Madagascar	42.52	57.48	0.60	23.97	0.30
Malawi	45.05	54.95	0.60	20.79	0.30
Mali	46.80	53.20	0.60	20.86	0.30
Mauritania	46.77	53.23	0.60	22.08	0.30
Mauritius	20.33	79.67	0.80	25.17	0.30
Mozambique	44.66	55.34	0.60	32.91	0.40
Namibia	38.09	61.91	0.70	26.76	0.30
Niger	47.09	52.91	0.60	21.43	0.30
Nigeria	62.34	37.66	0.40	41.12	0.50
RC	47.81	52.19	0.60	20.86	0.30
Rwanda	47.83	52.17	0.60	32.43	0.40
Senegal	39.93	60.07	0.70	32.40	0.40
Sierra Leone	40.82	59.18	0.60	37.70	0.40
Somalia	67.07	32.93	0.40	0.00	0.00
South Africa	45.87	54.13	0.60	38.03	0.40
South Sudan	65.59	34.41	0.40	13.83	0.20
Sudan	66.50	33.50	0.40	18.21	0.20
Swaziland	48.57	51.43	0.60	27.74	0.30
Tanzania	48.61	51.39	0.60	25.23	0.30
Togo	44.12	55.88	0.60	16.28	0.20
Uganda	48.81	51.19	0.60	39.63	0.40
Zambia	44.59	55.41	0.60	29.11	0.30
Zimbabwe	47.06	52.94	0.60	18.26	0.20

Table 28

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$FSACC^C$(Sub-Saharan Africa)	0.9891	0.9668	0.8470	0.5108	0	∞	∞	∞	∞	∞

Table 29

8. Analysis

MENA = Middle East and North Africa

α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$FSACC^C$(Asia Pacific)	0.9623	0.9428	0.8819	0.7201	0.5000	0.2182	∞	∞	∞	∞
$FSACC^C$(Russia and Eurasia)	1	1	1	0.8165	0.5000	∞	∞	∞	∞	∞
$FSACC^C$(MENA)	0.8819	0.8498	0.8498	0.5270	0.3651	∞	∞	∞	∞	∞
$FSACC^C$(The Americas)	1	1	0.9813	0.9230	0.7454	0.3849	0.2182	0.4082	∞	∞
$FSACC^C$(Europe)	1	1	1	0.9864	0.8699	0.4932	0.2357	∞	∞	∞
$FSACC^C$(Sub-Saharan Africa)	0.9891	0.9668	0.8470	0.5108	0	∞	∞	∞	∞	∞

Table 30

Thus, The Americas and Europe have the highest correlation levels in this case.

9. Sociological views on vulnerability and government responses to human trafficking

Human slavery is probably present in all nations, but to highly varying degrees. Between-nation trafficking networks involve almost all nations as either sources or destinations, but to highly varying degrees. The United Nations report focuses on factors that make some nations more vulnerable as sources of slaves, and describes variations in national public-policy responses to inter-national trafficking across nations. Both concepts - vulnerability and government response - are complex. Whether a nation is seen as vulnerable or not, and whether that nation's governmental response is seen as extensive or not, are uncertain due to definitional complexity. Assessing the general magnitude of the association between vulnerability and governmental response, measured across nations, is therefore also made uncertain because of the complexity of the concepts.

Many of the conditions that are identified as indications of vulnerability to human trafficking (weak civil and political protections; limited social, health, and economic rights; personal insecurity; and high levels of refugees and conflict), may both produce a need for governmental response and, at the same time, limit the capacity of government to respond. Governmental responses may be of multiple sorts, including survivor support, criminal justice, coordination and accountability, and risk assessment. Overall, across all nations, and by most measures, there is a negative association between vulnerability and response. That is, nations that display greater vulnerability to involvement in international human trafficking also display weaker public policy responses. The correlation, however, is far from perfect, and may vary across regions.

Because of geographical, cultural, and other factors, the strength of the association may differ from region to region. In some regions, certain combinations of factors producing vulnerability may be more common than in other regions; yet mean vulnerabilities may be similar. In some regions patterns of governmental responses may differ from the patterns in other regions; yet the mean responsiveness might be regarded as similar. To assess whether the association between the two concepts is equally strong across regions, methods are needed to deal with definitional uncertainty. We shall assume that other sources of uncertainty sampling and instrumentation are absent.

10. Fuzzy soft set analysis compared to "conventional" analysis in sociology

"The Global Slavery Index 2016" [4] data on four parameters of vulnerability and five parameters of government response for 167 nations, are divided into six regions. The five government responsiveness parameters (f) are measured quantitatively and is scaled such that high scores indicate greater responsiveness. The four vulnerability parameters (e) are measured quantitatively as scores on factors from 24 indicators, and is scaled such that high scores indicate greater vulnerability. One seeks to examine the distribution of case's vulnerabilities and responsiveness, as well as the association between these two concepts across the universe of nations (and within regions). Table 31 shows the raw data for the Asia region. We see, for examples, that Australia ranks high on responsiveness, but low on vulnerability; Afghanistan displays high levels of vulnerability and low responsiveness

Name	f_1	f_2	f_3	f_4	f_5	e_1	e_2	e_3	e_4
Afghanistan	0	0	0	0	0	83	47.39	53.31	84.55
Australia	64.44	81.85	56.25	69.05	25	15.14	19.85	17.45	35.49
Bangladesh	39.44	60.37	68.75	59.52	0	46.78	46.04	33.63	50.02
Brunei	7.41	35.74	12.5	30.95	0	60.78	30.99	99.99	63.93
Cambodia	25.19	33.33	37.5	64.29	0	53.68	42.96	57.4	12
China	35.56	23.7	31.25	52.38	0	55.12	26.9	43.84	52.78
HSC	5.93	5.19	0	30.95	0	42.28	17.55	21.44	35.65
India	44.07	45	43.75	45.24	0	37.07	36.68	43.88	87.78
Indonesia	37.59	40.56	50	54.76	0	39.15	43.35	50.38	36.01
Japan	42.59	19.44	18.75	45.24	0	25.23	19.09	22.16	19.13
Laos	28.7	26.48	31.25	50	0	56.64	34.01	53.98	1.19
Malaysia	36.48	51.48	31.25	35.71	0	34.9	32.43	46.39	40.33
Mongolia	27.78	31.67	31.25	47.62	0	39.22	36.74	40.87	3.54
Myanmar	41.11	8.89	50	50	0	57.81	50.11	50.53	66.99
Nepal	42.78	38.15	75	61.9	0	42.3	43.22	34.74	41.21
New Zealand	53.7	47.96	43.75	88.1	0	13.31	22.24	16.09	21.51
North Korea	0	13.89	0	-7.14	0	71.2	48.27	62.88	1
Pakistan	28.52	37.04	25	76.19	0	58.4	41.98	52.7	96.79
PNG	6.48	23.7	25	14.29	0	50.12	62.85	99.99	23.1
Philippines	46.48	62.78	50	78.57	0	44.76	39.62	52.34	53.95
Singapore	36.11	22.41	0	42.86	0	29.85	22.11	20.58	1
South Korea	35.93	31.85	12.5	33.33	0	38.2	34.64	28.98	17.32
Sri Lanka	25.93	38.52	37.5	83.33	0	47.01	35.12	31.82	31.08
Taiwan	50.56	23.62	43.75	42.86	0	34.91	33.38	22.34	1.76
Thailand	35.19	35.93	56.25	61.9	0	49.23	28.62	48.97	63.33
Timor-Leste	25.93	25.93	0	40.48	0	38.88	48.07	68.55	1
Vietnam	45.19	34.07	62.5	66.67	0	51.19	29.94	35.22	1

Table 31

Table 32 shows Pearson's zero-order product-moment correlations.

	f_1	f_2	f_3	f_4	f_5	e_1	e_2	e_3	e_4
f_1	1.00								
f_2	0.68	1.00							
f_3	0.68	0.63	1.00						
f_4	0.71	0.69	0.66	1.00					
f_5	0.39	0.50	0.21	0.18	1.00				
e_1	-0.72	-0.57	-0.25	-0.47	-0.39	1.00			
e_2	-0.39	-0.21	0.04	-0.32	-0.30	0.55	1.00		
e_3	-0.62	-0.21	-0.24	-0.41	-0.26	0.59	0.65	1.00	
e_4	-0.05	0.16	0.16	0.08	0.00	0.32	0.13	0.19	1.00

Table 32

The associations among government responsiveness indicators (f) are positive, substantial, and similar to one another. The associations among the indictors of vulnerability (e) also tend to be positive, but display a great deal of variation. Nations that are high on one aspect of vulnerability are not necessarily high on others. The associations between the set of responsiveness indicators and the set of vulnerability indicators are generally negative, consistent with the notion that many of the same factors that produce vulnerability to human trafficking, may limit governmental responses to it. The associations are far from homogeneous.

Overall, the pattern of associations, as measured by traditional "hard" statistics, suggest that, if we were to divide nations in to "high-risk" versus "low-risk" for trafficking, we would get different answers depending on which parameter we used. The same is true for classifying national government responses. It follows that the strength of the association between the two concepts depends considerably on which parameters are examined.

A conventional approach to summarizing the overall pattern of association between the two sets of indicators is to identify components or factors in each indicator, and to examine the correlation between the factors. For the 27 nations in the Asia region, the 1^{st} canonical correlation is 0.85, suggesting a very strong association between the underling concepts of vulnerability and governmental response.

How certain should we be, on the basis of such analysis that there really is a strong negative association between vulnerability and governmental responsiveness to human trafficking across the Asian nations? There are many heroic assumptions built into the conventional "hard" statistical approach.

References

[1] S. Acharjee and J. N. Mordeson, Soft statistics with respect to utility and application to human trafficking, New Math. Nat. Comput. **13(03)**, pp. 289-310, (2017).

[2] N. Çağman, S. Karataş and S. Enginoglu, Soft topology, Comput. Math. Appl., **62**, pp. 351-358, (2011).

[3] D. Chen, The parametrization reduction of soft sets and its application, Comput. Math. Appl., **49**, pp. 757-763, (2005).

[4] http://www.globalslaveryindex.org/, The Global Slavery index, (2016).

[5] http://www.unodc.org/undoc/en/humantrafficking/whatishumantrafficking, United Nations Office on Drug and Crime

[6] Y. Leung, A fuzzy set analysis of sociometric structure, Jour. Math. Socio. **7**, pp. 159-180, (1980).

[7] H. Li and Y. Shen, Similarity measures of fuzzy soft sets based on different distances, Fifth Int. Symp. Comput. Intel. and Design, pp. 527- 529, (2012).

[8] S. Li, Measuring the fuzziness of human thoughts: an application of fuzzy sets to sociological research, Jour. Math. Socio. **14(1)**, pp. 67-84, (1989).

[9] P. K. Maji, R. Biswas and A. R. Roy, Fuzzy soft sets, Jour. Fuzzy Math. **9(3)**, pp. 589-602, (2001).

[10] P. K. Maji, R. Biswas and A. R. Roy, Soft set theory, Comput. Math. Appl. **45**, pp. 555-562, (2003).

[11] P. Majumdar and S. K. Samanta, On similarity measures of fuzzy soft sets, Int. Jour. Adv. Soft. Comput. Appl. **3(2)**, 8pages, (2011).

[12] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. **37**, pp. 19-31,(1999).

[13] J. N. Mordeson, M. Mallenby, S. Mathew and S. Acharjee, Human trafficking: Policy intervention, New Math. Nat. Comput. **13(03)**, pp. 341-358, (2017).

[14] D. Pei and D. Miao, From soft sets to information systems, Proceedings of the IEEE international conference on granular computing, **2**, pp. 617-621, (2005).

[15] Trafficking in Persons: Global Patterns, United Nations Office on Drugs and Crime, Trafficking in Persons Citation Index.

[16] Z. Xiao and Y. Zou, A comparative study of soft sets with fuzzy sets and rough sets, Jour. Intel. Fuzzy Syst. **27**, pp. 425-434, (2014).

[17] L. A. Zadeh, Fuzzy sets, Inform. Control. **8(3)**, pp. 338-353, (1965).

Santanu Acharjee

Economics and Computational Rationality Group

Department of Mathematics

Debraj Roy College

Golaghat-785621, Assam

India

e-mail : sacharjee326@gmail.com

Diganta Jyoti Sarma

Department of Mathematics

Central Institute of Technology

BTAD, Kokrajhar-783370, Assam

India

e-mail : dj.sarma@cit.ac.in

Robert A. Hanneman
Department of Sociology
University of California
Riverside California-92521
U. S. A.
e-mail : hanneman@ucr.edu

John N. Mordeson
Center for Mathematics of Uncertainty
Department of Mathematics
Creighton University
Omaha, Nebraska,
U. S. A.
e-mail : JohnMordeson@creighton.edu

and

Davender S. Malik
Center for Mathematics of Uncertainty
Department of Mathematics
Creighton University
Omaha, Nebraska
U. S. A.
e-mail : DavenderMalik@creighton.edu