Proyecciones Journal of Mathematics
Vol. 37, N^o 3, pp. 583-591, September 2018.
Universidad Católica del Norte
Antofagasta - Chile

A new type of difference class of interval numbers

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Received : January 2018. Accepted : January 2018

Abstract

In this article we introduce the notation difference operator Δ_m *($m \geq 0$ be an integer) for studying some properties defined with interval numbers. We introduced the classes of sequence* $\bar{\ell}(p)(\Delta_m), \bar{c}(p)(\Delta_m)$ *and* $\bar{c}_0(p)(\Delta_m)$ *and investigate different algebraic properties like completeness, solidness, convergence free etc.*

Key Words : *Interval number, Completeness, Solid, Convergence free.*

AMS Subject Classification : *40C05, 40J05, 46A45.*

1. Introduction

The concept of interval arithmetic was first suggested by Dwyer [15] in 1951. Thereafter the concept has been using in area of science and technology. The evidence of its development as a formal system and application in computational device is found in Moore [8], Moore and Yang [9] and others ([15], [16], [17] and [20]). Different mathematical concepts were introduced and studied with interval numbers by several researchers across the globe. Chiao [13] introduced sequence of interval numbers and defined usual convergence of sequences of interval number. Sengnl and Eryilmaz [14] introduced and studied bounded and convergent sequence spaces of interval numbers and proved that these spaces are complete metric space. Recently Esi [1-8], Esi and Braha [18], Esi and Esi [19], Esi and Hazarika [20] and Esi and Catalbas [21] introduced and studied strongly almost-convergence and statistically almost-convergence of interval numbers.

A set consisting of a closed interval of real numbers x such that $a \leq x \leq b$ is called an interval number. A real interval can also be considered as a set. We can investigate some properties of interval numbers, for instance arithmetic properties or analysis properties. We denote the set of all real valued closed intervals by R. Any elements of R is called closed interval and denoted by $\bar{x}$, that is $\bar{x} = \{x \in R : a \leq x \leq b\}$. An interval number $\bar{x}$ is a closed subset of real numbers [15]. Let x_l and x_r be first and last points of interval number $\bar{x}$, respectively then we have for $x_1, x_2 \in R$,

i) $\bar{x_1} = \bar{x_2} \Leftrightarrow x_{1_\ell} = x_{2_\ell}, x_{1_r} = x_{2_r}$

ii) $\bar{x_1} + \bar{x_2} = \{x \in R : x_{1_\ell} + x_{2_\ell} \leq x \leq x_{1_r} + x_{2_r}\}$

iii) $\alpha\bar{x} = \{x \in R : \alpha x_{1_\ell} \leq x \leq \alpha x_{1_r}\}$, for $\alpha \geq 0$.and $\alpha x = \{x \in R : \alpha x_{1_r} \leq x \leq \alpha x_{1_l}\}$, for $\alpha < 0$.

iv) $$\bar{x_1}\bar{x_2} = \{x \in R : min\{x_{1_\ell}x_{2_\ell}, x_{1_\ell}x_{2_r}, x_{1_r}x_{2_l}, x_{1_r}x_{2_r}\} \leq x \leq max\{x_{1_\ell}x_{2_\ell}, x_{1_\ell}x_{2_r}, x_{1_r}x_{2_\ell}, x_{1_r}x_{2_r}\}$$

The set of all interval numbers R is a complete metric space defined by

$$d(\bar{x_1}, \bar{x_2}) = max\{|x_{1_\ell} - x_{2_\ell}|, |x_{1_r} - x_{2_r}|\}$$

.

In the special case, $\bar{x}_1 = [a, a]$ and $\bar{x}_2 = [b, b]$, we obtain usual metric of R.

Consider the transformation $f : N \to R$, by $k \to f(k) = \bar{x}, x = (x_k)$, then $\bar{x} = (\bar{x}_k)$ is called sequence of interval numbers. The term $\bar{x}_k$ is called the kth term of sequence $(\bar{x}) = (\bar{x}_k)$.

By w^i we denotes the set of all interval numbers with real terms. We give the following definitions of convergence of interval numbers.

A sequence $\bar{x} = (\bar{x}_k)$ of interval numbers is said to be convergent to the interval number $\bar{x}_0$ if for each $\varepsilon > 0$ there exists a positive integer k_0 such that $d(\bar{x}_k, \bar{x}_0) < \varepsilon$ for all $k \geq k_0$, denoted by $\lim_k \bar{x}_k = \bar{x}_0$. This imply that

$$lim_k \bar{x}_k = \bar{x}_0 \Leftrightarrow lim_k x_{k_\ell} = x_{0_\ell} \text{ and } \lim_k x_{k_r} = x_{0_r}.$$

An interval valued sequence space $\bar{E}$ is said to be solid if $\bar{y} = (\bar{y}_k) \in \bar{E}$ whenever $|\bar{y}_k| \leq |\bar{x}_k|$, for all $k \in N$ and $\bar{x} = (\bar{x}_k) \in \bar{E}$.

An interval valued sequence space $\bar{E}$ is said to be monotone if $\bar{E}$ contains the canonical pre- image of all its step spaces.

An interval valued sequence space $\bar{E}$ is said to be convergence free if $\bar{y} = (\bar{y}_k) \in \bar{E}$ whenever $\bar{x} = (\bar{x}_k) \in \bar{E}$ and $\bar{x}_k = \bar{0}$ implies $\bar{y}_k = \bar{0}$.

Throughout the paper, $p = (p_k)$ is a sequence of bounded strictly positive numbers.

Esi[1] define the following interval valued sequence space:

$$\bar{\ell}(p) = \left\{ \bar{x} = (\bar{x}_k) : \sum_{k=1}^{\infty} [d(\bar{x}_k, \bar{0})]^{p_k} < \infty \right\},$$

for $p_k = 1$ for all $k \in N$, we have

$$\bar{\ell}(p) = \left\{ \bar{x} = (\bar{x}_k) : \sum_{k=1}^{\infty} [d(\bar{x}_k, \bar{0})] < \infty \right\}.$$

Kizmaz [12] defined the sequence space for crisp set. The concept further generalized by Tripathy and Esi [12] as follows:

Let $m > 0$ be an integer then $Z_1(\Delta_m) = \{(\bar{x}_k) \in w : (\Delta_m x_k) \in Z_1\}$, for $Z_1 = \ell_\infty, c$ and c_0. Where $\Delta_m x_k = x_k - x_{k+m}$, for all $k \in N$ and they showed that these are Banach spaces under the norm $||x||_{\Delta_m} = \sum_{r=1}^{m} |x_r| + \sup_k |\Delta_m x_k|$. For $m = 1$, the sequence spaces $\ell_\infty(\Delta), c(\Delta)$ and $c_0(\Delta)$ are studied by Kizmaz [12].

In this paper we introduce the difference operator for sequence of interval numbers generalized by Tripathy and Esi [22] as follows:

Let $\bar{x} = (\bar{x}_k)$ be a sequence of interval numbers and $p = (p_k)$ is a sequence of bounded strictly positive numbers. Let $m \geq 0$ be an integer then

$$Z(\Delta_m) = \{(\bar{x}_k) \in w^i : (\Delta_m \bar{x}_k) \in Z\} \text{ for } Z = \bar{\ell}_p(\Delta_m), \bar{c}(p)(\Delta_m) \text{ and } \bar{c_0}(p)(\Delta_m),$$

where $\Delta_m x_k = x_k - x_{k+m}$, for all $k \in N$.

2. Main Results

Theorem 2.1: The sequence spaces $\bar{\ell}(p)(\Delta_m), \bar{c}(p)(\Delta_m)$ and $\bar{c_0}(p)(\Delta_m)$ are complete metric space with respect to the metric defined by

$$\rho(\bar{x}, \bar{y}) = \sum_{k=1}^{\infty} [d(\bar{x}_k, \bar{y}_k)]^{p_k} + \sup_k [d(\Delta_m \bar{x}_k, \Delta_m \bar{y}_k)]$$

Proof: Let $(\bar{x}^i)$ be a Cauchy sequence in $\bar{\ell}(p)(\Delta_m)$ such that $\bar{x}^i = (\bar{x}^i_k) = (\bar{x}^i_1, \bar{x}^i_2, \bar{x}^i_3, ...) \in \bar{\ell}(p)(\Delta_m)$ for each $i \in N$. Then for a given $\varepsilon > 0$, there exists $n_0 \in N$, such that

$$\rho(\bar{x}^i, \bar{x}^j) = \sum_{k=1}^{\infty} [d(\bar{x}^i_k, \bar{x}^i_j)]^{p_k} + \sup_k d(\Delta_m \bar{x}^i_k, \Delta_m \bar{x}^j_k) < \varepsilon, \text{ for all } i, j \geq n_0$$

(2.1)

Then

$$\sum_{k=1}^{\infty} [d(\bar{x}^i_k, \bar{x}^j_k)]^{p_k} < \varepsilon, \text{ for all } i, j \geq n_0$$

$$\Rightarrow d(\bar{x}^i_k, \bar{x}^j_k) < \varepsilon, \text{ for all } i, j \geq n_0 \text{ and for all } k \in N.$$

$\Rightarrow (\bar{x}^i_k)$ is a Cauchy sequence in R and for all $k \in N$.

$\Rightarrow$ $(\bar{x}^i_k)$ Converges in R and for all $k \in N$ as R is a Banach space.

Let $\lim\limits_j \bar{x}_k^j = \bar{x}_k$ (say) for each $k \in N$ and $\bar{x} = (\bar{x}_k)$.

From definition (2.1) we have

$$d(\Delta_m \bar{x}_k^i, \bar{x}_k^j) < \varepsilon, \text{ for } i, j \geq n_0 \text{ and for } k \in N.$$

$\Rightarrow (\Delta_m \bar{x}_k^j)$ is a Cauchy sequence in R for all $k \in N$.

$\Rightarrow (\Delta_m \bar{x}_k^j)$ converges in R for all $k \in N$.

Let $\lim\limits_j \Delta_m \bar{x}_k^j = \bar{y}_k$, for each $k \in N$.

Since $\lim\limits_j \bar{x}_k^j = \bar{x}_k$, for each $k \in N$, therefore $\lim\limits_j \bar{x}_k^j = \bar{x}_k$ exist for each $k \in N$.

We have

$$\lim_{j\to\infty} \sum_{k=1}^{\infty} d(\bar{x}_k^i, \bar{x}_k^j) = \sum_{k=1}^{\infty} d(\bar{x}_k^i, \bar{x}_k) < \varepsilon, \text{for all } i \geq n_0$$

and

$$\lim_{j\to\infty} d\left((\bar{x}_{k+m}^i - \bar{x}_{k+m}^j), (\bar{x}_k^i - \bar{x}_m^j)\right) = d\left((\bar{x}_{k+m}^i - \bar{x}_{k+m}), (\bar{x}_k^i - \bar{x}_m)\right) < \varepsilon,$$

for all $i \geq n_0$ and $k \in N$.

Hence for all $i \geq n_0$, $\quad \sup\limits_k d(\Delta_m \bar{x}_k^i, \Delta_m \bar{x}_k) < \varepsilon.$

Thus we have

$$\sum_{k=1}^{\infty} [d(\bar{x}_k^i, \bar{x}_k)]^{p_k} + \sup_k (d(\Delta_m \bar{x}_k^i, \Delta_m \bar{x}_k)) < 2\varepsilon, \text{ for all } i \geq n_0.$$

$$\Rightarrow \rho(\bar{x}^i, \bar{x}) < 2\varepsilon, \text{ for all } i \geq n_0.$$

i.e. $\bar{x}^i \to \bar{x}$, as $i \to \infty$ in $\bar{\ell}(p)(\Delta_m)$.

And for $i \geq n_0$,

$$\sup_k (d(\Delta_m \bar{x}_k, \bar{0})) \leq \sup_k \left(d(\Delta_m \bar{x}_k, \Delta_m \bar{x}_k^i)\right) + \sup_k \left(d(\Delta_m \bar{x}_k^i, \bar{0})\right) < \infty.$$

This completes the proof.

Theorem 2.2: The sequence spaces $\bar{\ell}(p)(\Delta_m), \bar{c}(p)(\Delta_m)$ and $\bar{c}_0(p)(\Delta_m)$ are solid.

Proof: Let $\bar{x} = (\bar{x}_k) \in \bar{\ell}(p)(\Delta_m)$ and $\bar{y} = (\bar{y}_k) \in \bar{\ell}(p)(\Delta_m)$ be interval valued sequences such that $|\bar{y}_k| \leq |\bar{x}_k|$ for all $k \in N$.

Then

$$\sum_{k=1}^{\infty}[d(\Delta_m \bar{x}_k, \bar{0})]^{p_k} < \infty$$

and

$$\sum_{k=1}^{\infty}[d(\Delta_m \bar{y}_k, \bar{0})]^{p_k} \leq \sum_{k=1}^{\infty}[d(\Delta_m \bar{x}_k, \bar{0})]^{p_k} < \infty.$$

Thus $\bar{y} = (\bar{y}_k) \in \bar{\ell}(p)(\Delta_m)$ and hence $\bar{\ell}(p)(\Delta_m)$ is solid.

This complete the proof.

Theorem 2.3: The sequence spaces $\bar{\ell}(p)(\Delta_m), \bar{c}(p)(\Delta_m)$ and $\bar{c}_0(p)(\Delta_m)$ are not convergence free.

Proof: Let $m = 2$, we consider the interval sequence $\bar{x} = (\bar{x}_k)$ as follows

$$\bar{x}_k = \left[\frac{-1}{k^2}, 0\right], \Delta_2 \bar{x} = \left[\frac{-1}{k^2}, \frac{1}{(k+2)^2}\right], \text{ for all } k \in N.$$

Then, for $p_k = 1$

$$\sum_{k=1}^{\infty}[d(\Delta_2 \bar{x}_k, \bar{0})] < \sum_{k=1}^{\infty}\left(\frac{1}{k^2}\right) < \infty.$$

Thus $\bar{x} = (\bar{x}_k) \in \bar{\ell}(p)(\Delta_m)$.

Now let us define $\bar{y} = (\bar{y}_k)$ as follows

$$\bar{y}_k = [-k^2, 0], \text{ then } \Delta_2 \bar{y}_k = [-k^2, (k+2)^2], \text{ for all } k \in N.$$

Then

$$\sum_{k=1}^{\infty}[d(\Delta_2 \bar{y}_k, \bar{0})] \leq \sum_{k=1}^{\infty}(k+2)^2 = \infty.$$

Thus $\bar{y} = (\bar{y}_k) \notin \bar{\ell}(p)(\Delta_m)$.

Hence $\bar{\ell}(p)(\Delta_m)$ is not convergence free.

This completes the proof.

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