Proyecciones Journal of Mathematics
Vol. 37, N^o 3, pp. 405-413, September 2018.
Universidad Católica del Norte
Antofagasta - Chile

Multiple solutions of stationary Boltzmann equation

Rafael Galeano Andrade
University of Cartagena, Colombia
and
José de los Santos Marrugo Pérez
University of Cartagena, Colombia
Received : October 2016. Accepted : July 2018

Abstract

We find two fixed points differents of cero of the operator in an Sobolev Spaces in $L^1(\Omega)$ with $\Omega \subseteq \mathbf{R}^n$ and they are solutions of Boltzmann equation.

Key Words : *Fixed points, multiple solutions, stationary Boltzmann equation.*

Classification AMS : *35Q20 - 46T20*

1. Introduction

Let $u : \Omega \times \mathbf{R}^n \longrightarrow \mathbf{R}^+$,with $u(x,v) \geq 0, \Omega \subseteq \mathbf{R}^n$ satisfying the following problem:

$$\begin{aligned} v \cdot \nabla_x u(x,v) &= Q(u,u)(v) \qquad x \in \Omega,\ v \in \mathbf{R}^n \\ u(x,v) &= e^{-|v|^2} \qquad x \in \partial\Omega,\ v \in \mathbf{R}^n \end{aligned}$$

where $Q(u,h)(v) = \begin{cases} Q(u,u)(v) & if \quad u = h \\ \\ 0 & if \quad u \neq h \end{cases}$

$Q(u,u)(v) = \int_{\mathbf{R}^n} \int_{S^2_+} B(n, w-v)[u(x,v')u(x,w') - u(x,v)u(x,w)]dndw$ being $v' = v + [(w-v).n]n$ and $w' = w - [(w-v).n]n$, n is the unitary vector in the direction of the bisecting the angle formed by $v - w$ and $w' - v'$ and $S^2_+ = \{n \in \mathbf{R}^n : \|n\| = 1,\ [n.(w-v)] \geq 0\}$, also B satisfies the following conditions:

i) $B(n, w-v) \geq 0$ yand B depends only of $\|w-v\|$ and $|(w-v,n)|$.

ii) $B \in L^\infty_{loc}(\mathbf{R}^n, S^2_+)$.

iii) $B(n, w-v) \leq b_1 \frac{|(w-v,n)|}{\|w-v\|}[1 + \|w-v\|^{\sqsubseteq}]$.

iv) $\int_{\mathbf{R}^n} \int_{S^2_+} B(w-v,n)dndw < \infty$ being $v \in [0,1]$ and $b_i (i = 0,1)$ are positive constants.

v) $Q(u,u)(v) = 0$ if u is a maxwellian $(u(x,v) = ke^{-|v|^2},\ k = cte)$

vi) $Q(-u,-u)(v) = Q(u,u)(v)$

The stationary Boltzmann equation has been studied in approach of L^1 by the method of weak compactness see [1], [3]and [7] in the literature,in [8] and [9] classic considerations of the stationary Boltzmann equation are made, Leray-Schauder alternative was studied in the paper [7], where the existence of solutions is demostrated, this paper is a continuation of [7]. In [5] and [7], a problem is studied in L^2 and in [10] in L^∞.

2. Development

Definition: Let $E = \{u \in L^1(\overline{\Omega}) : v_i \frac{\lceil u}{\lceil x_i} \in L^1(\overline{\Omega})\}$
$\|u\|_E = \max\{\ \|u\|_{L^1(\bar{\Omega})},\ \|v_i \frac{\lceil u}{\lceil x_i}\|_{L^1(\overline{\Omega})}\}$; being $\|u\|_{L^1(\overline{\Omega})} = \int_{\overline{\Omega}} |u(x,v)|dx$ for any $v \in \mathbf{R}^n$.

Observation: $(E, \|.\|)$ is a Banach Space.

Definition: $C = \{u \in E : v.\nabla_x u(x,v) \geq u(x,v) \geq Q(u,u)(v) \geq 0\}$.

Properties of C:

i) $C \neq \emptyset$. Indeed $u(x,v) = x.\langle e^{v^2}, e^{v^2}, ..., e^{v^2}\rangle \in C$, with $v_i \geq x_i$, because $u(x,v) = \sum_{i=1}^n x_i e^{v^2}$, then $\nabla_x u(x,v) = \langle e^{v^2}, e^{v^2}, ..., e^{v^2}\rangle$, so $v.\nabla_x u(x,v) = \sum_{i=1}^n v_i e^{v^2} \geq x.\langle e^{v^2}, e^{v^2}, ..., e^{v^2}\rangle = \sum_{i=1}^n x_i e^{v^2} \geq 0 = Q(u,u)$.

ii) If $u \in C$ and $-u \in C$, then $u = 0 \in C$.

Proof: It we have that $v \cdot \nabla_x u(x,v) \geq u(x,v) \geq Q(u,u)(v) > 0$ and $-v \cdot \nabla_x u(x,v) \geq -u(x,v) \geq Q(-u,-u)(v) = Q(u,u)(v) > 0$ and this is true if and only if $u = 0$.

iii) Q is continuous in $B_E(0,R)$ if $\int_{\mathbf{R}^n}\int_{S_+^2} b_1 \frac{|(w-v,n)|}{\|w-v\|}[1+\|w-v\|^{\sqsubseteq}]dndw \leq \frac{k}{4R}$, with $m(\Omega) < \infty$. Indeed,

$|Q(u_n,u_n)(v) - Q(u,u)(v)| =$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)[u_n(x,v')u_n(x,w') - u_n(x,v)u_n(x,w)]dndw \; -$
$\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)[u(x,v')u(x,w') - u(x,v)u(x,w)]dndw]| \; \leq$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)[u_n(x,v')u_n(x,w') - u(x,v')u(x,w')]dndw| \; +$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)[u(x,v)u(x,w) - u_n(x,v)u_n(x,w)]dndw| \; =$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)[u_n(x,v')u_n(x,w') - u_n(x,v')u(x,w') + u_n(x,v')u(x,w') - u(x,v')u(x,w')]dndw| \; +$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)[u(x,v)u(x,w) - u(x,v)u_n(x,w) + u(x,v)u_n(x,w) - u_n(x,v)u_n(x,w)]dndw| \; \leq$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)u_n(x,v')[u_n(x,w') - u(x,w')]dndw| \; +$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)u(x,w')[u_n(x,v') - u(x,v')]dndw| \; +$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)u(x,v)[u(x,w) - u_n(x,w)]dndw| \; +$
$|\int_{\mathbf{R}^n}\int_{S_+^2} B(n,w-v)u_n(x,w)[u(x,v) - u_n(x,v)]dndw|$

applying the hypothesis ii), iii) on B we get it:

$|Q(u_n,u_n)(v) - Q(u,u)(v)| \leq$

$| \int_{\mathbf{R}^n} \int_{S_+^2} b_1 \frac{|(w-v,n)|}{\|w-v\|}[1+\|w-v\|^{\sqsubseteq}]u_n(x,v')[u_n(x,w')-u(x,w')]dndw| +$
$| \int_{\mathbf{R}^n} \int_{S_+^2} b_1 \frac{|(w-v,n)|}{\|w-v\|}[1+\|w-v\|^{\sqsubseteq}]u(x,w')[u_n(x,v')-u(x,v')]dndw| +$
$| \int_{\mathbf{R}^n} \int_{S_+^2} b_1 \frac{|(w-v,n)|}{\|w-v\|} [1+||w-v||^{\sqsubseteq}]\, u(x,v)\, [u(x,w)-u_n(x,w)]\, dndw| +$
$| \int_{\mathbf{R}^n} \int_{S_+^2} b_1 \frac{|(w-v,n)|}{\|w-v\|}[1+\|w-v\|^{\sqsubseteq}]u_n(x,w)[u(x,v)-u_n(x,v)]dndw|$

so $\int_{\mathbf{R}^n} |Q(u_n,u_n)(v)-Q(u,u)(v)|dv \leq k\|u_n-u\|_E$, being
$k \geq 4R \int_{\mathbf{R}^n} \int_{S_+^2} b_1 \frac{|(w-v,n)|}{\|w-v\|}[1+\|w-v\|^{\sqsubseteq}]dndw$

iv) C is closed. Indeed: let $u_n \longrightarrow u$ in $B_E(0,R)$, with $u_n \in C$, see that $u \in C$. As $\lim_{n\to\infty} u_n = u$, and $v \cdot \nabla_x u_n(x,v) \geq Q(u_n,u_n)(v) > 0$ then $v \cdot \nabla_x u = v \cdot \nabla_x (u - u_n + u_n) = v \cdot \nabla_x (u-u_n) + v \cdot \nabla_x (u_n) \geq v \cdot \nabla_x (u-u_n) + Q(u_n,u_n)(v)$ so $\lim_{n\to\infty} v \cdot \nabla_x u(x,v) \geq \lim_{n\to\infty} v \cdot \nabla_x (u-u_n)(x,v) + \lim_{n\to\infty} Q(u_n,u_n)(v) > 0$, then $v \cdot \nabla_x u(x,v) \geq \lim_{n\to\infty} Q(u_n,u_n)(v) = Q(u,u)(v)$ (Q is continuous in $B_E(0,R)$) so $u \in C$, and C is closed.

v) C is convex: let $0 \leq t \leq 1$ and let $u_1 \in C$ and $u_2 \in C$ that is to say $v \cdot \nabla_x u_1(x,v) \geq u_1(x,v) \geq Q(u_1,u_1)(v) > 0$ and $v \cdot \nabla_x u_2(x,v) \geq u_2(x,v) \geq Q(u_2,u_2)(v) > 0$ then:
$v \cdot \nabla_x [tu_1 + (1-t)u_2](x,v) = tv \cdot \nabla_x u_1(x,v) + (1-t)v \cdot \nabla_x u_2(x,v) \geq tQ(u_1,u_1)(v) + (1-t)Q(u_2,u_2)(v) \geq t^2 Q(u_1,u_1)(v) + (1-t)^2 Q(u_2,u_2)(v) = Q(tu_1,tu_1)(v) + Q((1-t)u_2,(1-t)u_2)(v) = Q(tu_1+(1-t)u_2, tu_1+(1-t)u_2)(v)$, as $Q(u,h) = 0$ if $u \neq h$ so C is convex.

Definition: If $u,h \in E$, $u \leq h$ if and only if $h - u \in C$.

Lema Let $u,h \in C$ y $u \leq h$, then $\|u\|_E \leq \|h\|_E$.

Proof: As $h-u \in C$, we have:
$v \cdot \nabla_x (h)(x,v) - v \cdot \nabla_x (u)(x,v) \geq (h)(x,v) - (u)(x,v) \geq 0$ this is
$v \cdot \nabla_x h(x,v) - (h)(x,v) \geq v \cdot \nabla_x u(x,v) - (u)(x,v) \geq 0$ and $v \cdot \nabla_x h(x,v) \geq (h)(x,v)$
We have that $\|u\|_E = max\{\|u\|_{L^1(\overline{\Omega})}, \|v_i \frac{\partial u}{\partial x_i}\|_{L^1(\overline{\Omega})}\}$. That is, $\|h\|_E \geq \|h\|_{L^1(\overline{\Omega})}$
If $\|u\|_E = \|u\|_{L^1(\overline{\Omega})}$ then $\|u\|_E \leq \|h\|_E$ is valid.
Now, if $\|u\|_E = \|v_i \frac{\partial u}{\partial x_i}\|_{L^1(\overline{\Omega})}$, then $\|u\|_E = \int_{\overline{\Omega}} v_i \frac{\partial u}{\partial x_i} dx$ and

$\sum_{i=1}^{n} \|u\|_E = \sum_{i=1}^{n} \int_\Omega v_i \frac{\partial u}{\partial x_i} dx \le \sum_{i=1}^{n} \int_\Omega v_i \frac{\partial h}{\partial x_i} dx \le \sum_{i=1}^{n} \|h\|_E$. Then $n\|u\|_E \le n\|h\|_E$, that is, $\|u\|_E \le \|h\|_E$.

Definition:

Let $F : B_E(0,R) \cap C \longrightarrow C$

$$u \longrightarrow F(u) := \begin{cases} u - v \cdot \nabla_x u + Q(u,u) & if \quad u \in B_E(0,r) \cap C \\ \frac{1}{2}u - v \cdot \nabla_x u + Q(u,u) & if \quad u \in \partial B_E(0,r) \cap C \\ u + v \cdot \nabla_x u - Q(u,u) & if \quad u \in [\overline{B_E(0,R)} - \overline{B_E(0,r)}] \cap C \\ \frac{3}{2}u + v \cdot \nabla_x u - Q(u,u) & if \quad u \in \partial B_E(0,R) \cap C \end{cases}$$

being $0 < r < R$

Properties of F:

i) If $u \in \partial_E B(0,r) \cap C$, then:
$F(u) = \frac{1}{2}u - v \cdot \nabla_x u + Q(u,u)$, now $v \cdot \nabla_x u \ge Q(u,u)$, so
$F(u) + v \cdot \nabla_x u - Q(u,u) = \frac{1}{2}u \quad \Rightarrow \|F(u)\| \le \|\frac{1}{2}u\| < \|u\|$
if $u \in C \cap \partial B_E(0,r)$.

ii) If $u \in \partial B_E(0,R) \cap C$, then $F(u) = \frac{3}{2}u + v \cdot \nabla_x u - Q(u,u)$, as $u \in C$, then $\|F(u)\| \ge \|\frac{3}{2}u\| > \|u\|$

iii) $u \ne F(u)$ for all $u \in \partial_E B(0,r) \cap C$. Indeed, if exist $u_0 \in \partial B_E(0,r) \cap C$ such that $F(u_0) = u_0$ then $u_0 = \frac{1}{2}u_0 - v \cdot \nabla_x u_0 + Q(u_0,u_0) \quad \Rightarrow \quad \frac{1}{2}u_0 = -v \cdot \nabla_x u_0 + Q(u_0,u_0)$, as $u_0 \in C$, then $v \cdot \nabla_x u_0 \ge Q(u_0,u_0) > 0$ that is to say $-v \cdot \nabla_x u_0 + Q(u_0,u_0) < 0$ and $u_0 > 0$ which is a contradiction.

iv) $F(u) \in C$, with $u \in B_E(0,R) \cap C$. Indeed, if $u \in \overline{B_E(0,r)} \cap C$, then $F(u) \le u$, that is to say $u - F(u) \in C$, exist $c \in C : \; u - F(u) = c \quad \Rightarrow \quad F(u) = u - c \in C$. If $u \in B_E(0,R) \cap C$, $F(u) \ge u$, then $F(u) - u \in C$, that is to say, there exist $c' \in C : \; F(u) - u = c' \quad \Rightarrow$ $F(u) = u + c' \in C$. In other cases we proceed identically.

Lema: There exist $k' \ge 0$ such that if $\int_{\mathbf{R}^n} \int_{S_+^2} |B(n, w-v)| dn\, dw < \infty$, then
$|Q(u,u)(v)| \le k' \|u\|_E^2$.

Proof: $|Q(u,u)(v)| \leq \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)||u(x,v')u(x,w') - u(x,v)u(x,w)|dn\, dw$
as $u(x,v) \geq 0$,$u(x,w) \geq 0$, $u(x,v') \geq 0$ and $u(x,w') \geq 0$ then
$|Q(u,u)(v)| \leq \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)||u(x,v')u(x,w')|dn\ dw\ +$
$\int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)||u(x,v)u(x,w)|dn\ dw$
Integrating over x
$|Q(u,u)(v)| \leq \frac{1}{m(\Omega)} \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)|dn\ dw\ \int_\Omega u(x,v')u(x,w')dx$
$.\qquad\qquad + \frac{1}{m(\Omega)} \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)|dn\ dw\ \int_\Omega u(x,v)u(x,w)dx$
Now
$0 \leq [u(x,v') - u(x,w')]^2 = u^2(x,v') - 2u(x,v')u(x,w') + u^2(x,w')$

Then
$u(x,v')u(x,w') \leq \frac{1}{2}u^2(x,v') + \frac{1}{2}u^2(x,w')$
Analogously
$u(x,v')u(x,w') \leq \frac{1}{2}u^2(x,v) + \frac{1}{2}u^2(x,w)$
So
$|Q(u,u)(v)| \leq \frac{1}{m(\Omega)} \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)|dn\ dw\ [\int_\Omega \frac{1}{2}u^2(x,v')dx + \int_\Omega \frac{1}{2}u^2(x,w')dx]$
$.\qquad\qquad + \frac{1}{m(\Omega)} \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)|dn\ dw\ [\int_\Omega \frac{1}{2}u^2(x,v)dx + \int_\Omega \frac{1}{2}u^2(x,w)dx]$
$.\qquad\qquad = \frac{2}{m(\Omega)} \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)|dn\ dw[u^2_E]$
Then
$|Q(u,u)(v)| \leq k'u^2_E = k'u^2_E$ being $k' = \frac{2}{m(\Omega)} \int_{\mathbf{R}^n} \int_{S^2_+} |B(n, w-v)|dn\ dw$

Lema: F is continuous in $B_E(0,R)$, being valid the hypothesis about $B(n, w-v) < \int_{\mathbf{R}^n} \int_{S^2_+} b_1 \frac{|(w-v,n)|}{\|w-v\|}[1 + \|w-v\|^{\sqsubseteq}]dn\ dw$ and $m(\Omega) < \infty$.

Proof:

i) Let $u, h \in [\overline{B_E(0,R)} - \overline{B_E(0,r)}] \cap C$, then
$F(u) = u + v \cdot \nabla_x u - Q(u,u)$
$F(h) = h + v \cdot \nabla_x h - Q(h,h)$
$F(u) - F(h) = (u-h) + v \cdot \nabla_x (u-h) + Q(h,h) - Q(u,u)$
$|F(u) - F(h)| \leq |(u-h)| + |v \cdot \nabla_x (u-h)| + |Q(h,h) - Q(u,u)|$
$|F(u) - F(h)| \leq |(u-h)| + |\sum_{i=1}^n v_i \frac{\lceil (u-h)}{\lceil x_i}| + |Q(h,h) - Q(u,u)|$
$\|F(u) - F(h)\|_E \leq \|u-h\|_E + n\|u-h\|_E + k\|h-u\|^2_E$
and this shows that F is continuous in
$[\overline{B_E(0,R)} - \overline{B_E(0,r)}]$

ii) If $u, h \in \overline{B_E(0,r)} \cap C$, then:
$F(u) = u - v \cdot \nabla_x u + Q(u,u)$

$F(h) = u - v \cdot \nabla_x h + Q(h,h)$
$|F(u) - F(h)| \leq |(u-h)| + |v \cdot \nabla_x h - v \cdot \nabla_x u| + |Q(u,u) - Q(h,h)|$
$\Rightarrow \quad \|F(u) - F(h)\|_E \leq \|u-h\|_E + \sum_{i=1}^{N} \|v_i \frac{\lceil (h-u)}{\lceil x_i}\|_{L^1{}_\Omega} + k\|h-u\|_E^2,$
which also implies that F is continuous in $\overline{B_E(0,r)}$.The other cases are similar .

Lema: F is compact.

Proof: Apply the Dunford-Pettis's criterion, see [4].
Let $\mathcal{F} = \{F(u) \in C : u \in C \cap B_R(0)\}$.

i) See that $\mathcal{F}$ is equintegrable.
Let $\Omega \subseteq \mathbf{R}^n$ a mesurable set and calculate $\int_\Omega |F(u)|dx$, for this, suppose (1) that $u \in [\overline{B_R(0)} - \overline{B_r(0)}] \cap C$, then $\int_\Omega |F(u)(x,v)|dx = \int\limits_\Omega |u(x,v) + v \cdot \nabla_x u - Q(u,u)|dx$, also $u \in C$, $Q(u,u) > 0$ then $\int_\Omega |F(u)(x,v)|dx \leq \int_\Omega |u(x,v)|dx + \int_\Omega |v \cdot \nabla_x u|dx + \int_\Omega |Q(u,u)|dx$
$\Rightarrow \quad \int_\Omega |F(u)(x,v)|dx \leq \|u\|_E + N\|u\|_E + k'\|u\|_E^2 m(\Omega)$, is there exist a $\delta > 0$ such that $m(\Omega) < \delta$ then $\int_\Omega |F(u)(x,v)|dx \leq (1+N)\|u\|_E + k'\delta\|u\|_E^2$, as $u \in B_R(0)$ then $\int_\Omega |F(u)(x,v)|dx \leq (1+N)R + k'\delta R^2$ and if we define
$\epsilon = \{(1+N)R + k'\delta R^2\}$, it holds that $\forall \varepsilon > 0, \exists \delta > 0 \ \ \int_\Omega |F(u)(x,v)|dx \leq \epsilon$.

Now, if $u \in \overline{B_r(0)} \cap C$, then,
$\int_\Omega |F(u)(x,v)|dx \leq \int_\Omega |u(x,v)|dx - \int_\Omega |v \cdot \nabla_x u|dx + \int_\Omega |Q(u,u)|dx$ as $u \in C$, we have that $v \cdot \nabla_x u > 0$, we have $\int_\Omega |F(u)(x,v)|dx \leq \int_\Omega |u(x,v)|dx + \int_\Omega |v \cdot \nabla_x u|dx + \int_\Omega |Q(u,u)|dx$
If $u \in \partial_E B(0,r) \cap C$ and $u \in \partial_E B(0,R) \cap C$,we proceed analogously.

As Ω is measurable, there is a closed subset $\Omega', \Omega' \subseteq \Omega$ with $med(\Omega - \Omega') < \delta$, then

$$\begin{aligned}\int_{\Omega-\Omega'} |F(u)(x,v)|dx &\leq \|u\|_E + N\|u\|_E + k'\|u\|_E^2 m(\Omega - \Omega') \\ . \qquad &\leq (1+N)\|u\|_E + k'\delta\|u\|_E^2 \\ . \qquad &\leq (1+N)r + k'\delta r^2\end{aligned}$$

We define $\epsilon = \{(1+N)r + k'\delta r^2\}$ then $\forall \epsilon, \exists \delta : \int_{\Omega-\Omega'} |F(u)(x,v)|dx \leq \epsilon$

So,by Theorem 7.8 pag 84 [2], there exist to fixed points u_0 and u_1 with $u_0 \in B_E(0,r) \cap C$ y $u_1 \in C \cap [\overline{B_R(0)} - \overline{B_r(0)}]$ and this to fixed points are not trivial. Also, if $\in \partial\Omega, v \in \mathbf{R}^n, F(u) = u(x,v) = e^{-|v|^2}$ with $u \in B_E(0,r) \cap C$ and analogously $F(u) = u(x,v) = e^{-|v|^2}$ if $u \in [\overline{B_E(0,R)} - \overline{B_E(0,r)}] \cap C$

References

[1] Arkeryd, L. On the stationary Boltzman equation", Seminary E.D.P. expose N 3, pp 11 (2001-2002).

[2] Agarwal R., Meehan M., Orégan D. Fixed point theory and applications, C. T. M., (2004).

[3] Arkeryd, L. and Nouri, A. The stationary Boltzman equation in $\mathbf{R}^n$ with given data, Annals Scuola Normal Superior de Pisa. (2002).

[4] Brezis Haim Functional Analysis, Sobolev spaces and partial dofferential equations, Sprinjer Verlag, (2011).

[5] Galeano R, almanza M, A variational approach of stationary Boltzmann under a condition of poisson type, Revista Tecnica Facultad de Ingenieria, Zulia, (2013).

[6] Galeano R., Almanza M., Ortega P. Stationary Boltzmann Equation with Boundary Data Depending on the Maxwellian, Matemática Enseñanza Universitaria, Vol 20, No. 2, (2012).

[7] Galeano R., Cantillo J., Ortega P. Stationary Boltzmann Equation and the Nonlinear Alternative of Leray-Schauder, Cubo journal of mathematics, Vol 16, (2014).

[8] Maslova N. Nonlinear Evolution Equation, Series on Advances in Mathematics for Applied Series, Vol 10, (1993).

[9] Uray S., Yang T., Stationary Problems of Boltzmann Equation", Hanbook of Differential Equations, Vol 5, Elsevier, 2008.

[10] Wu Lei Hidrodinamic Limit with Geometric Correction of Stationary Boltzmann Equation, Arxiv: 14093945v5, October 2015.

Rafael Galeano Andrade
Institute of Applied Mathematics
University of Cartagena
Colombia
e-mail : rgaleanoa@unicartagena.edu.co

and

Jos de los Santos Marrugo Pérez
Institute of Applied Mathematics
University of Cartagena
Colombia
e-mail : jmarrugop@unicartagena.edu.co