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On velocity bimagnetic biharmonic particles with energy on Heisenberg space

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Abstract

In this work, we consider velocity bimagnetic biharmonic particle on 3D Heisenberg space in the magnetic field $\mathcal{B}$ and we give the concept of energy. Moreover, we characterize energy conditions of velocity bimagnetic biharmonic particles with Frenet vector field. Therefore, we obtain energy results for bimagnetic particles by Frenet fields in the Heisenberg space.

Keywords : *Biharmonic particle, Energy, Heisenberg space, Magnetic fields, Symmetries.*

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1. Introduction

In the literary works, certainly one of the significant aim is to acquire the magnetic curves connected to magnetic field B on Riemannian manifold. Therefore, implicit geometrical highlights in the Riemannian manifolds may be utilized to decide the curvature in the magnetic curves. As a result, magnetic curves can easily be determined totally based in any individual manifold. By way of model, [1-7] exhibited that trajectories involved with magnetic fields described with Riemannian surface region developing a constant Gaussian curvature K might very conveniently be determined. This excellent kind of study heve been prolonged to specific background spaces. As an illustration, [8,9] acquired specific trajectories connected by way of Kahler magnetic fields by means of presuming background space is a complex type from space. Practically, [10] provided comprehensive recommendation intended for regular flowlines in the contact magnetic field through presuming normal space is simple contact manifold.

Research upon the basic principle has prolonged by means of determining a Killing magnetic fields with the support of Killing vector fields. A variational strategy method on magnetic flows of the Killing magnetic field on 3D was created. Therefore, [11] researched that solution of the formula of Lorentz force can be regarded as Kirchoff elastic rods and the other way round by simply studying magnetic flow on a Killing magnetic field in 3D. After that, [12] handled the magnetic flowlines connected with Killing magnetic fields on the unit sphere in 3D. Finally, [13] explained N and B-magnetic curves as the trajectories of the particular magnetic field and revealed their magnetic flows connected with Killing magnetic field on space. Also some curves and surfaces with energy studied in [14-25].

2. Heisenberg Space and Frenet Frame

Heisenberg group is given by with following multipilcation [26].:

$$(\overline{x},\overline{y},\overline{z})(x,y,z)=(\overline{x}+x,\overline{y}+y,\overline{z}+z-\frac{1}{2}\overline{x}y+\frac{1}{2}x\overline{y}).$$

Heisenberg metric g is given by

$$g=dx^2+dy^2+(dz-xdy)^2.$$

Heisenberg group's Lie algebra has following orthonormal basis

$$\mathbf{d}_1 = \frac{\partial}{\partial x}, \quad \mathbf{d}_2 = \frac{\partial}{\partial y} + x\frac{\partial}{\partial z}, \quad \mathbf{d}_3 = \frac{\partial}{\partial z}.$$

Assume that ζ be a regular curve in Heisenberg space. Let $\mathbf{e}^{\mu}_{(0)}$, $\mathbf{e}^{\mu}_{(1)}$, and $\mathbf{e}^{\mu}_{(2)}$ represent the tangent vector, principal normal vector, and secondary normal vector, respectively. For all the curves in Heisenberg space, by transforming the time parameter t into an arc length parameter s, the Frenet frame of a curve is defined by

$$\frac{d\alpha}{ds} = \mathbf{e}^{\mu}_{(0)},$$

$$\begin{bmatrix} \nabla_{\mathbf{e}^{\mu}_{(0)}} \mathbf{e}^{\mu}_{(0)} \\ \nabla_{\mathbf{e}^{\mu}_{(0)}} \mathbf{e}^{\mu}_{(1)} \\ \nabla_{\mathbf{e}^{\mu}_{(0)}} \mathbf{e}^{\mu}_{(2)} \end{bmatrix} = \begin{bmatrix} & \kappa & \\ -\kappa & & \tau \\ & -\tau & \end{bmatrix} \begin{bmatrix} \mathbf{e}^{\mu}_{(0)} \\ \mathbf{e}^{\mu}_{(1)} \\ \mathbf{e}^{\mu}_{(2)} \end{bmatrix},$$

where κ and τ are curvatures of the Frenet frame.

3. Velocity (Tangent) Bimagnetic Particles of Biharmonic Curves

Lorentz force ϕ connected with a magnetic field $\mathcal{B}$ may easily be calculated simply by

$$\phi(\mathcal{Y}) = \mathcal{B} \times \mathcal{Y},$$

and consequently [2-8], Lorentz force equation meant for the magnetic particles ζ may easily be described by way of

$$\nabla_{\zeta'}\zeta' = \phi(\zeta') = \mathcal{B} \times \zeta'.$$

ζ is called a velocity bimagnetic partcle if ζ is satisfy following 2-Lorentz force equation

$$\nabla_{\zeta'}\nabla_{\zeta'}\zeta' = \phi(\nabla_{\zeta'}\zeta') = \mathcal{B} \times \nabla_{\zeta'}\zeta'.$$

Definition 3.1 *Energy of a differentiable map* $f : (M, \rho) \to \left(N, \tilde{h}\right)$ *is defined as*

$$\varepsilon(f) = \frac{1}{2}\int_M \sum_{a=1}^{n} \tilde{h}\left(df\left(e_a\right), df\left(e_a\right)\right) v,$$

where $\{e_a\}$ *is a local basis of the tangent space,* v *is the canonical volume form* $[2, 27, 28]$ *.*

From above equations Lorentz force of Frenet vector fields of ζ in Heisenberg space are given by

$$\begin{aligned}\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(0)}) &= [\tau\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]-\kappa^2\cos\mathcal{A}]\mathbf{d}_1\\ &+[-\tau\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})-\kappa^2\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]\mathbf{d}_2\\ &+[\tau\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]-\kappa^2\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2]]\mathbf{d}_3,\end{aligned}$$

$$\begin{aligned}\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(1)}) &= \tfrac{1}{\kappa}\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]\,\mathbf{d}_1\\ &-\tfrac{1}{\kappa}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})\mathbf{d}_2+\tfrac{1}{\kappa}\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\,\mathbf{d}_3,\end{aligned}$$

$$\begin{aligned}\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(2)}) &= [-\tfrac{\tau\rho}{\kappa}[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1\\ &+\cos\mathcal{A})]-\kappa\tau\cos\mathcal{A}]\mathbf{d}_1+[\tfrac{\tau\rho}{\kappa}[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\\ &-\sin^3\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin^2[\mathcal{R}_1 s+\mathcal{R}_2]]-\kappa\tau\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]\mathbf{d}_2\\ &+[\tfrac{\tau\rho}{\kappa}[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})\\ &+\sin^3\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]-\kappa\tau\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2]]\mathbf{d}_3.\end{aligned}$$

4. Energy of T-Bimagnetic Fields

In this section, we define energy of velocity bimagnetic particles in the Heisenberg space. Moreover, we give some characterizations and figures of the particle.

Firstly we compute energy *of* $\{\mathbf{e}^{\mu}_{(0)}, \mathbf{e}^{\mu}_{(0)}, \mathbf{e}^{\mu}_{(0)}\}$.

$$\begin{aligned}\varepsilon(\mathbf{e}^{\mu}_{(0)}) &= \tfrac{1}{2}\textstyle\int_0^s(1+[\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]^2+[\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1\\ &+\cos\mathcal{A})]^2+[\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]^2)ds,\\ \varepsilon(\mathbf{e}^{\mu}_{(1)}) &= \tfrac{1}{2}\textstyle\int_0^s(1+[\tfrac{\tau}{\kappa}[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1 s+\mathcal{R}](\mathcal{R}_1\\ &+\cos\mathcal{A})]-\kappa\cos\mathcal{A}]^2+[-\tfrac{\tau}{\kappa}[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\\ &-\sin^3\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin^2[\mathcal{R}_1 s+\mathcal{R}_2]]-\kappa\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]^2\\ &+[-\tfrac{\tau}{\kappa}[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})\\ &+\sin^3\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]-\kappa\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2]]^2)ds,\\ \varepsilon(\mathbf{e}^{\mu}_{(2)}) &= \tfrac{1}{2}\textstyle\int_0^s(1+[\tfrac{\tau}{\kappa}\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]^2+[\tfrac{\tau}{\kappa}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1\\ &+\cos\mathcal{A})]^2+[\tfrac{\tau}{\kappa}\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]^2)ds.\end{aligned}$$

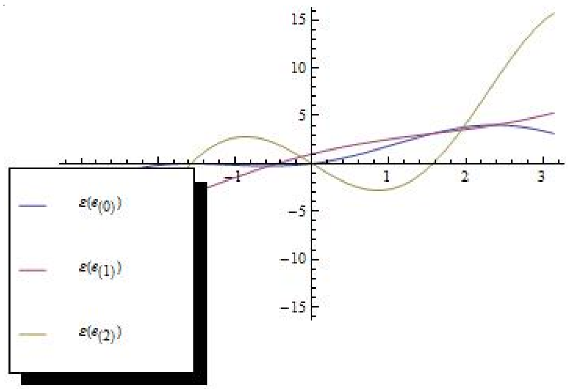

Figure 1. Energy of frenet vectors.

Theorem 4.1 *Energy for the particle in the 2-Lorentz force* $\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(0)})$ *is given by*

$$\varepsilon(\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(0)}))=\frac{1}{2}\int_0^s(1+[\tau^2[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1s+\mathcal{R}_2]+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1s+\mathcal{R}_2](\mathcal{R}_1 + \cos\mathcal{A})]-\kappa^2\sin^2\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]\sin[\mathcal{R}_1s+\mathcal{R}_2]-\kappa^2\tau\cos\mathcal{A}]^2 + [\kappa^2\sin\mathcal{A}\sin[\mathcal{R}_1s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})-\tau^2[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2] - \sin^3\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]\sin^2[\mathcal{R}_1s+\mathcal{R}_2]]-\kappa^2\tau\sin\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]]^2 + [-\tau^2[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})+\sin^3\mathcal{A}\cos^2[\mathcal{R}_1s+\mathcal{R}_2] \times\sin[\mathcal{R}_1s+\mathcal{R}_2]]-\kappa^2\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]-\kappa^2\tau\sin\mathcal{A}\sin[\mathcal{R}_1s+\mathcal{R}_2]]^2)ds.$$

Proof. Assume that Γ be a moving charged velocity bimagnetic biharmonic curve. Then, we can write

$\mathrm{d}(\omega)\circ d(\phi(\mathbf{T}))=d(\omega\circ\phi(\mathbf{T}))=d(id_C)=id_{TC}.$

Moreoever it is clear that

$$\mathrm{Q}(\phi\left(\mathbf{e}^{\mu}_{(0)}\right)(\mathbf{e}^{\mu}_{(\mathbf{0})}))=[\tau^2[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1s+\mathcal{R}_2]+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1s+\mathcal{R}_2](\mathcal{R}_1 + \cos\mathcal{A})]-\kappa^2\sin^2\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]\sin[\mathcal{R}_1s+\mathcal{R}_2]-\kappa^2\tau\cos\mathcal{A}]\mathbf{d}_1 + [\kappa^2\sin\mathcal{A}\sin[\mathcal{R}_1s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})-\tau^2[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2] - \sin^3\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]\sin^2[\mathcal{R}_1s+\mathcal{R}_2]]-\kappa^2\tau\sin\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]]\mathbf{d}_2 + [-\tau^2[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})+\sin^3\mathcal{A}\cos^2[\mathcal{R}_1s+\mathcal{R}_2] \times\sin[\mathcal{R}_1s+\mathcal{R}_2]]-\kappa^2\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1s+\mathcal{R}_2]-\kappa^2\tau\sin\mathcal{A}\sin[\mathcal{R}_1s+\mathcal{R}_2]]\mathbf{d}_3.$$

Then, it is easy to see that $\varepsilon(\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(0)}))$.

Theorem 4.2 *Energy for the particle in the 2-Lorentz force* $\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(1)})$ *is given by*

$\varepsilon(\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(1)}))=\frac{1}{2}\int_0^s(1+[\frac{1}{\kappa}\tau(-\kappa^2-\tau\rho)[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]$
$+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})]-\frac{\tau\rho'}{\kappa}\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]$
$-\kappa(-\kappa^2-\tau\rho)\cos\mathcal{A}]^2+[-\frac{1}{\kappa}\tau(-\kappa^2-\tau\rho)[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]-$
$\sin^3\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]$

$\times\sin^2[\mathcal{R}_1 s+\mathcal{R}_2]]+\frac{\tau\rho'}{\kappa}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})-\kappa(-\kappa^2-\tau\rho)$
$\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]^2+[-\frac{1}{\kappa}\tau(-\kappa^2-\tau\rho)[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2]$
$(\mathcal{R}_1+\cos\mathcal{A})+\sin^3\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]-\frac{\tau\rho'}{\kappa}\mathcal{R}_1\sin\mathcal{A}\cos$
$[\mathcal{R}_1 s+\mathcal{R}_2]-\kappa(-\kappa^2-\tau\rho)\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2]]^2)ds.$

Proof. By using Frenet frame, we obtain
$\phi'(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(1)})=[\frac{1}{\kappa}\tau(-\kappa^2-\tau\rho)[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]+\sin^2\mathcal{A}$
$\sin^2[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})]$
$-\frac{\tau\rho'}{\kappa}\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]-\kappa(-\kappa^2-\tau\rho)\cos\mathcal{A}]\mathbf{d}_1$
$+[-\frac{1}{\kappa}\tau(-\kappa^2-\tau\rho)[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]-\sin^3\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]$
$\sin^2[\mathcal{R}_1 s+\mathcal{R}_2]]+\frac{\tau\rho'}{\kappa}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})-\kappa(-\kappa^2-\tau\rho)$
$\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]]\mathbf{d}_2+[-\frac{1}{\kappa}\tau(-\kappa^2-\tau\rho)[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+$
$\cos\mathcal{A})+\sin^3\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]-\frac{\tau\rho'}{\kappa}\mathcal{R}_1\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]-$
$\kappa(-\kappa^2-\tau\rho)\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2]]\mathbf{d}_3.$

Theorem 4.3 *Energy for the particle in the 2-Lorentz force* $\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(2)})$ *is given by*
$\varepsilon(\phi(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(2)}))=\frac{1}{2}\int_0^s(1+[\frac{\tau^2\rho-\kappa^2\tau}{\kappa}\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]$
$-\frac{\tau\rho'}{\kappa}[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})]]^2$
$+[-\frac{\tau^2\rho-\kappa^2\tau}{\kappa}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})+\frac{\tau\rho'}{\kappa}[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos$
$[\mathcal{R}_1 s+\mathcal{R}_2]-\sin^3\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin^2[\mathcal{R}_1 s+\mathcal{R}_2]]]^2+[\frac{\tau^2\rho-\kappa^2\tau}{\kappa}\mathcal{R}_1$
$\sin\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]+\frac{\tau\rho'}{\kappa}[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})+$
$\sin^3\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]]]^2)ds.$

Proof. From Theorem 3.4, we get
$\phi'(\nabla_{\mathbf{e}^{\mu}_{(0)}}\mathbf{e}^{\mu}_{(2)})=[\frac{\tau^2\rho-\kappa^2\tau}{\kappa}\sin^2\mathcal{A}\cos[\mathcal{R}_1 s+\mathcal{R}_2]\sin[\mathcal{R}_1 s+\mathcal{R}_2]$
$-\frac{\tau\rho'}{\kappa}[\mathcal{R}_1\sin^2\mathcal{A}\cos^2[\mathcal{R}_1 s+\mathcal{R}_2]+\sin^2\mathcal{A}\sin^2[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})]]\mathbf{d}_1$
$+[-\frac{\tau^2\rho-\kappa^2\tau}{\kappa}\sin\mathcal{A}\sin[\mathcal{R}_1 s+\mathcal{R}_2](\mathcal{R}_1+\cos\mathcal{A})$
$+\frac{\tau\rho'}{\kappa}[\mathcal{R}_1\cos\mathcal{A}\sin\mathcal{A}\cos[\mathcal{R}_\infty\int+\mathcal{R}_\in]$
$-\sin^\ni\mathcal{A}\cos[\mathcal{R}_\infty\int+\mathcal{R}_\in]\sin^\in[\mathcal{R}_\infty\int+\mathcal{R}_\in]]]\mathbf{d}_\in$
$+[\frac{\tau^\in\rho-\kappa^\in\tau}{\kappa}\mathcal{R}_\infty\sin\mathcal{A}\cos[\mathcal{R}_\infty\int+\mathcal{R}_\in]$

$+\frac{\tau\rho'}{\kappa}[\cos\mathcal{A}\sin\mathcal{A}\sin[\mathcal{R}_\infty\int+\mathcal{R}_\in](\mathcal{R}_\infty+\cos\mathcal{A})+\sin^\ni\mathcal{A}\cos^\in$
$[\mathcal{R}_\infty\int+\mathcal{R}_\in]\sin[\mathcal{R}_\infty\int+\mathcal{R}_\in]]]\mathbf{d}_\ni$ is obtained which gives the result.

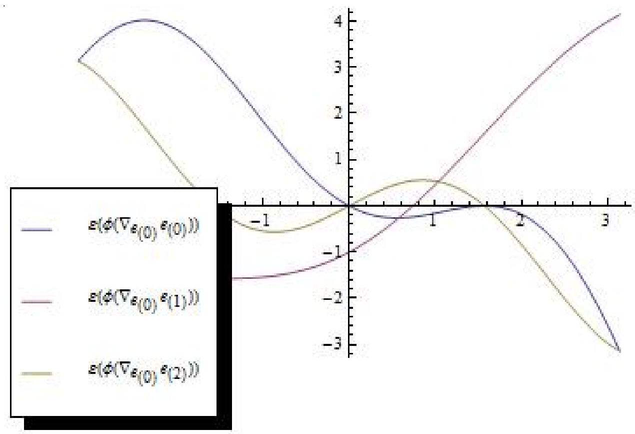

Figure 2. Energy of magnetic frenet vectors.

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