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A new type of generalized closed set via γ-open set in a fuzzy bitopological space

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Abstract:

This paper aims to present the notion of (i, j)-fuzzy γ-open set in a fuzzy bitopological space as a parallel form of (i, j)-fuzzy γ-open set due to Tripathy and Debnath (2013) [17] and show that both of them are independent concepts. Then we extend our study to (i, j)*-generalized fuzzy γ-closed set and (i, j)*-γ-generalized fuzzy closed set. We show that (i, j)*-γ-generalized fuzzy closed set and (i, j)*-generalized fuzzy γ-closed set are also independent of each other in nature. Though every (i, j)*-fuzzy γ-closed set is a (i, j)*-generalized fuzzy γ-closed set but with (i, j)*-γ-generalized fuzzy closed set, the same relation is not linear. Similarly though every (i, j)*-fuzzy closed set is (i, j)*-γ-generalized fuzzy closed set but it is independent to (i, j)*-generalized fuzzy γ-closed set. Various properties related to (i, j)*-generalized fuzzy γ-closed set are also studied. Finally, (i, j)*-generalized fuzzy γ-continuous function and (i, j)*-generalized fuzzy γ-irresolute functions are introduced and interrelationships among them are established. We characterized these functions in different directions for different applications.*

Keywords: (i, j)*-fuzzy γ-open set; (i, j)*-generalized fuzzy γ-closed set; (i, j)*-γ-generalized fuzzy closed set; (i, j)*-generalized fuzzy γ-continuous function; (i, j)*-generalized fuzzy γ-irresolute function.

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1. Introduction

Levine (1970) first initiated the concept of generalized closed set in a topological space. Cao, Ganster and Reilly (2002) further analyzed this set and found interesting results. In the meanwhile, further research has been carried out on the same concept in fuzzy topological space. Balasubramaniam and Sundaram (1997) first studied generalized fuzzy closed set and generalized fuzzy continuity in fuzzy topological space. Palaniappan and Rao (1993) came up with the notion of regular generalized closed set in a topological space and Park and Park (2003) extended this work in fuzzy environment. Introducing a new approach, Bhattacharya (2011) defined generalized regular closed set, which is different from regular generalized closed set and explored various characterizations of this set in an ordinary topological space. Very recently, Bhattacharya and Chakraborty (2015) extended this work in fuzzy topological space. Bitopological space was first introduced by Kelly (1963) and till now various constructive works have been going on in this particular field viz. Tripathy and Acharjee (2014), Tripathy and Sarma (2011, 2012, 2013, 2014), Tripathy and Debnath (2015, 2019). Also we can recollect that Fukutake (1986) introduced generalized closed set in bitopological space. But no report has been found till date, in showing the interest for extending this study in fuzzy bitopological space, which motivated us to define the concept of $(i,j)^*$-generalized fuzzy closed set and $(i,j)^*$-generalized fuzzy γ-closed set therein.

Throughout this paper, we denote a fuzzy bitopological space $(X,\tau_i,\tau_j), i \neq j, i,j = 1,2$, by fbts which is given by (X,τ_i,τ_j) and we simply denote this fbts as X. Some important related definitions are recalled below as ready references of our research work.

1.1. Definition (Kandil, Nouh, El-Sheikh, 1995)

Let X be a non-empty set and τ_i,τ_j be two fuzzy topologies defined on X. Then, (X,τ_i,τ_j) is called a fbts.
Bin Sahana (1991), as well as Singal and Prakash (1991) initiated the notion of fuzzy pre-open set in a fuzzy topological space and using this set Bhattacharya (2017) defined fuzzy γ^*-open set therein. Following these definitions, here we introduce $(i,j)^*$-fuzzy pre-open set and $(i,j)^*$-fuzzy γ-open set in a fbts.

1.2. Definition (Bhattacharya, 2017)

A fuzzy subset λ of a fuzzy topological space (X, τ) is said to be fuzzy γ^*-open if $\lambda \wedge \mu \in FPO(X)$, for each $\mu \in FPO(X)$, where $FPO(X)$ is the family of all fuzzy pre-open sets in X.

1.3. Definition (Kumar, 1994)

A fuzzy subset λ of a fbts (X, τ_i, τ_j) is called a (i, j)-fuzzy pre-open set if $\lambda \leq i\text{-}int(j\text{-}cl(\lambda)$, where $i \neq j, i, j = 1, 2$.
The collection of all (i, j)-fuzzy pre-open sets is denoted by (i, j)-$FPO(X)$.

1.4. Definition (Tripathy, Debnath, 2013)

A fuzzy subset λ of a fbts (X, τ_i, τ_j) is called a (i, j)-fuzzy γ-open set if $\lambda \wedge \mu$ is a (i, j)-fuzzy pre-open set, for every (i, j)-fuzzy pre-open set μ in X. A fuzzy subset η of X is called (i, j)-fuzzy γ-closed set if its complement, $1_X - \eta$ in X is a (i, j)-fuzzy γ-open set. The collection of all (i, j)-fuzzy γ-open set and (i, j)-fuzzy γ-closed set is denoted by (i, j)- F_γ-$O(X)$ and (i, j)- F_γ-$C(X)$ respectively.

1.5. Definition (Balasubramaniam, Sundaram, 1997)

Let (X, τ) be a fuzzy topological space. A fuzzy set λ in X is called a generalized fuzzy closed (in short, gf closed) set if $cl(\lambda) \leq \mu$ whenever, $\lambda \leq \mu$ and μ is a fuzzy open set in X.

1.6. Definition (Paul, Bhattacharya, Chakraborty, 2017)

A fuzzy subset μ in a fbts (X, τ_i, τ_j) is called a (i, j)-fuzzy generalized closed set if τ_j-$cl(\mu) \leq \eta$, whenever $\mu \leq \eta$ and $\eta \in \tau_i$-$FO(X)$, where τ_i-$FO(X)$ is the family of all τ_i fuzzy open sets.

1.7. Definition (Balasubramaniam, Sundaram, 1997)

A map $f : X \to Y$ from a fuzzy topological space (X, τ) into another fuzzy topological space (Y, σ) is called generalized fuzzy continuous (in short gf-continuous) if the inverse image of every fuzzy closed set in Y is gf-closed in X.

2. $(i, j)^*$ Generalized Fuzzy γ-Closed Sets

Analogues to the idea $\tau_{1,2}$-open set in a bitopological space (Thivagar, Ravi, 2004) we introduce the notion of $(i, j)^*$ fuzzy open set in a fbts. With the help of $(i, j)^*$ fuzzy open set, first we define $(i, j)^*$ fuzzy pre-open set and then $(i, j)^*$ fuzzy γ-open set. Furthermore, we propose the concepts of $(i, j)^*$ generalized fuzzy closed set and $(i, j)^*$ generalized fuzzy γ-closed set and investigate interrelationships between these sets. In addition, we study various characterizations of the newly introduced $(i, j)^*$ generalized fuzzy γ-closed set. We are to introduce another type of generalized fuzzy closed set called $(i, j)^*$-γ-generalized fuzzy closed set to find few interesting results. In particular, it is proved that every $(i, j)^*$-fuzzy closed set is a $(i, j)^*$-γ-generalized fuzzy closed set though the converse is not true and every $(i, j)^*$-fuzzy γ-closed set is a $(i, j)^*$-generalized fuzzy γ-closed set but not necessarily a $(i, j)^*$-γ-generalized fuzzy closed set. However, the detailed study on $(i, j)^*$- γ-generalized fuzzy closed set is out of the scope of this paper and may be considered as another scope for further exploration.

We start this section by recalling $\tau_{1,2}$-open set, due to Thivagar and Ravi (2004) in a bitopological space.

Let (X, τ_1, τ_2) be a bitopological space. A subset $A \subseteq X$ is said to be $\tau_{1,2}$-open set if $A = A_1 \cap A_2$, where $A_1 \in \tau_1$ and $A_2 \in \tau_2$. We start this section by recalling $\tau_{1,2}$-open set, due to Thivagar and Ravi (2004) in a bitopological space.

Let (X, τ_1, τ_2) be a bitopological space. A subset $A \subseteq X$ is said to be $\tau_{1,2}$-open set if $A = A_1 \cap A_2$, where $A_1 \in \tau_1$ and $A_2 \in \tau_2$.

2.1. Definition

A fuzzy subset λ in a fbts (X, τ_i, τ_j) is said to be $(i, j)^*$-fuzzy open if λ can be expressed as a union of two fuzzy sets λ_1, λ_2, where $\lambda_1 \in \tau_i$ and $\lambda_2 \in \tau_j$. The complement of a $(i, j)^*$-fuzzy open set is called a $(i, j)^*$-fuzzy closed set.
The family of all $(i, j)^*$-fuzzy open (resp. $(i, j)^*$-fuzzy closed) sets in a fbts is denoted by $(i, j)^*$-$FO(X)$ (resp. $(i, j)^*$-$FC(X)$).

2.2. Definition

The $(i,j)^*$-interior of any fuzzy set λ in a fbts means the union of all $(i,j)^*$-fuzzy open sets contained in λ and it is denoted by $(i,j)^*$-$int(\lambda)$. The $(i,j)^*$-closure of the fuzzy set λ is the intersection of all $(i,j)^*$-fuzzy closed sets containing λ and it is denoted by $(i,j)^*$-$cl(\lambda)$.

2.3. Definition

A fuzzy subset λ in a fbts (X, τ_i, τ_j) is said to be $(i,j)^*$-fuzzy pre-open if $\lambda \leq (i,j)^*\text{-}int((i,j)^*\text{-}cl(\lambda))$.
The collection of all $(i,j)^*$-fuzzy pre-open sets in a fbts is denoted by $(i,j)^*$-$FPO(X)$.

2.4. Definition

A fuzzy subset λ of a fbts (X, τ_i, τ_j) is called $(i,j)^*$-fuzzy γ-open if $\lambda \wedge \mu$ is $(i,j)^*$-fuzzy pre-open for every $(i,j)^*$-fuzzy pre-open set μ in X. A fuzzy subset η of X is called $(i,j)^*$-fuzzy γ-closed set if its complement, $1_X - \eta$ in X is a $(i,j)^*$-fuzzy γ-open set.
The collection of all $(i,j)^*$-fuzzy γ-open (resp. $(i,j)^*$-fuzzy γ-closed) sets in a fbts is denoted by $(i,j)^*$-$F\gamma$-$O(X)$ (resp. $(i,j)^*$-$F\gamma$-$C(X)$).
The $(i,j)^*$-γ-interior and $(i,j)^*$-γ-closure of a fuzzy set λ is denoted by $(i,j)^*$-$int_\gamma(\lambda)$ and $(i,j)^*$-$cl_\gamma(\lambda)$ and they are defined respectively as follows:

$$(i,j)^*\text{-}int_\gamma(\lambda) = \bigvee\{\mu : \mu \leq \lambda, \mu \text{ is a } (i,j)^*\text{-fuzzy } \gamma\text{-open set}\}$$
$$(i,j)^*\text{-}cl_\gamma(\lambda) = \bigwedge\{\mu : \lambda \leq \mu, \mu \text{ is a } (i,j)^*\text{-fuzzy } \gamma\text{-closed set}\}.$$

2.5. Definition

Let λ be any fuzzy set in a fbts (X, τ_i, τ_j). Then, λ is said to be a $(i,j)^*$-generalized fuzzy closed set (in short, $(i,j)^*$-gf closed set) if for any $(i,j)^*$-fuzzy open set μ in $X, \lambda \leq \mu$ implies $(i,j)^*$-$cl(\lambda) \leq \mu$. The family of all $(i,j)^*$-gf closed sets is denoted by $(i,j)^*$-$GFC(X)$.

2.6. Example

Consider a fbts (X, τ_i, τ_j) with $X = \{x\}, \tau_i = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.3)\}\}, \tau_j = \{0_X, 1_X, \{(x, 0.6)\}\}$. Here $(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.3)\}, \{(x, 0.6)\}\}$ and $(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$. Therefore

$(i,j)^*$-$GFC(X) = \{0_X, 1_X, \{(x,\alpha) : 0.3 < \alpha \leq 0.4$ or $\alpha > 0.6\}\}$. Thus the fuzzy set $\{(x, 0.7)\}$ is a $(i,j)^*$-gf closed set.

2.7. Definition

A fuzzy set λ in a fbts (X, τ_i, τ_j) is said to be a $(i,j)^*$-generalized fuzzy γ-closed set (in short, $(i,j)^*$-$gf\gamma$-closed set) if for a $(i,j)^*$-fuzzy open set μ in $X, \lambda \leq \mu$ implies $(i,j)^*$-$cl_\gamma(\lambda) \leq \mu$. The family of all $(i,j)^*$-$gf\gamma$-closed sets is denoted by $(i,j)^*$-GF_γ-$C(X)$.

2.8. Example

Take the same fbts given in example 2.6. Here
$(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x,\alpha) : \alpha \leq 0.3$ or $0.4 < \alpha \leq 0.6$ or $\alpha > 0.8\}\}$ and $(i,j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x,\alpha) : \alpha < 0.2$ or $0.4 \leq \alpha < 0.6$ or $\alpha \geq 0.7\}\}$. Then $(i,j)^*$-GF_γ-$O(X) = \{0_X, 1_X, \{(x,\alpha) : \alpha < 0.2$ or $0.3 < \alpha < 0.6$ or $\alpha > 0.6\}\}$. From the above collection it is obvious that the fuzzy set $\{(x, 0.8)\}$ is a $(i,j)^*$-$gf\gamma$-closed set in X.

2.9. Remark

The notions of $(i,j)^*$-fuzzy γ-open set and (i,j)-fuzzy γ-open set due to Tripathy et. al are completely independent of each other. The fact is demonstrated in the examples given below.

2.10. Example

Suppose a fbts (X, τ_i, τ_j) with $X = \{x\}, \tau_i = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.6)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x, 0.9)\}\}$. Then
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.6)\}, \{(x, 0.9)\}\}$ and
$(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.1)\}, \{(x, 0.4)\}, \{(x, 0.8)\}\}$. Here
(i,j)-$F\gamma$-$O(X) = \{0_X, 1_X, \{(x,\alpha) : \alpha > 0.1\}\}$ and
$(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x,\alpha) : 0.1 < \alpha \leq 0.2$ or $0.4 < \alpha \leq 0.6$ or $\alpha > 0.8\}\}$. Then the fuzzy set $\{(x, 0.3)\}$ is a (i,j)-fuzzy γ-open set but it is not a $(i,j)^*$-fuzzy γ-open set.

2.11. Example

Consider (X, τ_i, τ_j) with
$X = \{x, y\}, \tau_i = \{0_X, 1_X, \{(x, 0.2), (y, 0.3)\}, \{(x, 0.4), (y, 0.4)\}\}$ and
$\tau_j = \{0_X, 1_X, \{(x, 0.6), (y, 0.7)\}\}$. Then

$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x,0.2),(y,0.3)\}, \{(x,0.4),(y,0.4)\}, \{(x,0.6),(y,0.7)\}\}$ and
$(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x,0.8),(y,0.7)\}, \{(x,0.6),(y,0.6)\}, \{(x,0.4),(y,0.3)\}\}$.
Then,
$(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta)\} : \alpha > 0.2, \beta > 0.7\}$. Take a fuzzy set $\mu = \{(x,0.3),(y,0.8)\}$. Obviously, μ is a $(i,j)^*$-fuzzy γ-open set in X. Now (i,j)-$FPO(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta)\} : \alpha \leq 0.2, \beta \leq 0.3$ or $\alpha \leq 0.2, \beta > 0.4$ or $\alpha > 0.2, \beta > 0.3$ or $\alpha > 0.3\}$. The fuzzy set $\delta = \{(x,0.6),(y,0.3)\} \in (i,j)$-$FPO(X)$ but $\mu \wedge \delta = \{(x,0.3),(y,0.3)\} \notin (i,j)$-$FPO(X)$. So μ is not a (i,j)-fuzzy γ-open set in X.

2.12. Theorem

If λ_1 and λ_2 are two $(i,j)^*$-$gf\gamma$-closed set in a fbts X, then their union $\lambda_1 \vee \lambda_2$ is also $(i,j)^*$-$gf\gamma$-closed set in X.

Proof: Consider two $(i,j)^*$-$gf\gamma$-closed sets λ_1 and λ_2 such that $\lambda_1 \vee \lambda_2 \leq \mu$, where μ is a $(i,j)^*$-fuzzy open set. Then, $\lambda_1 \leq \mu, \lambda_2 \leq \mu$ and thus, $(i,j)^*$-$cl_\gamma(\lambda_1) \leq \mu$, $(i,j)^*$-$cl_\gamma(\lambda_2) \leq \mu$. So, $(i,j)^*$-$cl_\gamma(\lambda_1 \vee \lambda_2) = (i,j)^*$-$cl_\gamma(\lambda_1) \vee (i,j)^*$-$cl_\gamma(\lambda_2) \leq \mu$. As a consequence, $\lambda_1 \vee \lambda_2$ is a $(i,j)^*$-$gf\gamma$-closed set in X.

2.13. Remark

However, the intersection of any two $(i,j)^*$-$gf\gamma$-closed sets in a fbts X may not be a $(i,j)^*$-$gf\gamma$-closed set therein.

2.14. Example

Consider a fbts (X, τ_i, τ_j) with $X = \{x,y\}, \tau_i = \{0_X, 1_X, \{(x,0.2),(y,0)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x,0),(y,0.3)\}\}$. Then,
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x,0.2),(y,0.3)\}, \{(x,0),(y,0.3)\}, \{(x,0.),(y,0)\}\}$ and so
$(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x,0.8),(y,0.7)\}, \{(x,1),(y,0.7)\}, \{(x,0.8),(y,1)\}\}$.
Then
$(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta)\} : \alpha \leq 0.2, \beta \leq 0.3$ or $\alpha > 0.8, \beta > 0.7\}$ and
$(i,j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta)\} : \alpha < 0.2, \beta < 0.3$ or $\alpha \geq 0.8, \beta \geq 0.7\}$. Let $\lambda_1 = \{(x,0.8),(y,0.3)\}$ and $\lambda_2 = \{(x,0.2),(y,0.5)\}$. It can be easily verified that λ_1, λ_2 are $(i,j)^*$-$gf\gamma$-closed sets. Again,

$\lambda_1 \wedge \lambda_2 = \{(x, 0.2), (y, 0.3)\} \leq \{(x, 0.2), (y, 0.3)\}$, which is a $(i, j)^*$-fuzzy open set, but $(i, j)^*$-$cl_\gamma(\lambda_1 \wedge \lambda_2) = \{(x, 0.8), (y, 0.7)\}$ is not a fuzzy subset of $\{(x, 0.2), (y, 0.3)\}$. Therefore, $\lambda_1 \wedge \lambda_2$ is not a $(i, j)^*$-$gf\gamma$-closed set in X.

2.15. Remark

The notions of $(i, j)^*$-fuzzy closed set and $(i, j)^*$-fuzzy γ-closed set are no way related. It is exclusively illustrated in the following example.

2.16. Example

Take a fbts (X, τ_i, τ_j) with $X = \{x, y\}, \tau_i = \{0_X, 1_X, \{(x, 0.2), (y, 0.3)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x, 0.4), (y, 0.4)\}, \{(x, 0.6), (y, 0.7)\}\}$. Then, $(i, j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.2), (y, 0.3)\}, \{(x, 0.4), (y, 0.4)\}, \{(x, 0.6), (y, 0.7)\}\}$ and $(i, j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.8), (y, 0.7)\}, \{(x, 0.6), (y, 0.6), \{(x, 0.4), (y, 0.3)\}\}$. Thus $(i, j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x, \alpha), (y, \beta)\} : \alpha > 0.2, \beta > 0.7\}$ and $(i, j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x, \alpha), (y, \beta)\} : \alpha < 0.8, \beta < 0.3\}$. Here the fuzzy set $\lambda_1 = \{(x, 0.8), (y, 0.7)\}$ is a $(i, j)^*$-fuzzy closed set but it is not a $(i, j)^*$-fuzzy γ-closed set. Again, the fuzzy set $\lambda_2 = \{(x, 0.1), (y, 0.1)\}$ is a $(i, j)^*$-fuzzy γ-closed set in X but it is not $(i, j)^*$-fuzzy closed.

2.17. Theorem

Every $(i, j)^*$-fuzzy closed set in a fbts X is a $(i, j)^*$-gf closed set.

Proof: Let λ be a $(i, j)^*$-fuzzy closed set in a fbts X. Consider a fuzzy set λ such that $\lambda \leq \mu$, where μ is a $(i, j)^*$-fuzzy open set. Then, $(i, j)^*$-$cl(\lambda) = \lambda \leq \mu$. Therefore, λ is a $(i, j)^*$-gf closed set in X.

2.18. Remark

A $(i, j)^*$-gf closed set in a fbts X may not be a $(i, j)^*$-fuzzy closed set.

2.19. Example

Consider a fbts (X, τ_i, τ_j) with $X = \{x\}, \tau_i = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x, 0.8)\}, \{(x, 0.9)\}\}$. We have
$(i, j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.)\}, \{(x, 0.9)\}\}$ and so
$(i, j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.1)\}, \{((x, 0.2)\}, \{(x, 0.3)\}, \{(x, 0.6)\}\}$. Then
$(i, j)^*$-$GFC(X) = \{0_X, 1_X, \{(x, \alpha)\} : \alpha \leq 0.3$ or $0.4 < \alpha \leq 0.6$ or $\alpha > 0.9\}$.

Here the fuzzy set $\lambda = \{(x, 0.95)\}$ is a $(i,j)^*$-gf closed set but it is not a $(i,j)^*$-fuzzy closed set.

2.20. Remark

Both the notions of $(i,j)^*$-fuzzy γ-closed set and $(i,j)^*$-gf closed set are independent to each other. It is clarified in the following example.

2.21. Example

Consider a fbts (X, τ_i, τ_j) with $X = \{x\}, \tau_i = \{0_X, 1_X, \{(x, 0.3)\}, \{x, 0.6)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x, 0.8)\}\}$ which gives
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.3)\}, \{(x, 0.6)\}, \{x, 0.8)\}\}$ and accordingly $(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.4)\}, \{(x, 0.7)\}\}$. Thus calculation for $(i,j)^*$-pre-open sets provides that $(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x, \alpha) : 0.2 < \alpha \leq 0.3$ or $0.4 < \alpha \leq 0.6$ or $\alpha > 0.7\}\}$ and $(i,j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x, \alpha) : \alpha < 0.3$ or $0.4 \leq \alpha < 0.6$ or $0.7 \leq \alpha < 0.8\}\}$. Now $(i,j)^*$-$GFC(X) = \{0_X, 1_X, \{(x, \alpha) : \alpha \leq 0.2$ or $0.3 < \alpha \leq 0.4$ or $0.6 < \alpha \leq 0.7$ or $\alpha > 0.8\}\}$. Consider two fuzzy sets $\lambda_1 = \{(x, 0.21)\}$ and $\lambda_2 = \{(x, 0.35)\}$. It is obvious that λ_1 is a $(i,j)^*$-fuzzy γ-closed set but it is not a $(i,j)^*$-gf closed set. On the other hand, λ_2 is a $(i,j)^*$-gf closed set but it is not $(i,j)^*$-fuzzy γ-closed.

2.22. Theorem

Every $(i,j)^*$-fuzzy γ-closed set in a fbts X is a $(i,j)^*$-$gf\gamma$-closed set.

Proof: Let λ be a $(i,j)^*$-fuzzy γ-closed set in a fbts X and $\lambda \leq \mu$, where μ is a $(i,j)^*$-fuzzy γ-open set. Then, $(i,j)^*$-$cl_\gamma(\lambda) = \lambda \leq \mu$. Consequently, λ is a $(i,j)^*$-$gf\gamma$-closed set in X and hence our claim is accordingly proved.

2.23. Remark

A $(i,j)^*$-$gf\gamma$-closed set in a fbts may not be a $(i,j)^*$-fuzzy γ-closed set, as it is verified in the following example.

2.24. Example

Take the fbts given in the example 2.14 and consider the fuzzy set $\mu = \{(x, 0.8), (y, 0.3)\}$, which is a $(i,j)^*$-$gf\gamma$-closed set in X but it is not a $(i,j)^*$-fuzzy γ-closed set.

2.25. Remark

The concept of $(i, j)^*$-fuzzy closed set and $(i, j)^*$-$gf\gamma$-closed set are independent to each other. It is verified in the following two consecutive examples.

2.26. Example

Consider the same fbts from example 2.16. Here the fuzzy set $\lambda_1 = \{(x, 0.4), (y, 0.3)\}$ is a $(i, j)^*$-fuzzy closed set. Now $\lambda_1 \leq \{(x, 0.4), (y, 0.4)\}$, which is a $(i, j)^*$-fuzzy closed set, but $(i, j)^*$-$cl_\gamma(\lambda_1) = 1_X$ which is not a fuzzy subset of $\{(x, 0.4), (y, 0.4)\}$. Therefore λ_1 is not a $(i, j)^*$-$gf\gamma$-closed set.

2.27. Example

Consider a fbts (X, τ_i, τ_j) with $X = \{x, y\}, \tau_i = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x, 0.8)\}\}$. Then
$(i, j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$ and so $(i, j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.3)\}, \{(x, 0.6)\}\}$. Here the fuzzy set $\{(x, 0.9)\}$ is a $(i, j)^*$-$gf\gamma$-closed set but it is not a $(i, j)^*$-fuzzy closed set in X.

2.28. Theorem

If λ is a $(i, j)^*$-$gf\gamma$-closed set in a fbts X and $\lambda \leq \delta \leq (i, j)^*$-$cl_\gamma(\lambda)$, then δ is also a $(i, j)^*$-$gf\gamma$-closed set in X.

Proof: Let $\delta \leq \mu$, where μ is a $(i, j)^*$-fuzzy open set in X. Now, since $\lambda \leq \delta$ and λ is a $(i, j)^*$-$gf\gamma$-closed set in X, so $(i, j)^*$-$cl_\gamma(\lambda) \leq \mu$. But $(i, j)^*$-$cl_\gamma(\delta) \leq (i, j)^*$-$cl_\gamma(\lambda)$, since $\delta \leq (i, j)^*$-$cl_\gamma(\lambda)$. Evidently, $(i, j)^*$-$cl_\gamma(\delta) \leq \mu$ and hence δ is a $(i, j)^*$-$gf\gamma$-closed set in X.

2.29. Remark

Both the ideas of $(i, j)^*$-gf closed set and $(i, j)^*$-$gf\gamma$-closed set are also not related to each other.

2.30. Example

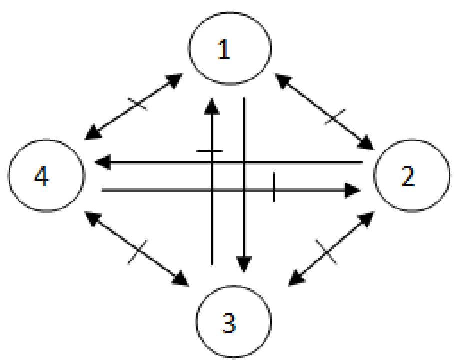
Let us consider the fbts taken in example 2.14. Here $(i, j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x, \alpha), (y, \beta)\} : \alpha \leq 0.2, \beta \leq 0.3 \text{ or } \alpha > 0.8, \beta > 0.7\}$ and so $(i,j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta)\} : \alpha < 0.2, \beta < 0.3$ or $\alpha \geq 0.8, \beta \geq 0.7\}$. Consider $\mu = \{(x,0.1),(y,0.1)\}$, which is clearly a $(i,j)^*$-$gf\gamma$-closed set in X. The fuzzy set $\mu \leq \{(x,0.2),(y,0.3)\}$ is a $(i,j)^*$-fuzzy open set. Now $(i,j)^*$-$cl(\mu)$ is not a fuzzy subset of $\{(x,0.2),(y,0.3)\}$. Hence μ is not a $(i,j)^*$-gf closed set.

2.31. Example

Consider the fbts of example 2.16 and consider the fuzzy set $\mu = \{(x,0.3),(y,0.3)\}$. Then, obviously μ is a $(i,j)^*$-gf closed set. Now $\mu \leq \{(x,0.4),(y,0.4)\}$, which is a $(i,j)^*$-fuzzy open set, but $(i,j)^*$-$cl_\gamma(\mu) = 1_X$, which is not a fuzzy subset of $\{(x,0.4),(y,0.4)\}$. Therefore, μ is not a $(i,j)^*$-$gf\gamma$-closed set.

2.32. Remark

The interrelationships between the above discussed different types of sets is pictured in the following diagram:
Suppose (1) $(i,j)^*$-fuzzy closed set $(i,j)^*$-fuzzy γ-closed set.
(3) $(i,j)^*$-gf closed set (4) $(i,j)^*$-$gf\gamma$-closed set.



We now define another type of generalized fuzzy closed set namely $(i,j)^*$-γ-gf closed set via $(i,j)^*$-fuzzy γ-open set in a different approach to show the kind of conventional result that every $(i,j)^*$-fuzzy closed set is a $(i,j)^*$-γ-gf closed set.

2.33. Definition

A fuzzy set λ in a fbts (X, τ_i, τ_j) is said to be a $(i,j)^*$-γ-generalized fuzzy closed (in short, $(i,j)^*$-γ-gf closed) set if $(i,j)^*$-$cl(\lambda) \leq \mu$, whenever $\lambda \leq \mu$, for any $(i,j)^*$-fuzzy γ-open set μ in X. The collection of all $(i,j)^*$-γ-gf closed sets is denoted by $(i,j)^*$-γ-$GFC(X)$.

2.34. Theorem

Every $(i,j)^*$-fuzzy closed set is a $(i,j)^*$-γ-gf closed set.

Proof: Let λ be a $(i,j)^*$-fuzzy closed set in a fbts (X, τ_i, τ_j) and $\lambda \leq \mu$, where μ is a $(i,j)^*$-fuzzy γ-open set in X. Now $(i,j)^*$-$cl(\lambda) = \lambda \leq \mu$. Therefore, λ is a $(i,j)^*$-γ-gf closed set.

2.35. Remark

A $(i,j)^*$-fuzzy γ-closed set may not be a $(i,j)^*$-γ-gf closed set.

2.36. Example

Take a fbts (X, τ_i, τ_j) with $X = \{x\}, \tau_i = \{0_X, 1_X, \{(x, 0.3)\}, \{(x, 0.4)\}\}$ and $\tau_j = \{0_X, 1_X, \{(x, 0.7)\}, \{(x, 0.8)\}\}$. Then,
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.3)\}, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$ and $(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.3)\}, \{(x, 0.6)\}, \{(x, 0.7)\}\}$. Thus $(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x, \alpha) : 0.2 < \alpha \leq 0.4 \text{ or } \alpha > 0.6\}\}$ and $(i,j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x, \alpha) : \alpha < 0.4 \text{ or } 0.6 \leq \alpha < 0.8\}\}$. Here the fuzzy set $\{(x, 0.35)\}$ is a $(i,j)^*$-fuzzy γ-closed set but it is not a $(i,j)^*$-γ-gf closed set in X.

Detailed study on $(i,j)^*$-γ-gf closed set is out of the scope of this paper.

2.37. Definition

A fuzzy set λ in a fbts (X, τ_i, τ_j) is called a $(i,j)^*$-generalized fuzzy γ-open set (in short, $(i,j)^*$-$gf\gamma$-open set) iff its complement is a $(i,j)^*$-$gf\gamma$-closed set. The family of all $(i,j)^*$-$gf\gamma$-open sets is $(i,j)^*$-GF_γ-$O(X)$.

We state the following result without proof, which can be established using standard technique.

2.38. Theorem

The intersection of any two $(i,j)^*$-$gf\gamma$-open sets is again a $(i,j)^*$-$gf\gamma$-open set.

2.39. Remark

However, union of two $(i,j)^*$-$gf\gamma$-open sets is not necessarily a $(i,j)^*$-$gf\gamma$-open set.

2.40. Example

Consider example 2.14. It is found that $\lambda_1 \vee \lambda_2 = \{(x, 0.8), (y, 0.5)\}$, which is not a $(i,j)^*$-$gf\gamma$-open set.

2.41. Theorem

In a fbts X, the following statements are equivalent:

(1) A fuzzy set λ in X is $(i,j)^*$-$gf\gamma$-open.

(2) For any $(i,j)^*$-fuzzy closed set δ in X with $\delta \leq \lambda$, $\delta \leq (i,j)^*$-$int_\gamma(\lambda)$.

Proof: Suppose λ is a $(i,j)^*$-$gf\gamma$-open set in X and δ is a $(i,j)^*$-fuzzy closed set such that $\delta \leq \lambda$. Then, $1_X - \lambda \leq 1_X - \delta$, where $1_X - \delta$ is a $(i,j)^*$-fuzzy open set. Now, $1_X - (i,j)^*$-$int_\gamma(\lambda) = (i,j)^*$-$cl_\gamma(1_X - \lambda) \leq 1_X - \delta$, which implies, $\delta \leq (i,j)^*$-$int_\gamma(\lambda)$.

Conversely, let λ be a fuzzy set in X such that $\delta \leq (i,j)^*$-$int_\gamma(\lambda)$, whenever δ is $(i,j)^*$-fuzzy closed and $\delta \leq \lambda$. If $1_X - \lambda \leq \mu$, where μ is $(i,j)^*$-fuzzy open set in X, then, $1_X - \lambda \leq \mu$ which means, $1_X - \mu \leq \lambda$. Hence by our assumption, $1_X - \mu \leq (i,j)^*$-$int_\gamma(\lambda)$. Thus, $1_X - (i,j)^*$-$int_\gamma(\lambda) \leq \mu$. Then, $(i,j)^*$-$cl_\gamma(1_X - \lambda) \leq \mu$, and hence $1_X - \lambda$ is $(i,j)^*$-$gf\gamma$-closed. Therefore, λ is a $(i,j)^*$-$gf\gamma$-open set in X.

We conclude this section with the following result:

2.42. Theorem

If $(i,j)^*$-$int_\gamma(\lambda) \leq \mu \leq \lambda$ and λ is a $(i,j)^*$-$gf\gamma$-open set, then μ is a $(i,j)^*$-$gf\gamma$-open set.

Proof: Here, $(i,j)^*$-$int_\gamma(\lambda) \leq \mu \leq \lambda$ implies $1_X - \lambda \leq 1_X - \mu \leq 1_X - (i,j)^*$-$int_\gamma(\lambda)$. Then, $1_X - \lambda \leq 1_X - \mu \leq (i,j)^*$-$cl_\gamma(1_X - \lambda)$. So, $1_X - \mu$ is a $(i,j)^*$-$gf\gamma$-closed set and thus μ is a $(i,j)^*$-$gf\gamma$-open set therein.

3. $(i,j)^*$-Generalized Fuzzy γ-Continuous Functions and $(i,j)^*$-Generalized Fuzzy γ-Irresolute Functions

The main goal of this section is to study various types of continuities between two fbts. We initiate the notions of $(i,j)^*$-fuzzy continuity, $(i,j)^*$-fuzzy γ-continuity, $(i,j)^*$-generalized fuzzy continuity and $(i,j)^*$-generalized fuzzy γ-continuity. We examine the interrelationships between these functions. Moreover, we characterize irresoluteness via $(i,j)^*$-fuzzy γ-open sets in the same environment.

3.1. Definition

Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function from a fbts X into another fbts Y. Then f is called a $(i,j)^*$-fuzzy continuous function if the inverse image of every $(i,j)^*$-fuzzy open set in Y is a $(i,j)^*$-fuzzy open set in X.

3.2. Example

Suppose two fbts (X, τ_i, τ_j) and (Y, σ_i, σ_j) with
$X = \{x\}, Y = \{y\}, \tau_i = \{0_X, 1_X, \{(x, 0.1)\}, \{(x, 0.7)\}\}, \tau_j = \{0_X, 1_X, \{(x, 0.8)\}\}, \sigma_i = \{0_Y, 1_Y, \{(y, 0.8)\}\}$ and $\sigma_j = \{0_Y, 1_Y\}$. Define a function
$f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ such that $f(x) = y$. Here
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.1)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$ and $(i,j)^*$-$FO(Y) = \{0_Y, 1_Y, \{(y, 0.8)\}\}$. Here inverse of every $(i,j)^*$-fuzzy open set in Y is a

$(i,j)^*$-fuzzy open set in X. Therefore f is a $(i,j)^*$-fuzzy continuous function.

3.3. Definition

Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be a function from a fbts X to another fbts Y. Then f is called a $(i,j)^*$-generalized fuzzy continuous (in short, $(i,j)^*$-gf continuous) function if the inverse image of every $(i,j)^*$-fuzzy closed set in Y is a $(i,j)^*$-gf closed set in X.

3.4. Example

Take fbts (X,τ_i,τ_j) and (Y,σ_i,σ_j) with
$X=\{x\}, Y=\{y\}, \tau_i=\{0_X,1_X,\{(x,0.1)\},\{(x,0.6)\}\}, \tau_j=\{0_X,1_X,\{(x,0.7)\}\}, \sigma_i=\{0_Y,1_Y,\{(y,0.8)\}\}$ and $\sigma_j=\{0_Y,1_Y\}$. Define a function
$f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ such that $f(x)=y$. Here
$(i,j)^*$-$FO(X)=\{0_X,1_X,\{(x,0.1)\},\{(x,0.6)\},\{(x,0.7)\}\}$ and $(i,j)^*$-$FO(Y)=\{0_Y,1_Y,\{(y,0.8)\}\}$. So $(i,j)^*$-$FC(X)=\{0_X,1_X,\{(x,0.3)\},\{(x,0.4)\},\{(x,0.9)\}\}$ and
$(i,j)^*$-$FC(Y)=\{0_Y,1_Y,\{(y,0.2)\}\}$. Now $(i,j)^*$-$GFC(X)=\{0_X,1_X,\{(x,\alpha):0.1<\alpha\le 0.4$ or $\alpha>0.7\}\}$. It is clear from the above that the inverse image of every $(i,j)^*$-fuzzy closed set in Y is a $(i,j)^*$-gf closed set in X. Therefore f is a $(i,j)^*$-gf continuous function.

3.5. Definition

A function $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ is called $(i,j)^*$-generalized fuzzy γ-continuous (in short, $(i,j)^*$-$gf\gamma$-continuous) if the inverse image of every $(i,j)^*$-fuzzy closed set in Y is a $(i,j)^*$-$gf\gamma$-closed set in X.

3.6. Example

Consider two fbts (X,τ_i,τ_j) and (Y,σ_i,σ_j) with
$X=\{x\}, Y=\{y\}, \tau_i=\{0_X,1_X,\{(x,0.1)\},\{(x,0.4)\}\}, \tau_j=\{0_X,1_X,\{(x,0.6)\}\}, \sigma_i=\{0_Y,1_Y,\{(y,0.8)\}\}$ and $\sigma_j=\{0_Y,1_Y\}$. Define a function
$f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ such that $f(x)=y$. Here
$(i,j)^*$-$FO(X)=\{0_X,1_X,\{(x,0.1)\},\{(x,0.4)\},\{(x,0.6)\}\}$ and $(i,j)^*$-$FO(Y)=\{0_Y,1_Y,\{(y,0.8)\}\}$. So $(i,j)^*$- $FC(X)=\{0_X,1_X,\{(x,0.4)\},\{(x,0.6)\},\{(x,0.9)\}\}$ and $(i,j)^*$-$FC(Y)=\{0_Y,1_Y,\{(y,0.2)\}\}$. Now $(i,j)^*$-F_γ-$0(X)=\{0_X,1_X,\{(x,\alpha):\alpha\le 0.6$ or $\alpha>0.9\}\}$ and so $(i,j)^*$-F_γ-$C(X)=\{0_X,1_X,\{(x,\alpha):\alpha<0.1$ or $\alpha\ge 0.4\}\}$. Then $(i,j)^*$-GF_γ-$C(X)=\{0_X,1_X,\{(x,\alpha):\alpha<0.1$ or

$0.1 < \alpha\}\}$. Now from the above we see that that the inverse image of every $(i,j)^*$-fuzzy closed set in Y is a $(i,j)^*$-$gf\gamma$-closed set in X. Therefore f is a $(i,j)^*$-$gf\gamma$-continuous function.

3.7. Proposition

A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is $(i,j)^*$-fuzzy continuous if the inverse image of every $(i,j)^*$-fuzzy closed set in Y is a $(i,j)^*$-fuzzy closed set in X.

Proof: It is straightforward from the definition of $(i,j)^*$-fuzzy continuity and hence ignored.

3.8. Theorem

Every $(i,j)^*$-fuzzy continuous function is necessarily a $(i,j)^*$-gf continuous function itself.

Proof: Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a $(i,j)^*$-fuzzy continuous function and μ be any $(i,j)^*$-fuzzy closed set in Y. Then $f^{-1}(\mu)$ is $(i,j)^*$-fuzzy closed in X and so it is a $(i,j)^*$-gf closed set in X and consequently f is a $(i,j)^*$-gf continuous function.

3.9. Remark

However, converse of this above theorem is not necessarily true in general. It can be verified in the following example.

3.10. Example

Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two fbts with $X = \{a, b\}, Y = \{c, d\}, \tau_i = \{0_X, 1_X, \{(a, 0.2), (b, 0.3)\}\}, \tau_j = \{0_X, 1_X, \{(a, 0.3), (b, 0.4)\}\}, \sigma_i = \{0_Y, 1_Y, \{(c, 0.8), (d, 0.7)\}\}$ and $\sigma_j = \{0_Y, 1_Y, \{(c, 0.1), (d, 0.2)\}\}$. Define a function $f : X \to Y$ in such a way that $f(a) = c, f(b) = d$. Now $(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(a, 0.2), (b, 0.3)\}, \{(a, 0.3), (b, 0.4)\}\}$ and $(i,j)^*$-$FO(Y) = \{0_Y, 1_Y, \{(c, 0.8), (d, 0.7)\}, \{(c, 0.1), (d, 0.2)\}\}$. So $(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(a, 0.8), (b, 0.7)\}, \{(a, 0.7), (b, 0.6)\}\}$ and thus $(i,j)^*$-$GFC(X) = \{0_X, 1_X, \{(x, \alpha), (y, \beta)\} : \alpha \leq 0.3, \beta \leq 0.4 \text{ or } \alpha > 0.8, \beta > 0.7\}$. Clearly f is a $(i,j)^*$-gf continuous function. But $f^{-1}\{(c, 0.9), (d, 0.8)\} = \{(a, 0.9), (b, 0.8)\} \notin (i,j)^*$-$FC(X)$. Therefore, f is not a $(i,j)^*$-fuzzy continuous function.

3.11. Remark

Both the notions of $(i,j)^*$-$gf\gamma$-continuous function and $(i,j)^*$-gf continuous function are independent of each other. This is discussed in details in the following two examples.

3.12. Example

Consider a fbts (X, τ_i, τ_j) with $X = \{x\}$ and $\tau_i = \{0_X, 1_X, \{(x, 0.1)\}, \{(x, 0.3)\}\}$, $\tau_j = \{0_X, 1_X, \{(x, 0.7)\}, \{(x, 0.8)\}\}$. Then,
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.1)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$. Thus, $(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.9)\}, \{(x, 0.3)\}, \{(x, 0.2)\}\}$. Let (Y, σ_i, σ_j) be another fbts with $Y = \{y\}, \sigma_i = \{0_Y, 1_Y, \{(y, 0.5)\}\}, \sigma_j = \{0_Y, 1_Y\}$. Consider a function
$f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ such that $f(x) = y$. Clearly, f is a $(i,j)^*$-$gf\gamma$-continuous function. One can easily verify that $f^{-1}\{(y, 0.5)\} = \{(x, 0.5)\} \notin (i,j)^*$-$GFC(X)$. Then f is not a $(i,j)^*$-gf continuous function.

3.13. Example

Consider two fbts $(X, \tau_i, \tau_j), (Y, \sigma_i, \sigma_j)$ with
$X = Y = \{a, b\}, \tau_i = \{0_X, 1_X, \{(a, 0.2), b, 0)\}\}, \tau_j = \{0_X, 1_X, \{(a, 0), (b, 0.3)\}\}, \sigma_i = \{0_Y, 1_Y, \{(a, 0.29), (b, 0.19)\}\}, \sigma_j = \{0_Y, 1_Y\}$. Define a function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ such that $f(a) = b, f(b) = a$. Here f is a $(i,j)^*$-gf continuous function. The inverse of the $(i,j)^*$ fuzzy closed set $\{(a, 0.71), (b, 0.81)\}$ in Y is $\{(a, 0.81), (b, 0.71)\}$, which is not a $(i,j)^*$-$gf\gamma$-closed set. Therefore, f is not a $(i,j)^*$-$gf\gamma$-continuous mapping.

3.14. Remark

The relation between $(i,j)^*$-fuzzy continuous function and $(i,j)^*$-$gf\gamma$-continuous function is non-linear in nature, that is they are no way related.

3.15. Example

Consider two fbts (X, τ_i, τ_j) and (Y, σ_i, σ_j) with
$X = \{x\}, Y = \{y\}, \tau_i = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}, \tau_j = \{0_X, 1_X\}, \sigma_i = \{0_Y, 1_Y\}$ and $\sigma_j = \{0_Y, 1_Y, \{(y, 0.5)\}\}$. Also suppose that $f : X \to Y$ be a function such that $f(x) = y$. Now $(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$ and so
$(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.3)\}, \{(x, 0.6)\}$. So $(i,j)^*$-F_γ-$O(X) =$

$\{0_X, 1_X, \{(x,\alpha) : 0.3 < \alpha \leq 0.4$ or $\alpha > 0.6\}\}, \{(x,\alpha) : \alpha < 0.4$ or $0.6 \leq \alpha < 0.7\}\}$. Then obviously f is a $(i,j)^*$-$gf\gamma$-continuous function. Again, $\{(y,0.5)\}$ is a $(i,j)^*$-fuzzy closed set in Y but $f^{-1}\{(y,0.5)\} = \{(x,0.5)\} \notin (i,j)^*$-$FC(X)$. Therefore, f is not a $(i,j)^*$-fuzzy continuous function.

3.16. Example

Take two fbts (X, τ_i, τ_j) and (Y, σ_i, σ_j) with
$X = \{x,y\}, Y = \{a,b\}, \tau_i = \{0_X, 1_X, \{(x,0.2),(y,0.3)\}, \{(x,0.4),(y,0.4)\}\}, \tau_j = \{0_X, 1_X, \{(x,0.6),(y,0.7)\}\}, \sigma_i = \{0_Y, 1_Y, \{(a,0.6),(b,0.7)\}\}$ and $\sigma_j = \{0_Y, 1_Y\}$.
Now define a function $f : X \to Y$ such that
$f(x) = a, f(y) = b$. Then
$(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(x,0.2),(y,0.3)\}, \{(x,0.4),(y,0.4)\}, \{((x,0.6,(y,0.7)\}\}$
and so
$(i,j)^*$-$FC(X) = \{0_X, 1_X, \{(x,0.8),(y,0.7)\}, \{(x,0.6),(y,0.6)\}, \{(x,0.4),(y,0.3)\}\}$.
Thus $(i,j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta)\} : \alpha > 0.2, \beta > 0.7\}$ and $(i,j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x,\alpha),(y,\beta) : \alpha < 0.8, \beta < 0.3\}\}$. Clearly f is a $(i,j)^*$-fuzzy continuous function. Now $\{(a,0.4),(b,0.3)\}$ is $(i,j)^*$-fuzzy closed in Y and $f^{-1}(\{(a,0.4),(b,0.3)\}) = \{(x,0.4),(y,0.3)\} \leq \{(x,0.4),(y,0.4)\}$, which is a $(i,j)^*$-fuzzy open set, but $(i,j)^*$-$cl_\gamma\{(x,0.4),(y,0.3)\} = 1_X$, which is not a fuzzy subset of $\{(x,0.4),(y,0.4)\}$. So $f^{-1}\{(a,0.4),(b,0.3)\}$ is not a $(i,j)^*$-$gf\gamma$-closed set in X and consequently f is not a $(i,j)^*$-$gf\gamma$-continuous function.

Following the definition of pairwise γ-continuity due to Tripathy and Debnath (2013), we introduce $(i,j)^*$-fuzzy γ-continuity between two fbts and study some related results.

3.17. Definition

Let $f : X \to Y$ be a function from a fbts X to another fbts Y. Then f is called a $(i,j)^*$-fuzzy γ-continuous if the inverse image of every $(i,j)^*$-fuzzy open set in Y is a $(i,j)^*$-fuzzy γ-open set in X.

3.18. Remark

The concepts of $(i,j)^*$-fuzzy continuity and $(i,j)^*$-fuzzy γ-continuity are independent of each other. It is demonstrated in the following example.

3.19. Example

Consider the same fbts (X, τ_i, τ_j) of example 2.14 and also two fbts (Y, σ_i, σ_j) and (Z, ρ_i, ρ_j) with
$X = \{x, y\}, Y = \{a, b\}, Z = \{p, q\}, \tau_i = \{0_X, 1_X, \{(x, 0.2), (y, 0.3)\}\}$,
$\tau_j = \{0_X, 1_X, \{(x, 0.4), (y, 0.4)\}, \{(x, 0.6), (y, 0.7)\}\}, \sigma_i = \{0_Y, 1_Y\}, \sigma_j = \{0_Y, 1_Y, \{(a, 0.2), (b, 0.3)\}\}, \rho_i = \{0_Z, 1_Z\}, \rho_j = \{0_Z, 1_Z, \{(p, 0.9), (q, 0.9)\}\}$.
Suppose two functions $f : X \to Y$ and $g : X \to Z$ such that $f(x) = y, g(x) = z$. Now, it is obvious that f is a $(i, j)^*$-fuzzy γ-continuous function and g is a $(i, j)^*$-fuzzy continuous function. Now $\{(a, 0.8), (b, 0.3)\}$ is a $(i, j)^*$-fuzzy closed set in Y and $f^{-1}\{(a, 0.8), (b, 0.3)\} = \{(x, 0.8), (y, 0.3)\}$, which is not a $(i, j)^*$-fuzzy closed set in X. Thus f is not a $(i, j)^*$-fuzzy continuous function. Further, the fuzzy set $\lambda = \{(p, 0.1), (q, 0.1)\}$ is a $(i, j)^*$-fuzzy closed set in Z but $g^{-1}(\lambda) = \{(x, 0.1), (y, 0.1)\}$, which is not a $(i, j)^*$-fuzzy γ-closed set. So g is not a $(i, j)^*$-fuzzy γ-continuous function.

3.20. Theorem

Every $(i, j)^*$-fuzzy γ-continuous function is necessarily a $(i, j)^*$-$gf\gamma$-continuous function.

Proof: Let $f : X \to Y$ be a $(i, j)^*$-fuzzy γ-continuous function and μ be a $(i, j)^*$-fuzzy closed set in Y. Then $f^{-1}(\mu)$ is $(i, j)^*$-fuzzy γ-closed in X and so it is a $(i, j)^*$-$gf\gamma$-closed set in X. Therefore, f is a $(i, j)^*$-$gf\gamma$-continuous function.

3.21. Remark

We claim that the converse of the above theorem is not necessarily true in general and we justify the same by employing the following example.

3.22. Example

Let (X, τ_i, τ_j) and (Y, σ_i, σ_j) be two fbts such that $X = \{x\}, Y = \{y\}, \tau_i = \{0_X, 1_X, \{(x, 0.4)\}$,
$\{(x, 0.7)\}, \{(x, 0.8)\}\}, \tau_j = \{0_X, 1_X\}, \sigma_i = \{0_Y, 1_Y\}$ and $\sigma_j = \{0_Y, 1_Y, \{(y, 0.1)\}\}$.
Now $(i, j)^*$-$FO(X) = \{0_X, 1_X, \{(x, 0.4)\}, \{(x, 0.7)\}, \{(x, 0.8)\}\}$ and so $(i, j)^*$-$FC(X) = \{0_X, 1_X, \{(x, 0.0.2)\}, \{(x, 0.3)\}, \{(x, 0.6)\}\}$. Then $(i, j)^*$-F_γ-$O(X) = \{0_X, 1_X, \{(x, \alpha) : 0.3 < \alpha \leq 0.4 \text{ or } \alpha > 0.6\}\}$ and $(i, j)^*$-F_γ-$C(X) = \{0_X, 1_X, \{(x, \alpha) : \alpha < 0.4 \text{ or } 0.6 \leq \alpha < 0.7\}\}$. Consider a function

$f : X \to Y$ such that $f(x) = y$. Here it is obvious that f is a $(i,j)^*$-$gf\gamma$-continuous function. Clearly, $\lambda = \{(y, 0.9)\}$ is a $(i,j)^*$-fuzzy closed set in Y and $f^{-1}(\lambda) = \{(x, 0.9)\}$, which is not a $(i,j)^*$- fuzzy γ-closed set in X. This shows that f is not a $(i,j)^*$-fuzzy γ-continuous function.

3.23. Remark

A $(i,j)^*$-gf continuous function between two fbts may not be a $(i,j)^*$-fuzzy γ-continuous function and vice versa. The following example will establish our claim.

3.24. Example

Consider the fbts (X, τ_i, τ_j) considered in example 3.22 and also take two other fbts (Y, σ_i, σ_j) and (Z, ρ_i, ρ_j) with $Y = \{y\}, Z = \{z\}, \sigma_i = \{0_Y, 1_Y\}, \sigma_j = \{0_Y, 1_Y, \{(y, 0.35)\}\}, \rho_i = \{0_Z, 1_Z, \{(z, 0.5)\}\}$ and $\rho_j = \{0_Z, 1_Z\}$. Define two functions $f : X \to Y$ and $g : X \to Z$ such that $f(x) = y, g(x) = z$. It is obvious that f is a $(i,j)^*$-fuzzy γ-continuous function and g is a $(i,j)^*$-gf continuous function. Here $\lambda = \{(y, 0.65)\}$ is $(i,j)^*$-fuzzy closed in Y but $f^{-1}(\lambda) = \{(x, 0.65)\}$ is not a $(i,j)^*$-gf closed set in X. Thus f is not a $(i,j)^*$-gf continuous function. On the other hand $\nu = \{(z, 0.5)\}$ is a $(i,j)^*$-fuzzy closed set in Z but $g^{-1}(\nu) = \{(x, 0.5)\}$ is not a $(i,j)^*$-fuzzy γ-closed set in X. Therefore, g is not a $(i,j)^*$-fuzzy γ-continuous function.

3.25. Theorem

Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be a function from a fbts X into another fbts Y. Then the following statements are equivalent:

(1) f is a $(i,j)^*$-$gf\gamma$-continuous function.

(2) The inverse image of every $(i,j)^*$-fuzzy open set in Y is $(i,j)^*$-$gf\gamma$-open in X.

Proof: The proof is easy and hence ignored.

3.26. Theorem

Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ and $g : (Y, \sigma_i, \sigma_j) \to (Z, \rho_i, \rho_j)$ be two functions. If f is $(i,j)^*$-$gf\gamma$-continuous and g is $(i,j)^*$-fuzzy continuous then the composition $g \circ f$ is a $(i,j)^*$-$gf\gamma$-continuous function.

Proof: Let η be a $(i,j)^*$-fuzzy closed set in Z. Then, $\mu = g^{-1}(\eta)$ is a $(i,j)^*$-fuzzy closed set in Y, since g is a $(i,j)^*$-fuzzy continuous function. Again from the assumption, f is a $(i,j)^*$-$gf\gamma$-continuous function, accordingly $\lambda = f^{-1}(\mu)$ is a $(i,j)^*$-$gf\gamma$-closed set in X. Now $(g \circ f)^{-1}(\eta) = f^{-1}(g^{-1}(\eta)) = f^{-1}(\mu) = \lambda$, which is a $(i,j)^*$-$gf\gamma$-closed set in X. Hence $g \circ f$ is a $(i,j)^*$-$gf\gamma$-continuous function.

3.27. Remark

The composition of two $(i,j)^*$-$gf\gamma$-continuous functions may not be a $(i,j)^*$-$gf\gamma$-continuous function.

3.28. Example

Suppose $(X, \tau_i, \tau_j), (Y, \sigma_i, \sigma_j)$ and (Z, ρ_i, ρ_j) be three fbts such that $X = \{x\}, Y = \{y\}, Z = \{z\}, \tau_i = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.4)\}\}, \tau_j = \{0_X, 1_X, \{(x, 0.7)\}\}, \sigma_i = \{0_Y, 1_Y, \{(y, 0.7)\}\}, \sigma_j = \{0_Y, 1_Y, \{(y, 0.1)\}\}, \rho_i = \{0_Z, 1_Z, \{(z, 0.6)\}\}, \rho_j = \{0_Z, 1_Z, \{(z, 0.8)\}\}$. Consider two fuzzy mappings $f : X \to Y$ and $g : Y \to Z$ defined by $f(x) = y$ and $g(y) = z$. Evidently, both f and g are two $(i,j)^*$-$gf\gamma$-continuous functions. Now, $(g \circ f)^{-1}\{(z, 0.4)\} = f^{-1}g^{-1}\{(z, 0.4)\} = f^{-1}\{(y, 0.4)\} = \{(x, 0.4)\}$. Here, $\{(z, 0.4)\}$ is a $(i,j)^*$ -fuzzy closed set in Z, but $(g \circ f)^{-1}\{(z, 0.4)\} = \{(x, 0.4)\}$ is not a $(i,j)^*$-$gf\gamma$-closed set in X. Consequently, $g \circ f$ is not a $(i,j)^*$-$gf\gamma$-continuous function.

3.29. Definition

A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is called a $(i,j)^*$-generalized fuzzy irresolute (in short, $(i,j)^*$-gf-irresolute) function if $f^{-1}(\lambda)$ is a $(i,j)^*$-gf open (resp. $(i,j)^*$-gf closed) set in X for every $(i,j)^*$-gf open (resp. $(i,j)^*$-gf closed) set λ in Y.

3.30. Definition

A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is called a $(i,j)^*$-fuzzy γ-irresolute function if $f^{-1}(\lambda)$ is a $(i,j)^*$-fuzzy γ-open (resp. $(i,j)^*$-fuzzy γ-closed) set in X for every $(i,j)^*$-fuzzy γ-open (resp. $(i,j)^*$-fuzzy γ-closed) set λ in Y.

3.31. Definition

A function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ is called a $(i,j)^*$-generalized fuzzy γ-irresolute (in short, $(i,j)^*$-$gf\gamma$-irresolute) function if $f^{-1}(\lambda)$ is a $(i,j)^*$-

$gf\gamma$-closed set in X for every $(i,j)^*$-$gf\gamma$-closed set λ in Y.

3.32. Remark

Both the notions of $(i,j)^*$-$gf\gamma$-irresolute functions and $(i,j)^*$-$gf\gamma$-continuous functions are independent of each other. It can be verified through the following examples.

3.33. Example

Consider two fbts $(X, \tau_i, \tau_j), (Y, \sigma_i, \sigma_j)$ with $X = \{x\}, Y = \{y\}, \tau_i = \{0_X, 1_X\}, \tau_j = \{0_X, 1_X, \{(x, 0.2)\}\}, \sigma_i = \{0_Y, 1_Y, \{(y, 0.2)\}, \{(y, 0.75)\}\}, \sigma_j = \{0_Y, 1_Y, \{(y, 0.8)\}\}$. Consider a function $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ such that $f(x) = y$. Here f is a $(i,j)^*$-$gf\gamma$-irresolute function. Now, $\lambda = \{(y, 0.2)\}$ is a $(i,j)^*$-fuzzy closed set in Y but the inverse of this set is $\{(x, 0.2)\}$, which is not a $(i,j)^*$-$gf\gamma$-closed set in X. Therefore, f is not a $(i,j)^*$-$gf\gamma$-continuous function.

3.34. Example

We consider two fbts $(X, \tau_i, \tau_j), (Y, \sigma_i, \sigma_j)$ with $X = \{x\}, Y = \{y\}, \tau_i = \{0_X, 1_X, \{(x, 0.2)\}, \{(x, 0.4)\}\}, \tau_j = \{0_X, 1_X, \{(x, 0.8)\}\}, \sigma_i = \{0_Y, 1_Y, \{(y, 0.2)\}, \{(y, 0.5)\}, \{(y, 0.75)\}\}, \sigma_j = \{0_Y, 1_Y, \{(y, 0.8)\}\}$. Let $f : (X, \tau_i, \tau_j) \to (Y, \sigma_i, \sigma_j)$ be defined by $f(x) = y$, which is a $(i,j)^*$-$gf\gamma$-continuous function. Now, the fuzzy set $\{(y, 0.4)\}$ is $(i,j)^*$-$gf\gamma$-closed in Y but the inverse of this set is $\{(x, 0.4)\}$, which is not a $(i,j)^*$-$gf\gamma$-closed set in X. Hence f is not a $(i,j)^*$-$gf\gamma$-irresolute function.

3.35. Remark

Both the notions of $(i,j)^*$-$gf\gamma$-irresolute function and $(i,j)^*$-gf irresolute function are independent of each other.

3.36. Example

In the example 3.33 the function f is a $(i,j)^*$-$gf\gamma$-irresolute function. Also, $\{(y, 0.1)\}$ is a $(i,j)^*$-gf closed set in Y. But the inverse of this set is $\{(x, 0.1)\}$, which is not $(i,j)^*$-gf closed in X. So, f is not a $(i,j)^*$-gf irresolute function.

3.37. Example

Let $(X,\tau_i,\tau_j),(Y,\sigma_i,\sigma_j)$ be any two fbts with $X=\{x\},Y=\{y\},\tau_i=\{0_X,1_X,\{(x,0.2)\},\{(x,0.6)\}\},\tau_j=\{0_X,1_X,\{(x,0.8)\}\},\sigma_i=\{0_Y,1_Y,\{(y,0.2)\}\},\sigma_j=\{0_Y,1_Y,\{(y,0.7)\}\}$. Let $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$ be a function such that $f(x)=y$. Here f is a $(i,j)^*$-gf irresolute function and $\{(y,0.6)\}$ is a $(i,j)^*$-$gf\gamma$-closed set in Y. But $f^{-1}\{(y,0.6)\}=\{(x,0.6)\}$, which is not a $(i,j)^*$-$gf\gamma$-closed set in X. Hence f is not a $(i,j)^*$-$gf\gamma$-irresolute function.

We conclude this section by stating the following two results without proof.

3.38. Theorem

For any function $f:(X,\tau_i,\tau_j)\to(Y,\sigma_i,\sigma_j)$, the following statements are equivalent.

(1) f is a $(i,j)^*$-$gf\gamma$-irresolute function.

(2) The inverse image of every $(i,j)^*$-$gf\gamma$-open set in Y is a $(i,j)^*$-$gf\gamma$-open set.

3.39. Theorem

Let $f:X\to Y$ and $g:Y\to Z$ be two functions. If f and g are both $(i,j)^*$-$gf\gamma$-irresolute functions, then the composition $g\circ f$ is also a $(i,j)^*$-$gf\gamma$-irresolute function.

4. Conclusion

In the literature, various research work have already been done on different kind of generalized fuzzy closed sets, viz. regular generalized fuzzy closed set (rgf-closed set) (Park and Park, 2003), generalized regular fuzzy closed set (grf-closed set) (Bhattachrya and Chakraborty, 2015), θ-generalized fuzzy closed set (El- Shafei and Zakari, 2006) etc. in a fuzzy topological space. In this paper, we initiated the notion of $(i,j)^*$-fuzzy open set, $(i,j)^*$-fuzzy γ-open set in a fuzzy bitopological space to show that the relation between these two sets is non-linear in nature that is, they are completely independent of each other. At this juncture we introduced $(i,j)^*$-generalized fuzzy closed set and extended this study via $(i,j)^*$-fuzzy γ-open set ($(i,j)^*$-generalized fuzzy γ-closed set). In our observation it is found that though every $(i,j)^*$-fuzzy γ-closed set is a $(i,j)^*$-generalized fuzzy γ-closed set, but a $(i,j)^*$-fuzzy closed set may not be a $(i,j)^*$-generalized fuzzy γ-closed set. Nonetheless we have defined a new type of generalized fuzzy closed set called $(i,j)^*$-γ-generalized fuzzy closed set, in a different approach by applying $(i,j)^*$-closure operator and $(i,j)^*$ fuzzy γ-open set in definition 2.33 to show the conventional relation found in the literature, since the inception of generalized closed set. One can study considering that idea in the same environment also. There is another scope to study generalized fuzzy closed set by using both $(i,j)^*$-γ-closure operator and $(i,j)^*$-fuzzy γ-open set simultaneously.

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