POSITIVE SOLUTIONS FOR THE 1-DIMENSIONAL GENERALIZED p -LAPLACIAN INVOLVING A REAL PARAMETER *

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Abstract

In this paper we study existence and multiplicity of positive solutions of the Dirichlet problem

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1. Introduction

In this paper we study positive solutions of the Dirichlet problem

$$(P)_\lambda \quad \begin{cases} -(\varphi(u'))' = \lambda f(u) & \text{in } (0, 1) \\ u(0) = u(1) = 0, \end{cases}$$

where $\lambda \in \mathbb{R}$, $\varphi : \mathbb{R} \rightarrow \mathbb{R}$ is an increasing odd homeomorphism and $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous. By a solution of problem $(P)_\lambda$ we will understand a function $u \in C^1(0, 1)$ such that $\varphi(u') \in C^1(0, 1)$ satisfying the equations in $(P)_\lambda$.

For the partial case, the existence of positive solutions for the problem $-\Delta u = \lambda f(u)$ in Ω , $u = 0$ on $\partial\Omega$, has been widely studied by many authors (see De Figueiredo, Lions and Nussbaum [2], Brezis and Turner [1], among others). An informative survey concerning this problem can be founded in the paper of Lions [6], where the author consider different behaviors of f in order to obtain the existence and multiplicity results of positive solutions. On the other hand, taking into account that f is superlinear and the partial differential operator is asymptotic to a power at zero and at infinity, Narukawa and Suzuki [7] studied the existence of positive solutions, and obtained the existence of a $\lambda^* > 0$, such that for any $0 < \lambda < \lambda^*$ the described problem has at least two positive solutions, nevertheless the existence of positive solutions for $\lambda \geq \lambda^*$ was not considered. In the 1-dimensional case is also possible to obtain existence results for $\lambda \geq \lambda^*$, thus for an operator satisfying the conditions (A_0) , (A_∞) and for the non-linearity satisfying appropriate jumping conditions (see (1.2)) we study the existence of positive solutions. In particular, in our main result (Theorem 3.2), besides proving that $(P)_\lambda$ has at least two positive solutions for any $0 < \lambda < \lambda^*$, we prove that it has at least one positive solution for $\lambda = \lambda^*$, and has no positive solution for any $\lambda > \lambda^*$. To this goal, first we construct the solutions of $(P)_\lambda$ using the Shooting Method (see Ubilla [10], and Guedda and Veron [5]), and briefly review and refine some results finded in [4] to obtain Lemma 2.1. Then we formulate and prove our results, and finally we present some examples.

Suppose that the function φ satisfies the next conditions,

$$(A_0) \quad \lim_{x \rightarrow 0} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{q-1} \quad \text{for some } q > 1 \text{ and for all } \sigma \in (0, 1),$$

$$(A_\infty) \quad \lim_{x \rightarrow +\infty} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{p-1} \quad \text{for some } p > 1 \text{ and for all } \sigma \in (0, 1),$$

that is, φ is a *regularly varying function of positive index at zero and at infinity*. In general this kind of functions are used in Probability: references may be found in Resnick [8], and Seneta [9]. Nevertheless, in the context that we will use here, these functions were considered recently in [3] and [4].

We also assume that f is an odd function which satisfies

$$(1.1) \quad xf(x) > 0 \quad \forall x \neq 0$$

$$(1.2) \quad \lim_{x \rightarrow 0} \frac{f(x)}{\varphi(x)} = a \quad \text{and} \quad \lim_{x \rightarrow +\infty} \frac{f(x)}{\varphi(x)} = A,$$

where $a, A \in [0, +\infty]$.

Finally we introduce some notation to be used in the subsequent sections. Let us set

$$\Phi(x) =: \int_0^x \varphi(s) ds, \quad \Phi_*(x) =: \int_0^x \varphi^{-1}(s) ds \quad \text{and} \quad F(x) =: \int_0^x f(t) dt.$$

Since φ and φ^{-1} are strictly increasing odd functions, Φ and Φ_* are strictly increasing even functions on $(0, +\infty)$. Then, the inverse of $\Phi_*|_{(0, +\infty)}$ is well-defined and we will write Φ_*^{-1} . Finally, we denote by $\lambda_k(p)$ the k -th eigenvalue of the problem

$$\begin{cases} -(|u'|^{p-2}u')' = \lambda|u|^{p-2}u, \\ u(0) = u(1) = 0, \end{cases}$$

where $p > 1$. One can prove that

$$\lambda_k(p) = k^p \lambda_1(p) = k^p (p-1) \left[2 \int_0^1 \frac{dz}{(1-z^p)^{1/p}} \right]^p,$$

using the ideas finded in [5] and [10].

2. Construction of the Solutions

We observe that if problem $(P)_\lambda$ has non-trivial solutions, then $\lambda > 0$. In fact, let u a solution of $(P)_\lambda$, then multiplying by u and integrating by parts over $(0, 1)$, yields

$$\int_0^1 u' \varphi(u') dt = \lambda \int_0^1 u f(u) dt.$$

Hence using (1.1) and the fact that φ is an increasing odd function we obtain $\lambda > 0$.

Now, we consider the initial value problem (I) associated to $(P)_\lambda$

$$(I) \quad \begin{cases} -(\varphi(u'))' = \lambda f(u) \\ u(0) = 0 \\ u'(0) = \alpha > 0. \end{cases}$$

Let u a solution of (I) , then multiplying (I) by u' and integrating we obtain

$$(\Phi_* \circ \varphi)(u'(t)) + \lambda F(u(t)) = C = \text{constant},$$

so, taking $W =: \Phi_* \circ \varphi$, yields

$$(2.1) \quad W(u'(t)) + \lambda F(u(t)) = C.$$

Setting $t = 0$ in (2.1), we obtain $C = W(\alpha)$, thus the energy relation for (I) is

$$(2.2) \quad W(u'(t)) = W(\alpha) - \lambda F(u(t)).$$

We will denote by $S(\alpha, \lambda)$ the unique positive zero of the function $W(\alpha) - \lambda F(\cdot)$, then

$$(2.3) \quad F(S(\alpha, \lambda)) = \frac{1}{\lambda} W(\alpha).$$

Thus we observe that $S(\alpha, \lambda) \rightarrow +\infty$ as $\alpha \rightarrow +\infty$ for each $\lambda > 0$.

Using similar arguments as in [4] and [10], we can prove the existence of solutions $\{-u, u\}$ for the problem (I) such that

$$(2.4) \quad u(2k\theta(\alpha, \lambda)) = 0 \quad , \quad k = 0, \pm 1, \pm 2, \dots,$$

where

$$(2.5) \quad \theta(\alpha, \lambda) = \int_0^{S(\alpha, \lambda)} \frac{dt}{W_+^{-1}(W(\alpha) - \lambda F(t))}, \quad \text{with } W_+ = W|_{[0, +\infty)}.$$

Therefore, the only zeros of the solutions of problem (I) are $\{2k\theta(\alpha, \lambda) \mid k \in \mathbb{Z}\}$, moreover, they are $4\theta(\alpha, \lambda)$ -periodic.

Now, we observe that if there exist positive constants α and λ such that

$$\frac{1}{2\theta(\alpha, \lambda)} \in \mathbb{N},$$

there exists a non-trivial solution of $(P)_\lambda$.

The next Lemma will give us information about the behavior of the function $\theta(\cdot, \lambda)$ for all $\lambda > 0$. Since the proof uses the same ideas of [4], it is omitted.

Lemma 2.1. *Let φ satisfy conditions (A_0) , (A_∞) and f satisfy conditions (1.1), (1.2). Then for all $\lambda > 0$ the function $\theta(\cdot, \lambda)$ is continuous in $\mathbb{R}^+$ and verify;*

- (i) $\lim_{\alpha \rightarrow +\infty} \theta(\alpha, \lambda) = 0$ for $A = +\infty$
- (ii) $\lim_{\alpha \rightarrow 0} \theta(\alpha, \lambda) = 0$ for $a = +\infty$
- (iii) $\lim_{\alpha \rightarrow +\infty} \theta(\alpha, \lambda) = +\infty$ for $A = 0$
- (iv) $\lim_{\alpha \rightarrow 0} \theta(\alpha, \lambda) = +\infty$ for $a = 0$
- (v) $\lim_{\alpha \rightarrow +\infty} \theta(\alpha, \lambda) = \frac{1}{2} \left(\frac{\lambda_1(p)}{\lambda A} \right)^{1/p}$ for $0 < A < +\infty$
- (vi) $\lim_{\alpha \rightarrow 0} \theta(\alpha, \lambda) = \frac{1}{2} \left(\frac{\lambda_1(q)}{\lambda a} \right)^{1/q}$ for $0 < a < +\infty$.

In the next section we will write $\theta(+\infty, \lambda)$ and $\theta(0, \lambda)$, instead of $\lim_{\alpha \rightarrow +\infty} \theta(c$ and $\lim_{\alpha \rightarrow 0} \theta(\alpha, \lambda)$ respectively.

3. Positive Solutions

We now state and prove our main results.

Theorem 3.1. *Suppose that φ satisfy (A_0) , (A_∞) and f satisfy (1.1), (1.2).*

- (i) *If $\frac{\lambda_1(p)}{A} < \frac{\lambda_1(q)}{a}$ then problem $(P)_\lambda$ has at least one positive solution for any $\frac{\lambda_1(p)}{A} < \lambda < \frac{\lambda_1(q)}{a}$.*
- (ii) *If $\frac{\lambda_1(q)}{a} < \frac{\lambda_1(p)}{A}$ then problem $(P)_\lambda$ has at least one positive solution for any $\frac{\lambda_1(q)}{a} < \lambda < \frac{\lambda_1(p)}{A}$.*

Proof. Let $\frac{\lambda_1(p)}{A} < \lambda < \frac{\lambda_1(q)}{a}$, it is immediate that $\frac{1}{2} \left(\frac{\lambda_1(p)}{\lambda A} \right)^{1/p} < \frac{1}{2} < \frac{1}{2} \left(\frac{\lambda_1(q)}{\lambda a} \right)^{1/q}$ then, by Lemma 2.1 we have $\theta(+\infty, \lambda) < \frac{1}{2} < \theta(0, \lambda)$. Since $\theta(\cdot, \lambda)$ is continuous the result of part (i) follows by application of the Intermediate Value Theorem. Part (ii) of the theorem can be proved in the same way.

Remark 3.1 Taking $A = +\infty$ (the superlinear case) one can prove that $(P)_\lambda$ has at least one positive solution for any $0 < \lambda < \frac{\lambda_1(q)}{a}$. If in addition we take $a = 0$, it also can be proved that $(P)_\lambda$ has at least one positive solution for all $\lambda > 0$. Similar results are possible to obtain in the sublinear case, that is, when $a = +\infty$.

Theorem 3.2. Suppose that φ satisfy $(A_0), (A_\infty)$ and f satisfy (1.1), (1.2). Assume that the function

$$(3.1) \quad \psi_t(x) = \frac{F(tx)}{F(x)}$$

is non-decreasing in $x > 0$, for all $t \in (0, 1)$.

- (i) If $a = A = +\infty$ then there exists a constant $\lambda^* > 0$ such that $(P)_\lambda$ has at least two positive solutions for any $0 < \lambda < \lambda^*$, has at least one positive solution for $\lambda = \lambda^*$ and has no positive solutions for any $\lambda > \lambda^*$.
- (ii) If $a = A = 0$ then there exists a constant $\lambda_* > 0$ such that $(P)_\lambda$ has no solutions for any $0 < \lambda < \lambda_*$, has at least one positive solution for $\lambda = \lambda_*$ and has at least two positive solutions for any $\lambda > \lambda_*$.

Proof. The function $\theta(\alpha, \cdot)$ is decreasing in $(0, +\infty)$ for all $\alpha > 0$. In fact, let $\lambda_1 < \lambda_2$, using (2.3) we have $S(\alpha, \lambda_1) > S(\alpha, \lambda_2)$. Now, from (2.5) and (2.3) it is clear that

$$(3.2) \quad \theta(\alpha, \lambda) = \int_0^1 \frac{S(\alpha, \lambda) dt}{W_+^{-1} \left(W(\alpha) \left(1 - \frac{F(tS(\alpha, \lambda))}{F(S(\alpha, \lambda))} \right) \right)},$$

and since $\psi_t(\cdot)$ is non-decreasing, the result is immediate.

Let $a = A = +\infty$, by Lemma 2.1 $\theta(0, \lambda) = \theta(+\infty, \lambda) = 0$. We observe that from (3.2) we obtain

$$\theta(\alpha, \lambda) \geq \frac{S(\alpha, \lambda)}{\alpha},$$

thus, if we take α and λ constants such that

$$(3.3) \quad \frac{S(\alpha, \lambda)}{\alpha} > \frac{1}{2},$$

there exist at least two positive solutions of $(P)_\lambda$.

On the other hand, (2.3) and (3.3) imply that

$$\lambda < \frac{W(\alpha)}{F\left(\frac{\alpha}{2}\right)}.$$

Thus, we define

$$(3.4) \quad \mu^* =: \max \left\{ \frac{W(\alpha)}{F\left(\frac{\alpha}{2}\right)} \mid \alpha > 0 \right\}$$

to obtain that,

- (i) For each $\lambda \in (0, \mu^*)$, the problem $(P)_\lambda$ has at least two positive solutions.
- (ii) For $\lambda = \mu^*$, the problem $(P)_\lambda$ has at least one positive solution.

Moreover, if $(P)_{\lambda_0}$ has a positive solution for some $\lambda_0 > 0$, then $(P)_\lambda$ has a positive solution for each $\lambda \in (0, \lambda_0]$, this is immediate using the monotonicity of $\theta(\alpha, \cdot)$.

Now, we consider the following set

$$S = \{\lambda > 0 \mid (P)_\eta \text{ has positive solution for all } \eta \leq \lambda\}.$$

Clearly $S \neq \emptyset$ and is upper bounded, so we can define

$$\lambda^* = \sup S.$$

We claim that for any $0 < \lambda < \lambda^*$ problem $(P)_\lambda$ has at least two positive solutions. In fact, from the definition of λ^* , and since $\theta(\alpha, \cdot)$ is decreasing, there exist $\lambda_0 \in (\lambda, \lambda^*)$ and $\alpha_{\lambda_0} \in (0, +\infty)$ such that

$$\frac{1}{2} = \theta(\alpha_{\lambda_0}, \lambda_0) < \theta(\alpha_{\lambda_0}, \lambda),$$

from the continuity of $\theta(\cdot, \lambda)$ the result is immediate.

On the other hand, if $\lambda = \lambda^*$ we know that $(P)_{\lambda^* - \frac{1}{n}}$ has a positive solution for each $n \in \mathbb{N}$, then we obtain a bounded sequence $(\alpha_n)_{n \in \mathbb{N}} \subset (0, +\infty)$ satisfying

$$\theta\left(\alpha_n, \lambda^* - \frac{1}{n}\right) = \frac{1}{2} \quad \text{for all } n \in \mathbb{N}.$$

Thus, there exists a convergent subsequence $(\alpha_{n_k})_{k \in \mathbb{N}}$ such that

$$\alpha_{n_k} \longrightarrow \alpha_{\lambda^*} \in (0, +\infty) \quad \text{when } k \rightarrow +\infty,$$

and then, using the continuity of $\theta(\cdot, \cdot)$ we obtain

$$\theta(\alpha_{\lambda^*}, \lambda^*) = \frac{1}{2},$$

that is, $(P)_{\lambda^*}$ has at least one positive solution.

Finally for any $\lambda > \lambda^*$ problem $(P)_{\lambda}$ has no positive solutions. In fact, if we suppose that $(P)_{\lambda}$ has positive solution for $\lambda > \lambda^*$, then $\lambda \in S$, but from the definition of the supremum $\lambda \leq \lambda^*$; which is a contradiction.

Thus, part (i) of theorem is proved.

Part (ii) of the theorem can be proved in the same way.

Remark 3.2 If we suppose that f is homogeneous (i.e. $f(x) = |x|^{p-2}x$, $p > 1$), then $\psi_t(x) = t^p$ is constant in $x > 0$, thus condition (3.1) in Theorem 3.2 is also satisfied. thus ce0.3cm

Remark 3.3 We know from Theorem 3.2(i) that $(P)_{\lambda}$ has no positive solutions for $\lambda > \lambda^*$, however it is possible to prove that in this case it has infinite solutions.

4. Examples

In the following examples, let $r, p, q > 1$.

Example 4.1 Let us set the functions

$$\varphi(x) = (1 + |x|^r)^{\frac{p}{r}-1} |x|^{r-2}x \quad \text{and} \quad f(x) = |x|^{q-2}x.$$

Then, conditions (A_0) and (A_{∞}) are satisfied since

$$\lim_{x \rightarrow 0} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{r-1} \quad \text{and} \quad \lim_{x \rightarrow +\infty} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{p-1}.$$

We consider the following problem

$$(4.1) \quad \begin{cases} -\left((1+|u'|^r)^{\frac{p}{r}-1}|u'|^{r-2}u'\right)' = \lambda u^{q-1} \\ u(0) = u(1) = 0. \end{cases}$$

When $p < q < r$ we have

$$\lim_{x \rightarrow 0} \frac{f(x)}{\varphi(x)} = +\infty = \lim_{x \rightarrow +\infty} \frac{f(x)}{\varphi(x)},$$

(i.e. $a = A = +\infty$) then by Theorem 3.2(i) there exists $\lambda^* > 0$ such that (4.1) has at least two positive solutions for any $0 < \lambda < \lambda^*$, has at least one positive solution for $\lambda = \lambda^*$ and has no positive solutions for any $\lambda > \lambda^*$.

When $r < q < p$ we have

$$\lim_{x \rightarrow 0} \frac{f(x)}{\varphi(x)} = 0 = \lim_{x \rightarrow +\infty} \frac{f(x)}{\varphi(x)},$$

(i.e. $a = A = 0$) then by Theorem 3.2(ii) there exists $\lambda_* > 0$ such that (4.1) has no solutions for any $0 < \lambda < \lambda_*$, has at least one positive solution for $\lambda = \lambda_*$ and has at least two positive solutions for any $\lambda > \lambda_*$.

We note that,

- (1) When $q < r$ and $q < p$, or, $q > r$ and $q > p$ it can be proved that (4.1) has at least one positive solution for all $\lambda > 0$.
- (2) When $p = r$ we obtain the p -Laplacian operator, then for $q < p$ ($p < q$) the function $\theta(\cdot, \lambda)$ is non-decreasing (resp. decreasing), in $(0, +\infty)$ therefore (4.1) has one positive solution for all $\lambda > 0$.
- (3) When $p = r = q$, we obtain from (4.1) the eigenvalue problem for the p -Laplacian, (see [5]).

Example 4.2 Let us set the functions

$$\varphi(x) = |x|^{r-2} \log(1 + |x|^{q-1}) x \quad \text{and} \quad f(x) = \frac{|x|^{q-2} x (q + p|x|^q)}{(1 + |x|^q)^{2-\frac{p}{q}}}.$$

Then, condition (A_0) and (A_∞) are satisfied since

$$\lim_{x \rightarrow 0} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{r+q-2} \quad \text{and} \quad \lim_{x \rightarrow +\infty} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{r-1}.$$

Now, when $q > p$ the function

$$\psi_t(x) = \frac{F(tx)}{F(x)}$$

is non-decreasing in $x > 0$ for all $t \in (0, 1)$. In this case

$$F(x) = \left(1 + |x|^q\right)^{\frac{p}{q}-1} |x|^q.$$

Now, if we suppose that $p > r$, we obtain

$$\lim_{x \rightarrow 0} \frac{f(x)}{\varphi(x)} = +\infty = \lim_{x \rightarrow +\infty} \frac{f(x)}{\varphi(x)}$$

(i.e. $a = A = +\infty$), then by Theorem 3.2 (i) there exists $\lambda^* > 0$ such that the problem

$$(4.2) \quad \begin{cases} -\left(|u'|^{r-2} \log(1 + |u'|^{q-1}) u'\right)' = \lambda \left(\frac{|u|^{q-2} u (q + p|u|^q)}{(1 + |u|^q)^{2-\frac{p}{q}}} \right) \\ u(0) = u(1) = 0 \end{cases}$$

has at least two positive solutions for any $0 < \lambda < \lambda^*$, has at least one positive solution for $\lambda = \lambda^*$, and has no positive solutions for any $\lambda > \lambda^*$.

Example 4.3 Let us set the functions

$$\varphi(x) = \begin{cases} \frac{x \log(1 + |x|^{q-1})}{|x| \log(1 + |x|^{1-p})} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

and

$$f(x) = |x|^{r-1} \log(1 + |x|^r) x.$$

Then, conditions (A_0) and (A_∞) are satisfied since

$$\lim_{x \rightarrow 0} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^q \quad \text{and} \quad \lim_{x \rightarrow +\infty} \frac{\varphi(\sigma x)}{\varphi(x)} = \sigma^{p-1}.$$

When $2r > q - 1$ and $r > p - 1$ we have that

$$\lim_{x \rightarrow 0} \frac{f(x)}{\varphi(x)} = 0 \quad \text{and} \quad \lim_{x \rightarrow +\infty} \frac{f(x)}{\varphi(x)} = +\infty,$$

(i.e. $a = 0$ and $A = +\infty$), then by Theorem 3.1 problem

$$(4.3) \quad \begin{cases} - \left(\frac{u' \log(1 + |u'|^{q-1})}{|u'| \log(1 + |u'|^{1-p})} \right)' = \lambda |u|^{r-1} \log(1 + |u|^r) u \\ u(0) = u(1) = 0 \end{cases}$$

has at least one positive solution for all $\lambda > 0$.

Let observe us that if we suppose that $2r > q - 1$ and $r < p - 1$ we obtain

$$\lim_{x \rightarrow 0} \frac{f(x)}{\varphi(x)} = 0 = \lim_{x \rightarrow +\infty} \frac{f(x)}{\varphi(x)},$$

(i.e. $a = A = 0$). But in this case it is not difficult to prove that there exists $\mu_* > 0$ such that for any $0 < \lambda < \mu_*$ problem (4.3) has no solution. Here μ_* is defined in a similar way as in (3.4).

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