Proyecciones
Vol. 16, Nº 1, pp. 49-65, July 1997
Universidad Católica del Norte
Antofagasta - Chile

DOI: 10.22199/S07160917.1997.0001.00005

RIEMANN SURFACE WITH CYCLIC AUTOMORPHISMS GROUP *

Gustavo Labbé M.
Universidad de La Serena, La Serena, Chile

Abstract

In this paper, we present the uniformization of $y^2 = x^p - 1$, with $p > 5$ and prime, i. e., the only hyperelliptic Riemann surface of genus $g = \frac{p-1}{2}$, which admit $Z/2pZ$ as automorphism group. This uniformization is found by using a fuchsian group which reflects the action of $Z/2pZ$ and is constructed starting of a triangle group of type $(0; p, p, p)$.

Moreover, we describe completely the action of the automorphism group in homology, so that we can describe the invariant subvariety for $Z/2pZ$ in $\mathcal{A}_g$ (principally polarized abelian varieties of dimension g), which is determined by the real Abel aplication from $\mathcal{M}_g$ in $\mathcal{A}_g$.

*This work was partially supported by DIULS, grant No. 120-2-01

1 Introduction

From [1] and [2], for to describe the compacts singular locus of M_g (moduli space for compact Riemann surface of genus g), are important the Riemann surfaces which admit conformal automorphism groups isomorphic to Z/pZ, with p prime.

By [2], it is possible to calculate the number of different classes for g and p fixes, but it is not known an explicit description and hyperbolic uniformization for these classes.

In this context, we present the uniformization of the only hyperelliptic Riemann surface of genus $g = \frac{p-1}{2}$, which admit $Z/2pZ$ as a automorphism group. Moreover, we describe completely the action of the automorphism group in homology, so that we can describe the invariant subvariety for $Z/2pZ$ in A_g.

2 Riemann Surfaces which Admit Z/2pZ.

Let S be the Riemann surface associated to the algebraic curve $C \subset P_2(C)$ given by

$$C: \; z^{p-2}y^2 - x^p + z^p = 0, \quad p \text{ prime}, p > 3.$$

Then:

(1) S is a compact Riemann surface which is a covering of the Riemann sphere C with $p+1$ branch points.

In fact, consider the affine curve $C_a : \; y^2 = x^p - 1$, and put $x_1 = 1$, $x_2, ..., x_p$ the $p^{\underline{th}}$-roots of 1. Then x_i, $i = 1, ..., p$ are branch points for $\pi : \; C_a \to C$. This may be obtained, writing $y^2 = \prod_{i=1}^{p}(x - x_i)$ and using the local parameters $w = x - x_i$, $i = 1, ..., p$ around x_i. Thus, $y = \sqrt{w}h(w)$, where $h(0) \neq 0$ and h is analytic about $w = 0$. Hence, y has two values near $w = 0$, therefore $x_i = 1, ..., p$ are branch points with branching order $b_\pi(x_i) = 1$.

Now, if we intersect C with $P_1^\infty = \{\overline{(x, y, 0)} \mid x, y \in C\}$, we obtain that $x^p = 0$, which implies $x = 0$.

Thus, $C \cap P_1^\infty = \{\overline{(x,y,0)} = w_{p+1}\}$ and w_{p+1} is singular, since

$$\frac{\partial C}{\partial x}(w_{p+1}) = \frac{\partial C}{\partial y}(w_{p+1}) = \frac{\partial C}{\partial z}(w_{p+1}) = 0$$

and $b_\pi(\pi(w_{p+1})) = 1$.

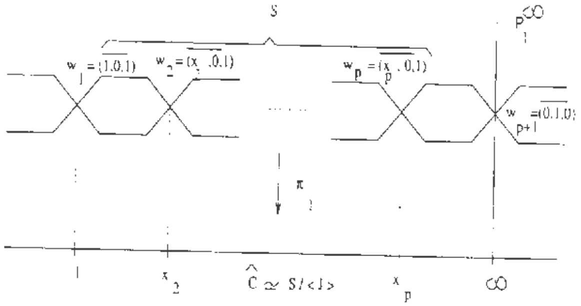

Figure 1:

(2) The genus of S is $g = \frac{p-1}{2}$.

In fact, by the Riemann-Hurwitz relation: $g = n(\gamma - 1) + 1 + \frac{B}{2}$ and for $n = 2$; $\gamma = 0$; $B = p + 1$ we have that $g = \frac{p-1}{2}$.

(3) S admit the hyperelliptic involution $J : S \to S$ defined by $J(\overline{(x,y,z)}) = \overline{(x,-y,z)}$ which fixes the $p+1$ points $w_i = \overline{(x_i,0,1)}$, $i = 1,...,p$, $w_{p+1} = \overline{(0,1,0)}$, and therefore S is a hyperelliptic Riemann surface.

(4) Now we consider the canonical projection $\pi_1 : S \to S/ < J >$.

Since $| < J > | = 2$, then π_1 is a branched analytic covering of degree two and branched only at the fixed points of J. The stability group of each point is $< J >$ and these points are inequivalent under $< J >$.

Thus, the total branch number of π_1 is

$$B = \sum_{i=1}^{p+1} \frac{N}{\nu_i}(\nu_i - 1) = p+1 \quad (N = 2,\ \nu_i = |<J>| = 2,\ i = 1, ..., p+1)$$

and by the Riemann-Hurwitz relation, the genus γ of $S/<J>$ is zero.

Therefore, the symbol associated to $\pi_1 : S \to S/<J>$ is $<(\frac{p-1}{2}, 0): \underbrace{2, 2, ..., 2}_{p+1}>$.

(5) Moreover S admit the automorphism of order p, $\varphi : S \to S$ given by $\varphi(\overline{(x, y, z)}) = \overline{(\xi x, y, z)}$ where $\xi^p = 1$ and $\xi \neq 1$.

The canonical projection $\pi_2 : S \to S/<\varphi>$ is a branched analytic covering of degree $|<\varphi>| = p$ and branched only at the fixes points of φ, they are $v_1 = \overline{(0, 1, i)}$, $v_2 = \overline{(0, -i, 1)}$, $w_{p+1} = \overline{(0, 1, 0)}$.

The stability group of each point is $<\varphi>$, and theses points are inequivalent under $<\varphi>$. Thus, the total branch number of π_2 is given by

$$B = \sum_{i=1}^{3} \frac{N}{\nu_i}(\nu_i - 1) = 3(p-1) \quad (N = p,\ \nu_i = |<\varphi>| = p,\ i = 1, 2, 3)$$

and by the Riemann-Hurwitz relation, the genus γ of $S/<J>$ is zero and therefore the symbol associated to π_2 is $<(\frac{p-1}{2}, 0); p, p, p>$.

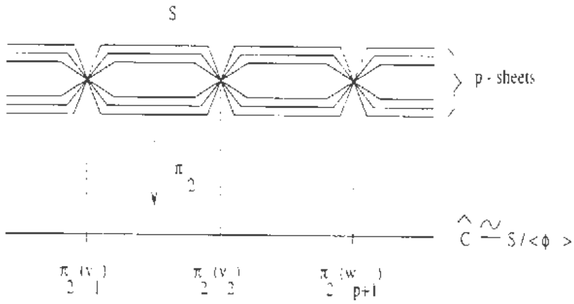

Figure 2:

(6) Moreover, S admit the automorphism of order $2p$, $\psi = \varphi \circ J : S \to S$ defined by $\psi(\overline{x, y, z}) = \overline{(\xi x, -y, z)}$ where $\xi^p = 1$ and $\xi \neq 1$.

The canonical projection $\pi_3 : S \to S/ < \psi >$ is a branched analytic covering of degree $| < \psi > | = 2p$, and branched only at the fixes points of $< \psi >$, they are:

(a) $v_1 = \overline{(0, i, 1)}$, $v_2 = \overline{(0, -i, 1)}$ with stability group $< \varphi >$ and equivalent under J in $S/ < \psi >$. Therefore, $\nu_1 = | < \varphi > | = p$.

(b) $w_1, w_2, \ldots, w_p$ with stability group $< J >$ and equivalent under φ in $S/ < \psi >$. Therefore, $\nu_2 = | < J > | = 2$.

(c) w_{p+1} with stability group $< \psi >$, therefore $\nu_3 = 2p$. Thus, in $S/ < \psi >$, $P = \pi_3(v_1)$, $Q = \pi_3(w_1)$ and $O = \pi_3(w_{p+1})$ are the branched points of π_3. The total branch number of π_3 is $B = 5p - 3$ and by the Riemann-Hurwitz relation, the genus γ of $S/ < \psi >$ is zero, and therefore the symbol associated to π_3 is $< (\frac{p-1}{2}, 0); 2p, p, 2 >$.

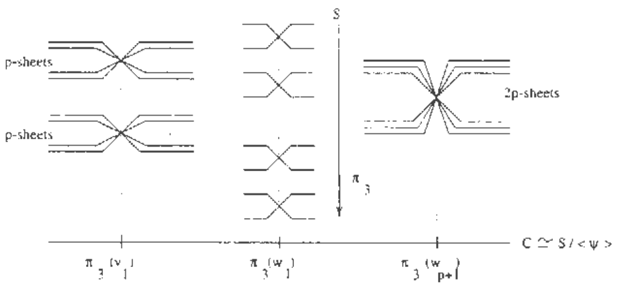

Figure 3:

Therefore, we have the following covering:

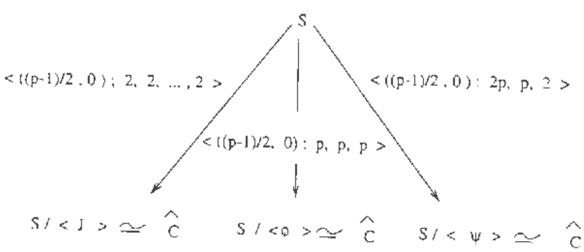

Figure 4:

3 Uniformization of S

By [3], if $\triangle$ denotes the disc in C, then there exists a Fuchsian group T uniformizing the Riemann sphere $\hat{C}$ with signature $(0\colon p, p, p)$ and T has a presentation $T =< a, b\colon a^p = b^p = (ab)^p = 1 >$.

Moreover, if $\phi : T \to Z/pZ$ is an epimorphism such that $\Gamma = \mathrm{Ker}(\phi)$ is a torsion-free Fuchsian group, then $S = \triangle/\Gamma$ is a compact Riemann surface of genus $\frac{p-1}{2}$ and the natural projection $S \to \triangle/T \simeq \hat{C}$ is a covering with associated signature $(0\colon p, p, p)$ and covering group $T/\Gamma \simeq Z/pZ$.

Thus, we have the short exact sequence

$$0 \longrightarrow \Gamma \longrightarrow T \xrightarrow{\phi} Z/pZ \longrightarrow 0$$

and the coverings following:

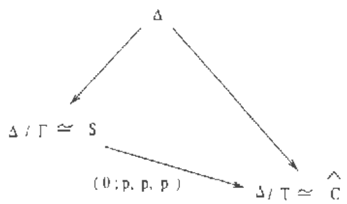

Figure 5:

Put $\Phi(\Gamma, T, Z/pZ) = \{\phi : T \to Z/pZ | \phi \ \text{epimorphism}, \mathrm{Ker}\phi \simeq \Gamma\}$ where $T =< a, b\colon a^p = b^p = (ab)^p = 1 >$ and Γ is a torsion-free group.

For $\phi_1, \phi_2 \in \Phi(\Gamma, T, Z/pZ)$ we define:

$\phi_1 \sim \phi_2$ if and only if there exists automorphisms $\left\{ \begin{array}{lcc} \alpha & : & T \to T \\ \beta & : & Z/pZ \to Z/pZ \end{array} \right.$

such that the following diagram is conmutative

$$\begin{array}{ccc} T & \xrightarrow{\phi_1} & Z/pZ \\ \alpha \downarrow & & \downarrow \beta \\ T & \xrightarrow{\phi_2} & Z/pZ \end{array}$$

We have that $\sim$ is an equivalence relation and if $\phi_1 \sim \phi_2$ then by [5], $\triangle/\mathrm{Ker}(\phi_1) = S_1$ and $\triangle/\mathrm{Ker}(\phi_2) = S_2$ are equivalent conformal.

Proposition 1. Let p be a number prime, $p \geq 5$. Then

$$N(3, p) = \mathrm{Card}(\Phi(\Gamma, T, Z/pZ)/\sim) = \left\{ \begin{array}{ll} \frac{p+1}{6} & \text{if} \quad 3 \not| (p-1) \\ \frac{p+5}{6} & \text{if} \quad 3 \mid (p-1) \end{array} \right.$$

Thus, there exists $N(3,p)$ Riemann surface S of genus $\frac{p-1}{2}$ which are conformally non-equivalent and such that the covering $S \to \hat{C}$ has signature $(0; p, p, p)$.

Proof By [2] and [4], $N(3,p)$ is the fourth coefficient of the Taylor series $G_p(z) = \sum_{j=0}^{\infty} N(j,p) z^j$, where

$$G_p(z) = \frac{1}{p-1}\{\frac{1}{p}[(1-z)^{-p+1} + (p-1)(1-z)(1-z)^{-1}] + \sum_l \phi(l)(1-z^l)^{-\frac{p-1}{l}}\}$$

and the summation being over l such that $l \neq 1$, $l|(p-1)$ and $\phi(l)$ is the Euler function.

Therefore $N(3,p) = \frac{G_p'''(0)}{6}$. We have two cases:

(a) If $3 \not| (p-1)$, put $G_p(z) = P_1(z) + R_1(z)$ where

$$P_1(z) = \frac{1}{(p-1)p}(1-z)^{-p+1} \quad \text{and}$$

$$R_1(z) = \frac{1}{p-1}\{\frac{1}{p}(p-1)(1-z)(1-z^p)^{-1} + \sum_l \phi(l)(1-z^l)^{-\frac{p-1}{l}}\},$$

then we obtain $P_1'''(0) = p+1$, $R_1'''(0) = 0$ and therefore $G_p'''(0) = p+1$.

(b) If $3 \mid (p-1)$, put $G_p(z) = P_2(z) + R_2(z)$ where

$$P_2(z) = \frac{1}{p-1}\{\frac{1}{p}(1-z)^{-p+1} + \phi(3)(1-z^3)^{-\frac{p-1}{3}}\} \quad \text{and}$$

$$R_2(z) = \frac{1}{p-1}\{\frac{1}{p}(p-1)(1-z)(1-z^p)^{-1} + \sum_{l, l\leq 3} \phi(l)(1-z^l)^{-\frac{p-1}{l}}\},$$

then we obtain that $P_2'''(0) = p+5$, $R'''(0) = 0$ and therefore $G_p'''(0) = p+5$.

4. Fundamental Polygon.

Now we will construct a fundamental polygon F for T using [1], where T is the same group considered previously.

We have that T is a tringle group of type (p,p,p) which is presented by $T = < a, b; a^p = b^p = (ab)^p = 1 >$.

Let q_1, q_2, q_3 be fixed points of a, b and ab respectively. If we put q_1 in the center of the disc Δ, q_2 on the positive real axis and q_3 such that $\Im q_3 < 0$, we obtain a fundamental polygon F for T as shown in Figure 6.

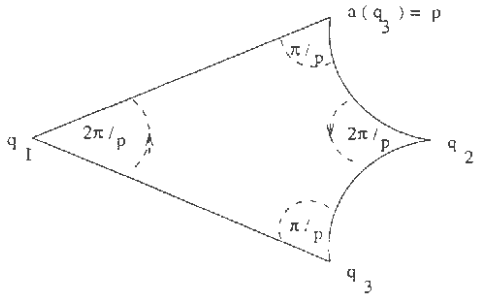

Figure 6:

Starting from F we build a fundamental region R for S, where S has genus $\frac{p-1}{2}$. The action of a on F with center in q_1 repeated p-times gives a closed polygon of $2p$-sides as shown in Figure 7.

Enumerating the sides of this polygon in the positive sense, the identifications of these sides are given by the kernel of the epimorphism $\phi \in \Phi(\Gamma, T, Z/pZ)$.

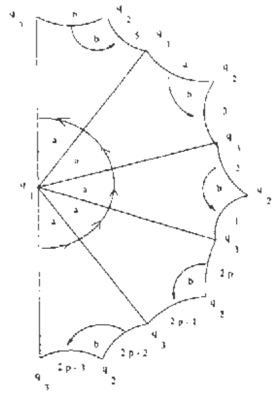

Figure 7:

For example, the epimorphism defined by $\phi(a) = 1$, $\phi(b) = \frac{p-1}{2}$, has $\mathrm{Ker}\phi = < a^{\frac{p-1}{2}} b^{-1} >$ which is torsion-free.

We look now at the identifications given by Kerϕ.

If z is a point which belong to side 1, then $b^{-1}(z)$ belong to the side 2 and $a^{\frac{p-1}{2}} b^{-1}(z)$ belong to the side $2(\frac{p-1}{2}) + 2 = p + 1$.

If z is a point belonging to side 2, then using $(a^{\frac{p-1}{2}} b^{-1})^{-1} = ba^{-\frac{p-1}{2}} = ba^{\frac{p+1}{2}}$ instead of $a^{\frac{p-1}{2}} b^{-1}$, we obtain that $a^{\frac{p+1}{2}}(z)$ belongs to the side $2(\frac{p-1}{2})+2 = p+3$ and $ba^{\frac{p+1}{2}}(z)$ belongs to the side $(p+3) - 1 = p + 2$.

Repeating the same argument, we obtain the identifications:

$$1 \leftrightarrow (p+1); \quad 2 \leftrightarrow (p+2); \cdots; \quad p \leftrightarrow 2p.$$

Hence, R is a fundamental region for S_ϕ and with the identification given above, we obtain that $R/\sim = S_\phi$.

We note that S_ϕ is hyperelliptic, since S_ϕ admit the hyperelliptic involution J given by a rotation of angle π around q_1 in R, which fixes the p mid-points of the sides of the polygon and also q_1.

Therefore, the Riemann surface S_ϕ is associated with the algebraic curve $z^{p-2}y^2 - x^p + z^p = 0$.

Moreover, S_ϕ admit an automorphism φ of order p given by a rotation of angle $\frac{2\pi}{p}$ around q_1 in R and admit another automorphism of order $2p$, $\psi = \varphi \circ J$ given by a rotation of angle $\frac{(p+2)\pi}{p}$ around q_1.

On the other hand, if we build another polygon R' of $2p$-sides centered in q_3 from R with the identifications given by Kerϕ, then the automorphism of order $2p$ is a rotation of angle $\frac{\pi}{p}$ around q_3 of R'.

5 Homology Basis of S_ϕ.

Let R be the polygon of $2p$-sides with the identifications given by Kerϕ, i. e., $j \leftrightarrow (j+p)$, $j = 1, ..., p$, which uniformizes the hyperelliptic Riemann surface $S = S_\phi$ of genus $\frac{p-1}{2}$.

For will obtain an homology basis of S_ϕ, we consider $U = \{q_2, q_3\}$ and the short exact sequence in homology:

$$0 \longrightarrow H_1(S) \longrightarrow H_1(S,U) \xrightarrow{\delta} H_0(U) \longrightarrow H_0(S) \longrightarrow 0$$

where $H_1(S)$ is the first homology group of S, $H_1(S,U)$ is the first relative homology group and δ is defined by $\delta(\gamma) = \gamma(1) - \gamma(0)$.

Therefore, the sequence in homology is

$$0 \longrightarrow H_1(S) \longrightarrow H_1(S,U) \longrightarrow Z \oplus Z \longrightarrow Z \longrightarrow 0,$$

and the homology basis for S is given by Kerδ.

If we identify the sides 1, 2,..., p by the polygon R with the paths β_1, β_2,..., β_p with the positive orientation, then $1+p$, $2+p$,..., $2p$ correspond respectively to β_1^{-1}, β_2^{-1},..., β_p^{-1} and β_1, β_2,..., β_p are generators for $H_1(S,U)$.

Let $\gamma \in \mathrm{Ker}\delta$ be, then $\gamma = \sum_{i=1}^{p} m_i\beta_i$, with $m_i \in Z$ and

$$\delta(\gamma) = q_3(\sum_{j=1}^{p}(-1)^j m_j) + q_2(\sum_{j=1}^{p}(-1)^{j+1} m_j = 0,$$

hence, $m_1 = \sum_{j=2}^{p}(-1)^j m_j$ and we obtain an homology basis for S, $\{\alpha_i\}_{i=1}^{p-1}$ where $\alpha_i = \beta_i + \beta_{i+1}$, with $i = 1, ..., p+1$ as shown in Figure 8.

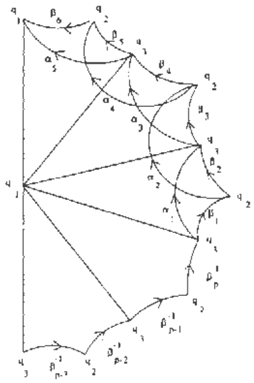

Figure 8:

Let $\psi = \varphi \circ J : S \longrightarrow S$ be the automorphism of order $2p$, then $\psi : H_1(S) \longrightarrow H_1(S)$ in term of the basis $\{\alpha_i\}_{i=1}^{p-1}$ is given by

$$\begin{cases} \psi(\alpha_i) & = & -\alpha_{i+2}, \quad i = 1, ..., p-3 \\ \psi(\alpha_{p-2}) & = & \sum_{j=1}^{p-1}(-1)^{j+1}\alpha_j \\ \psi(\alpha_{p-1}) & = & \alpha_1 \end{cases}$$

and therefore is represented for the matrix

$$A_{2p} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 1 & 1 \\ 0 & 0 & 0 & \cdots & 0 & -1 & 0 \\ -1 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & -1 & 0 & \cdots & 0 & -1 & 0 \\ 0 & 0 & -1 & \cdots & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & -1 & 0 \end{pmatrix}$$

The matrix A_{2p} satisfies: $A_{2p}^{p} = -I$ and $A_{2p}^{2p} = I$.

6. Example

Let S be the Riemann surface associated to the algebraic curve

$$C: \quad z^{11}y^2 - x^{13} + z^{13} = 0.$$

Then, S is a Riemann surface of genus six and S admit the hyperelliptic involution J, hence is a hyperelliptic Riemann surface.

Moreover, S admit another automorphism φ of order thirteen and the canonical projection $\pi : S \to S/ < \varphi >$ is a covering with signature $< (6,0): 13, 13, 13 >$.

Also, S admit another automorphism $\psi = \varphi \circ J$ of order twenty-six and the canonical projection $\pi : S \to S/ < \psi >$ is a covering with signature $< (6,0): 26, 13, 2 >$.

Let T be a triangle group of type $(13, 13, 13)$ presented by

$$T = < a, b;\ a^{13} = b^{13} = (ab)^{13} = 1 > .$$

Then, given an epimorphism $\phi : T \to Z/13Z$ such that $\Gamma = \mathrm{Ker}\phi$ is torsion-free. $S = \triangle/\Gamma$ is a Riemann surface of genus six and the canonical projection $S \to \triangle/T$ is a covering with signature $(0; 13, 13, 13)$ and covering group $T/\Gamma \simeq Z/13Z$.

Then, by lemma previous, there exists three Riemann surfaces S of genus six which signature $(0, 13, 13, 13)$, one of these is the hyperelliptic Riemann surface associated to algebraic curve $z^{11}y^2 - x^{13} + z^{13} = 0$.

These surfaces are given by the three different classes of epimorphisms $\phi : T \to Z/13Z$ with $\mathrm{Ker}\varphi$ torsion-free and $S = \triangle/\mathrm{Ker}\phi$.

Now, the fundamental polygon F for T is shown in the following Figure, where q_1 is the fixed point of a, q_2 is the fixed point of b and q_3 is the fixed point of ab.

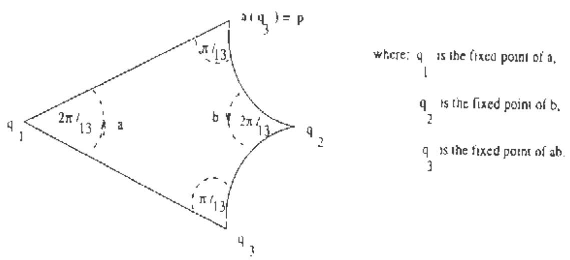

Figure 9:

Starting with F, we build a fundamental region R for S as shown in Figure 10.

The three different classes of epimorphisms are given by:

(i) $\phi_1(a) = 1$, $\varphi_1(b) = 6$, $\mathrm{Ker}\phi_1 = < a^6 b^{-1} >$ that gives the identifications

$$1 \leftrightarrow 14; \quad 2 \leftrightarrow 15; \quad 3 \leftrightarrow -16; \quad \cdots \quad ; 13 \leftrightarrow 26.$$

Therefore, $S_1 = \triangle/\mathrm{Ker}\phi_1$ is the hyperelliptic Riemann surface.

(ii) $\phi_2(a) = 1$, $\phi_2(b) = 2$, $\mathrm{Ker}\phi_2 = < a^2 b^{-1} >$ that gives the identifications

$$1 \leftrightarrow 6; \quad 2 \leftrightarrow 23; \quad 3 \leftrightarrow 8; \quad 4 \leftrightarrow 25; \quad 5 \leftrightarrow 10; \quad 7 \leftrightarrow 12; \quad 9 \leftrightarrow 14;$$

$$11 \leftrightarrow 16; \quad 13 \leftrightarrow 18; \quad 15 \leftrightarrow 20; \quad 17 \leftrightarrow 22; \quad 19 \leftrightarrow 24; \quad 21 \leftrightarrow 26.$$

(iii) $\phi_3(a) = 1$, $\phi_3(b) = 3$, $\mathrm{Ker}\phi_3 = < a^3b^{-1} >$ that gives the identifications

$$1 \leftrightarrow 8; \quad 2 \leftrightarrow 21; \quad 3 \leftrightarrow 10; \quad 4 \leftrightarrow 23; \quad 5 \leftrightarrow 12; \quad 6 \leftrightarrow 25; \quad 7 \leftrightarrow 14;$$

$$9 \leftrightarrow 16; \quad 11 \leftrightarrow 18; \quad 13 \leftrightarrow 20; \quad 15 \leftrightarrow 22; \quad 17 \leftrightarrow 24; \quad 19 \leftrightarrow 26.$$

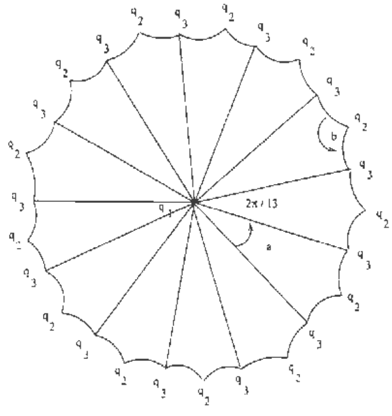

Figure 10:

An homology basis for the hyperelliptic Riemann surface is given by the paths $\{\alpha_i\}_{i=1}^{12}$, as shown in Figure 11.

The hyperelliptic involution J is given by a rotation of angle π around q_1, the automorphism φ of order thirteen is given by a rotation of angle $\frac{2\pi}{13}$ around q_1 and the automorphism $\psi = \varphi \circ J$ of order twenty-six is given by a rotation of angle $\frac{15\pi}{13}$ around q_1.

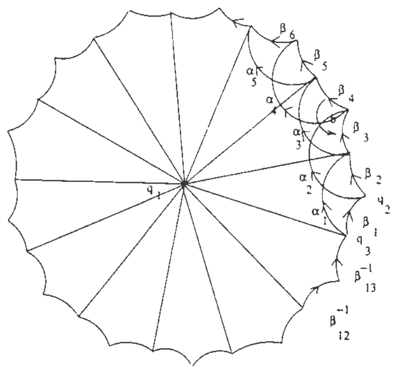

Figure 11:

The automorphism ψ in homology in the basis $\{\alpha_i\}_{i=1}^{12}$ is given by

$$A_{26} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{pmatrix}$$

References

[1] Cornalba M., *On the Locus of Curves with Automorphisms.* Annali di Mat. Pura et Appl., **149**, (1987), 135–151.

[2] Harvey, W. J., *On branch loci in Teichmuller space.* Transaction A. M. S. (1971)

[3] González V., Riera G. & Rodrí guez R., *On Riemann Matrices and Uniformazations lof Riemann Surfaces with Automorphism.* Complex Geometry Seminar, **Vol. II**, 1986, U.T.F.S.M.

[4] Lloyd, E. K., *Riemann Surface Transformation groups.* Ph. D. Thesis, Birmingham. 1967.

[5] Macbeath, A. M., *Proceeding of the Summer School in Geometry and Topology.* Queen's College, Dundee, 1961.

Received : 10 de Enero 1996.

Gustavo Labbé M
Departamento de Matemáticas
Facultad de Ciencias
Universidad de La Serena
Casilla 559
La Serena
Chile