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Pairwise generalized b-R_o spaces in bitopological spaces

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Abstract

The main purpose of this paper is to introduce pairwise generalized b-R_o spaces in bitopological spaces with the help of generalized b-open sets in bitopological spaces and give several characterizations of this spaces. We also introduce generalized b-kernel of a set and investigate some properties of it and study the relationship between this space and other bitopological spaces.

Key Words : *Bitopological spaces; pairwise gb-R_o spaces ; pairwise gb-R_1 spaces ; (i, j)-gb-kernel ; (i, j)-gb-open sets.*

AMS Classification : *54A10; 54C08; 54C10; 54D15.*

1. Introduction

A triplet (X, τ_1, τ_2), where X is a non-empty set and τ_1, τ_2 are topologies on X is called a bitopological space. Kelly [9] initiated the study of bitopological spaces. Later on several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces. Andrijevic [6] introduced a new class of generalized open sets called b-open sets in the field of topology and studied several fundamental and interesting properties. Later on Khadra and Nasef [1] and Al-Hawary and Al-Omari [4] defined the notions of b-open sets in bitopological spaces. Al-Hawary ([2], [3]) studied about the pre-open sets. Tripathy and Sarma ([18], [19], [20], [21], [22]) have done some works on bitopological spaces using this notion. Ganster and Steiner [8] introduced the concept of generalized b-closed sets in topological spaces. There after Tripathy and Sarma [22] have extended this notion to bitopological spaces. It is observed from literature that there has been a considerable work on different retatively weak form of seperation axiom like R_o axiom. For instance, semi-R_o, pre-R_o, b-R_o are some of the varient form of R_o property that have been investigated by different researchers as seperate entities. Khaleefa [10] has introduced and studied new types of seperation axioms termed by generalized b-R_o and generalized b-R_1 by using generalized b-open sets due to Ganster and Steiner [8]. Tripathy and Acharjee [20] have done some works on bitopological spaces.

In this paper, we introduce the notion of pairwise gb-R_o spaces and pairwise gb-R_1 spaces in bitopological spaces and investigate some of their properties. In particular, the notion of (i, j)-gb-kernel of a set is also defined in bitopological spaces.

2. Preliminaries

Throughout this paper, (X, τ_1, τ_2) denotes a bitopological space on which no separation axioms are assumed. For a subset A of X, i-$int(A)$ and j-$cl(A)$ denotes the i-interior and j-closure of A with respect to the topology τ_i and τ_j respectively, where $i, j \in \{1, 2\}, i \neq j$.

Definition 2.1. A subset A of (X, τ) is called b-open, if $A \subset int(cl(A)) \cup cl(int(A))$ and called b-closed if $X \setminus A$ is b-open.

One may refer to Andrijevic [6] for the above definition.

The following definitions are due to Al-Hawary and Al-Omari [4].

Definition 2.2. A subset A of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-b-open if $A \subset i\text{-}int(j\text{-}cl(A)) \cup j\text{-}cl(i\text{-}int(A))$. The complement of an (i, j)-b-open set is (i, j)-b-closed.
By (i, j) we mean the pair of topologies (τ_i, τ_j).

Definition 2.3. Let A be a subset of a bitopological space (X, τ_1, τ_2). Then

(i) the (i, j)-b-closure of A denoted by (i, j)-b $cl(A)$, is defined by the intersection of all (i, j)-b-closed sets containing A.

(ii) the (i, j)-b-interior of A denoted by (i, j)-b $int(A)$, is defined by the union of all (i, j)-b-open sets contained in A.

The following Definitions and results are due to Tripathy and Sarma [19].

Definition 2.4. A subset $A$of a bitopological space (X, τ_1, τ_2) is said to be (i, j)-generalized b-closed (in short, (i, j)-gb-closed) set if (j, i)-$bcl(A) \subset U$ whenever $A \subset U$ and U is τ_i-open in X.

We denote the family of all (i, j)-gb-closed sets and (i, j)-gb-open sets in(X, τ_1, τ_2) by $GBC(i, j)$ and $GBO(i, j)$ respectively.

Definition 2.5. The (i, j)-generalized b-closure of a subset A of a bitopological space (X, τ_1, τ_2) is the intersection of all (i, j)-gb-closed sets containing A and is denoted by (i, j)-$gbcl(A)$.

Lemma 2.1. A subset A of a bitopological space (X, τ_1, τ_2) is (i, j)-gb-open if and only if $U \subset (j, i)$-$bint(A)$, whenever U is τ_i-closed and $U \subset A$.

Lemma 2.2. For any subset A of a bitopological space (X, τ_1, τ_2), $A \subset (i, j)$-$gbcl(A)$.

Lemma 2.3. Let (X, τ_1, τ_2) be a bitopological space. If A is (i, j)-gb-closed subset of X, then $A = (i, j)$-$gbcl(A)$.

Lemma 2.4. A point $x \in (i, j)$-$gbcl(A)$ if and only if for every (i, j)-gb-open set U containing x, $U \cap A \neq \emptyset$.

3. Pairwise gb-R_o Spaces

Definition 3.1. A bitopological space (X, τ_1, τ_2) is said to be pairwise generalized b-R_o (in short, pairwise gb-R_o) spaces if (j, i)-$gbcl(\{x\}) \subset U$, for every (i, j)-gb-open set U containing x and $i, j = 1, 2, i \neq j$.

Theorem 3.1. Let (X, τ_1, τ_2) be a bitopological space. Then the following statements are equivalent :

(a) (X, τ_1, τ_2) is pairwise gb-R_o space.

(b) For any (i, j)-gb-closed set V and $x \notin V$, there exist a (j, i)-gb-open set U such that $x \notin U$ and $V \subset U$, for $i, j = 1, 2, i \neq j$.

(c) For any (i, j)-gb-closed set V and $x \notin V$, (j, i)-$gbcl(\{x\}) \cap V = \emptyset$ for $i, j = 1, 2, i \neq j$.

Proof. $(a) \Rightarrow (b)$ Let V be an (i, j)-gb-closed set and $x \notin V$. By (a), we have (j, i)-$gbcl(\{x\}) \subset X \setminus V$. Put $U = X \setminus (j, i)$-$gbcl(\{x\})$. Then U is (j, i)-gb-open and $V \subset X \setminus (j, i)$-$gbcl(\{x\}) = U$. Thus $V \subset U$ and $x \notin U$.

$(b) \Rightarrow (c)$ Let V be a (i, j)-gb-closed set and $x \notin V$. By hypothesis, there exists a (j, i)-gb-open set U such that $x \notin U$ and $V \subset U$. Which implies $U \cap (j, i)$-$gbcl(\{x\}) = \emptyset$ since U is (j, i)-gb-open. Hence $V \cap (j, i)$-$gbcl(\{x\}) = \emptyset$.

$(c) \Rightarrow (d)$ Let U be a (i, j)-gb-open set such that $x \in U$. Now, $X \setminus U$ is (i, j)-gb-closed and $x \notin X \setminus U$. By (c), (j, i)-$gbcl(\{x\}) \cap (X \setminus U) = \emptyset$. Which implies (j, i)-$gbcl(\{x\}) \subset U$. Hence (X, τ_1, τ_2) is pairwise gb-R_o space.

Theorem 3.2. Let (X, τ_1, τ_2) be a bitopological space. Then X is pairwise gb-R_o space if and only if for any two distinct points x and y of X, either (i,j)-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\}) = \emptyset$ or $\{x, y\} \subset (i,j)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\})$.

Proof. Let (i,j)-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\}) \neq \emptyset$ and {x, y} is not contained in (i,j)-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\})$. Let $z \in (i,j)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\})$ and $x \notin (i,j)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\})$. Now, $x \notin (j,i)$-$gbcl(\{y\})$ implies $x \in X \setminus (j,i)$-$gbcl(\{y\})$, which is a (j,i)-gb-open set containing x. Since $z \in (j,i)$-$gbcl(\{y\})$, so (i,j)-$gbcl(\{x\})$ is not contained in $X \setminus (j,i)$-$gbcl(\{y\})$. Hence (X, τ_1, τ_2) is not pairwise gb-R_o space.

Conversely, Let U be a (i,j)-gb-open set such that $x \in U$. Suppose (j,i)-$gbcl(\{x\})$ is not contained in U. Then there exists a $y \in (j,i)$-$gbcl(\{x\})$ such that $y \notin U$ and (i,j)-$gbcl(\{y\}) \cap U = \emptyset$, since $X \setminus U$ is (i,j)-gb-closed and $y \in X \setminus U$. Hence {x, y} is not contained in (i,j)-$gbcl(\{y\}) \cap (j,i)$-$gbcl(\{x\})$ and (i,j)-$gbcl(\{y\}) \cap (j,i)$-$gbcl(\{x\}) \neq \emptyset$.

Now, we introduce the concept of (i,j)-gb-kernel of a set and utilizing it to characterize the notion of pairwise gb-R_o space.

Definition 3.2. Let (X, τ_1, τ_2) be a bitopological space and $A \subset X$. The intersection of all (i,j)-gb-open sets containing A is called the (i,j)-gb-kernel of A and is denoted by (i,j)-gb-ker(A).

The (i,j)-gb-kernel of a point $x \in X$ is the set
(i,j)-gb-ker$(\{x\}) = \cap\{U : U$ is (i,j)-gb-open and $x \in U\}$.
$= \{y : x \in (i,j)$-$gbcl(\{y\})\}$.

Theorem 3.3. Let (X, τ_1, τ_2) be a bitopological space and A be a subset of X. Then (i,j)-gb-ker$(A) = \{x \in X : (i,j)$-$gbcl(\{x\}) \cap A \neq \emptyset\}$.

Proof. Let $x \in (i,j)$-gb-ker(A) and (i,j)-$gbcl(\{x\}) \cap A = \emptyset$. Therefore (i,j)-$gbcl(\{x\}) \subset X \setminus A$ and so $A \subset X \setminus (i,j)$-$gbcl(\{x\})$. But $x \notin X \setminus (i,j)$-$gbcl(\{x\})$, which is a (i,j)-gb-open sets containing A. Thus $x \notin (i,j)$-gb-ker(A) , a contradiction. Consequently, (i,j)-$gbcl(\{x\}) \cap A \neq \emptyset$.

Conversely, let (i,j)-$gbcl(\{x\}) \cap A \neq \emptyset$. If possible, let $x \notin (i,j)$-gb-ker(A). Then there exists $U \in GBO(i,j)$ such that $x \notin U$ and $A \subset U$. Let $y \in (i,j)$-$gbcl(\{x\}) \cap A$. Then $y \in (i,j)$-$gbcl(\{x\})$ and $y \in A \subset U$. Hence $U \in GBO(i,j)$ such that $y \in U$ and $x \notin U$, which is a contradiction, since $y \in (i,j)$-$gbcl(\{x\}) \in GBC(i,j)$. Therefore $x \in (i,j)$-gb-ker(A). Hence (i,j)-gb-ker$(A) = \{x \in X : (i,j)$-$gbcl(\{x\}) \cap A \neq \emptyset\}$.

Theorem 3.4. Let (X, τ_1, τ_2) be a bitopological space. Then $\bigcap\{(i,j)$-$gbcl(\{x\}) : x \in X\} = \emptyset$ if and only if (i,j)-gb-ker$(\{x\}) \neq X$, for every $x \in X$.

Proof. Assume that $\bigcap\{(i,j)$-$gbcl(\{x\}) : x \in X\} = \emptyset$. Let (i,j)-gb-ker$(\{x\}) = X$. If there is some $y \in X$, then X is the only (i,j)-gb-open set containing y. Which shows $y \in (i,j)$-$gbcl(\{x\})$, for every $x \in X$. Therefore $\bigcap\{(i,j)$-$gbcl(\{x\}) : x \in X\} \neq \emptyset$, a contradiction. Hence (i,j)-gb-ker$(\{x\}) \neq X$, for every $x \in X$.

Conversely assume that (i,j)-gb-ker$(\{x\}) \neq X$, for every $x \in X$. Let $\bigcap\{(i,j)$-$gbcl(\{x\}) : x \in X\} \neq \emptyset$. If there is some $y \in X$ such that $y \in \bigcap\{(i,j)$-$gbcl(\{x\}) : x \in X\}$, then every (i,j)-gb-open set containing y must contain every point of X. This shows that X is the only (i,j)-gb-open set containing y. Therefore (i,j)-gb-ker$(\{x\}) = X$, a contradiction. Hence $\bigcap\{(i,j)$-$gbcl(\{x\}) : x \in X\} = \emptyset$.

Theorem 3.5. Let (X, τ_1, τ_2) be a bitopological space. Then the following statements are equivalent :
(a) (X, τ_1, τ_2) is pairwise gb-R_o space.
(b) For any $x \in X$, (i,j)-$gbcl(\{x\}) = (j,i)$-gb-ker$(\{x\})$, for $i,j = 1,2$ and $i \neq j$.
(c) For any $x \in X$, (i,j)-$gbcl(\{x\}) \subset (j,i)$-$gb$-ker$(\{x\})$, for $i,j = 1,2$ and $i \neq j$.
(d) For any $x, y \in X$, $y \in (i,j)$-gb-ker$(\{x\})$ if and only if $x \in (j,i)$-gb-ker$(\{y\})$, for $i,j = 1,2$ and $i \neq j$.
(e) For any $x, y \in X$, $y \in (i,j)$-$gbcl(\{x\})$ if and only if $x \in (j,i)$-$gbcl(\{y\})$, for $i,j = 1,2$ and $i \neq j$.
(f) For any (i,j)-gb-closed set V and $x \notin V$, there exist a (j,i)-gb-open set U such that $x \notin U$ and $V \subset U$, for $i,j = 1,2$ and $i \neq j$.
(g) For each (i,j)-gb-closed set V, $V = \bigcap\{U : U$ is (j,i)-gb-open and

$V \subset U\}$, for $i, j = 1, 2$ and $i \neq j$.
(h) For each (i,j)-gb-open set U, $U = \bigcup\{V : V$ is (j,i)-gb-closed and $V \subset U\}$, for $i, j = 1, 2$ and $i \neq j$.
(i) For every non-empty subset A of X and for any (i,j)-gb-open set U such that $A \cap U \neq \emptyset$, there exists a (j,i)-gb-closed V such that $A \cap V \neq \emptyset$ and $V \subset U$, for $i, j = 1, 2$ and $i \neq j$.
(j) For any (j,i)-gb-closed set V and $x \notin V$, (j,i)-$gbcl(\{x\}) \cap V = \emptyset$, for $i, j = 1, 2$ and $i \neq j$.

Proof. $(a) \Rightarrow (b)$ Let $x, y \in X$. Then by Definition 3.2, $y \in (j,i)$-gb-$\ker(\{x\}) \Leftrightarrow x \in (j,i)$-$gbcl(\{y\})$. Since X is pairwise gb-Ro space, therefore by Theorem 3.2, we have $x \in (j,i)$-$gbcl(\{y\}) \Leftrightarrow y \in (i,j)$-$gbcl(\{x\})$. Thus $y \in (j,i)$-gb-$\ker(\{x\}) \Leftrightarrow x \in (j,i)$-$gbcl(\{y\}) \Leftrightarrow y \in (i,j)$-$gbcl(\{x\})$. Hence (i,j)-$gbcl(\{x\}) = (j,i)$-gb-$\ker(\{x\})$.

$(b) \Rightarrow (c)$ It is obvious.

$(c) \Rightarrow (d)$ Let $x, y \in X$ and $y \in (i,j)$-gb-$\ker(\{x\})$. Then by Definition 3.2, $x \in (i,j)$-$gbcl(\{y\})$. Therefore by (c), $x \in (i,j)$-$gbcl(\{y\}) \subset (j,i)$-$gb$-$\ker(\{y\})$. Thus $x \in (j,i)$-gb-$\ker(\{y\})$. Similarly, we can prove the other part also.

$(d) \Rightarrow (e)$ Let $x, y \in X$ and $y \in (i,j)$-$gbcl(\{x\})$. Then by Definition 3.2, $x \in (i,j)$-gb-$\ker(\{y\})$. Therefore by (d), $y \in (j,i)$-gb-$\ker(\{x\})$ and so $x \in (j,i)$-$gbcl(\{y\})$. Similarly, we can prove the other part also.

$(e) \Rightarrow (f)$ Let V be a (i,j)-gb-closed set and $x \notin V$. Then for any $y \in V$, we have (i,j)-$gbcl(\{y\}) \subset V$ and $x \notin (i,j)$-$gbcl(\{y\})$. Therefore by (e), $y \notin (j,i)$-$gbcl(\{x\})$. That is there exists a (j,i)-gb-open set U_y such that $y \in U_y$ and $x \notin U_y$. Let $U = \bigcup_{y \in V}\{U_y : U_y$ is (j,i)-gb-open, $y \in U_y$ and $x \notin U_y\}$. Hence U is (j,i)-gb-open set such that $x \notin U$ and $V \subset U$.

$(f) \Rightarrow (g)$ Let V be an (i,j)-gb-closed set in X and $W = \bigcap\{U : U$ is (j,i)-gb-open and $V \subset U\}$. Clearly, $V \subset W$. Suppose that, $x \notin V$. Therefore by (f), there is a (j,i)-gb-open set U such that $x \notin U$ and $V \subset U$. So $x \notin W$ and thus $W \subset V$. Hence $V = W = \bigcap\{U : U$ is (j,i)-gb-open and $V \subset U\}$.

$(g) \Rightarrow (h)$ It is obvious.

$(h) \Rightarrow (i)$ Let A be a non-empty subset of X and U be a (i,j)-*gb*-open set in X such that $A \cap U \neq \emptyset$. Let $x \in A \cap U$. By (h), $U = \bigcup\{V : V$ is (j,i)-*gb*-closed and $V \subset U\}$. Then there is a (j,i)-*gb*-closed V such that $x \in V \subset U$. Therefore $x \in A \cap V$ and so $A \cap V \neq \emptyset$.

$(i) \Rightarrow (j)$ Let V be a (i,j)-*gb*-closed set such that $x \notin V$. Then $X \setminus V$ is (i,j)-*gb*-open set containing x and $\{x\} \cap (X \setminus V) \neq \emptyset$. Therefore by (i), there is a (j,i)-*gb*-closed set W such that $W \subset X \setminus V$ and $\{x\} \cap W \neq \emptyset$. Hence (j,i)-$gbcl(\{x\}) \subset X \setminus V$ and so (j,i)-$gbcl(\{x\}) \cap V = \emptyset$.

$(j) \Rightarrow (a)$ Follows from Theorem 3.1.

Theorem 3.6. In a pairwise *gb*-R_o space (X, τ_1, τ_2), for any $x \in X$, (i,j)-$gbcl(\{x\}) \cap (j,i)$-*gb*-$\ker(\{x\}) = \{x\}$ holds for $i,j = 1,2$ and $i \neq j$, then (i,j)-$gbcl(\{x\}) = \{x\}$.

Proof. Since (X, τ_1, τ_2) is pairwise *gb*-R_o space, therefore by Theorem 3.5 (b), we have (i,j)-$gbcl(\{x\}) = (j,i)$-*gb*-$\ker(\{x\})$. Hence the result follows.

Theorem 3.7. If (X, τ_1, τ_2) is a pairwise *gb*-R_o space, then for any $x, y \in X$, either (i,j)-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{x\}) = (i,j)$-$gbcl(\{y\}) \cap (j,i)$-$gbcl(\{y\})$ or $\{(i,j)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{x\})\} \cap \{(i,j)$-$gbcl(\{y\}) \cap (j,i)$-$gbcl(\{y\})\} = \emptyset$.

Proof. Let (X, τ_1, τ_2) is a pairwise *gb*-R_o space. Suppose that $\{(i,j)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{x\})\} \cap \{(i,j)$-$gbcl(\{y\}) \cap (j,i)$-$gbcl(\{y\})\} \neq \emptyset$. Let $z \in \{(i,j)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{x\})\} \cap \{(i,j)$-$gbcl(\{y\}) \cap (j,i)$-$gbcl(\{y\})\}$. Then (i,j)-$gbcl(\{z\}) \subset (i,j)$-$gbcl(\{x\}) \cap (i,j)$-$gbcl(\{y\})$ and (j,i)-$gbcl(\{z\}) \subset (j,i)$-$gbcl(\{x\}) \cap (j,i)$-$gbcl(\{y\})$. Since $z \in (i,j)$-$gbcl(\{x\})$, we have by (e) of Theorem 3.5, $x \in (j,i)$-$gbcl(\{z\})$. Therefore (j,i)-$gbcl(\{x\}) \subset (j,i)$-$gbcl(\{z\}) \subset (j,i)$-$gbcl(\{y\})$. Similarly, $z \in (j,i)$-$gbcl(\{x\})$ implies (i,j)-$gbcl(\{x\}) \subset (i,j)$-$gbcl(\{y\})$, $z \in (i,j)$-$gbcl(\{y\})$ implies (j,i)-$gbcl(\{y\}) \subset (j,i)$-$gbcl(\{x\})$ and also from $z \in (j,i)$-$gbcl(\{y\})$ implies (i,j)-$gbcl(\{y\}) \subset (i,j)$-$gbcl(\{x\})$. Thus (i,j)-$gbcl(\{x\}) = (i,j)$-$gbcl(\{y\})$ and (j,i)-$gbcl(\{x\}) = (j,i)$-$gbcl(\{y\})$. Hence the result follows.

Theorem 3.8. If (X, τ_1, τ_2) is a pairwise gb-R_o space, then for any $x, y \in X$, either (i,j)-gb-ker$(\{x\}) \cap (j,i)$-gb-ker$(\{x\}) = (i,j)$-gb-ker$(\{y\}) \cap (j,i)$-gb-ker$(\{y\})$ or $\{(i,j)$-gb-ker$(\{x\}) \cap (j,i)$-gb-ker$(\{x\})\} \cap \{(i,j)$-gb-ker$(\{y\}) \cap (j,i)$-gb-ker$(\{y\})\} = \emptyset$.

Proof. The proof is similar to that of Theorem 3.7 which follows from Definition of (i,j)-gb-ker$(\{x\})$ and Theorem 3.5.

4. Pairwise gb-R_1 Spaces

Definition 4.1. A bitopological space (X, τ_1, τ_2) is said to be pairwise generalized b-R_1(in short, pairwise gb-R_1) if for every pair of distinct points x and y of X such that (i,j)-$gbcl(\{x\}) \neq (j,i)$-$gbcl(\{y\})$, there exists a (j,i)-gb-open set U and an (i,j)-gb-open set V such that $U \cap V = \emptyset$ and (i,j)-$gbcl(\{x\}) \subset U$, (j,i)-$gbcl(\{y\}) \subset V$, for $i, j = 1, 2$ and $i \neq j$.

Theorem 4.1. If (X, τ_1, τ_2) is pairwise gb-R_1 space, then it is pairwise gb-R_o space.

Proof. Suppose that (X, τ_1, τ_2) is pairwise gb-R_1 space. Let U be an (i,j)-gb-open set containing x. Then for each $y \in X \setminus U$, (j,i)-$gbcl(\{x\}) \neq (i,j)$-$gbcl(\{y\})$. Since (X, τ_1, τ_2) is pairwise gb-R_1, there exists an (i,j)-gb-open set Uy and a (j,i)-gb-open set V_y such that $U_y \cap V_y = \emptyset$ and (i,j)-$gbcl(\{y\}) \subset V_y$, (j,i)-$gbcl(\{x\}) \subset U_y$. Let $A = \bigcup\{V_y : y \in X \setminus U\}$. Then $X \setminus U \subset A$, $x \notin A$ and A is (j,i)-gb-open set. Therefore (j,i)-$gbcl(\{x\}) \subset X \setminus A \subset U$ and hence (X, τ_1, τ_2) is pairwise gb-R_o space.

Theorem 4.2. A bitopological space (X, τ_1, τ_2) is pairwise gb-R_1 if and only if for every $x, y \in X$ such that (i,j)-$gbcl(\{x\}) \neq (j,i)$-$gbcl(\{y\})$, there exists an (i,j)-gb-open set U and a (j,i)-gb-open set V such that $x \in V$, $y \in U$ and $U \cap V = \emptyset$, for $i, j = 1, 2$ and $i \neq j$.

Proof. Suppose that (X, τ_1, τ_2) is pairwise gb-R_1 space. Let $x, y \in X$ such that (i,j)-$gbcl(\{x\}) \neq (j,i)$-$gbcl(\{y\})$. Then there exists an (i,j)-gb-open set U and a (j,i)-gb-open set V such that $x \in (i,j)$-$gbcl(\{x\}) \subset V$ and $y \in (j,i)$-$gbcl(\{y\}) \subset U$.

Conversely, suppose that there exists an (i, j)-gb-open set U and a (j, i)-gb-open set V such that $x \in V$, $y \in U$ and $U \cap V = \emptyset$. Therefore (i, j)-$gbcl(\{x\}) \cap (j, i)$-$gbcl(\{y\}) = \emptyset$. So by Theorem 3.2, (X, τ_1, τ_2) is pairwise gb-R_o space. Then (i, j)-$gbcl(\{x\}) \subset V$ and (j, i)-$gbcl(\{y\}) \subset U$. Hence (X, τ_1, τ_2) is pairwise gb-R_1 space.

Theorem 4.3. Let (X, τ_1, τ_2) be a bitopological space. Then the following are equivalent:
(a) (X, τ_1, τ_2) is pairwise gb-R_1 space.
(b) For any $x, y \in X$, $x \neq y$ and (i, j)-$gbcl(\{x\}) \neq (j, i)$-$gbcl(\{y\})$ implies that there exists an (i, j)-gb-closed set G_1 and a (j, i)-gb-closed set G_2 such that $x \in G_1$, $y \notin G_1$, $y \in G_2$, $x \notin G_2$ and $X = G_1 \cup G_2$, for $i, j = 1, 2$ and $i \neq j$.

Proof. $(a) \Rightarrow (b)$ Suppose that (X, τ_1, τ_2) is pairwise gb-R_1 space. Let $x, y \in X$ such that (i, j)-$gbcl(\{x\}) \neq (j, i)$-$gbcl(\{y\})$. Therefore by Theorem 4.2, there exists an (i, j)-gb-open set V and a (j, i)-gb-open set U such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Then $G_1 = X \setminus V$ is (i, j)-gb-closed and $G_2 = X \setminus U$ is (j, i)-gb-closed set such that $x \in G_1$, $y \notin G_1$, $y \in G_2$, $x \notin G_2$ and $X = G_1 \cup G_2$.

$(b) \Rightarrow (a)$ Let $x, y \in X$ such that (i, j)-$gbcl(\{x\}) \neq (j, i)$-$gbcl(\{y\})$. Therefore for any $x, y \in X$, $x \neq y$, we have (i, j)-$gbcl(\{x\}) \cap (j, i)$-$gbcl(\{y\}) = \emptyset$. Then by Theorem 3.2, (X, τ_1, τ_2) is pairwise gb-R_o space. By (b), there is an (i, j)-gb-closed set G_1 and a (j, i)-gb-closed set G_2 such that $x \in G_1$, $y \notin G_1$, $y \in G_2$, $x \notin G_2$ and $X = G_1 \cup G_2$. Therefore $x \in X \setminus G_2 = U$, which is (j, i)-gb-open and $y \in X \setminus G_1 = V$, which is (i, j)-gb-open. Which implies that (i, j)-$gbcl(\{x\}) \subset U$, (j, i)-$gbcl(\{y\}) \subset V$ and $U \cap V = \emptyset$. Hence the result.

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