DOI: 10.4067/S0716-09172017000300545

Corrigendum

Corrigendum to "An Extension of Sheffer Polynomials"

[Proyecciones Journal of Mathematics, Vol. 30, No 2, pp. 265 – 275, 2011]

A. K. Shukla, S. J. Rapeli

Department of Mathematics,
S. V. National Institute of Technology, Surat-395 007, India
E-mail: ajayshukla2@rediffmail.com, shrinu0711@gmail.com

We regret to announce that there are some mathematical errors in theorem 2.3 and theorem 2.4. Our aim to correct and modify the theorem 2.3 and theorem 2.4.

Brown [1] stated that $\{B_n(x)\}$ is a polynomial sequence which is simple and of degree precisely n. $\{B_n(x)\}$ is a binomial sequence if

$$B_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} B_{n-k}(x) B_k(y) \qquad n = 0, 1, 2, ...$$

and a simple polynomial sequence $\{P_n(x)\}$ is a Sheffer sequence if there is a binomial sequence $\{B_n(x)\}$ such that

$$P_n(x+y) = \sum_{k=0}^{n} \binom{n}{k} B_{n-k}(x) P_k(y) \qquad n = 0, 1, 2, ...$$

Theorem 2.3: Let $p_n(x, y)$ be symmetric, a class of polynomials in two variables and Sheffer A-type zero which belong to the operator $J(D)$ and have the generating function as equation (1.11) of theorem 2.1 (See [2]). There exist sequences $\alpha_k^{(s)}, \mu_k$ and η_k, independent of x, y and n, such that for all $n \geq 1$,

$$np_n(x,y) = \sum_{k=0}^{n-1} \sum_{i=1}^{r} \left(\alpha_k^{(i)} + x\varepsilon_i^{k+1}\mu_k + y\varepsilon_i^{k+1}\eta_k \right) p_{n-k-1}(x,y), \qquad (1.15)$$

where $\mu_k = (k+1)g_k$ in terms of g_k of equation (1.8) and $\eta_k = (k+1)h_k$, in terms of h_k of equation (1.9).

Proof: Let (See [2])

$$\sum_{n=0}^{\infty} np_n(x,y)\,t^n = t\left[\sum_{i=1}^{r}\left\{A_i'(t) + x\varepsilon_i G'(\varepsilon_i t)\,A_i(t) + y\varepsilon_i H'(\varepsilon_i t)\,A_i(t)\right\}\right.$$
$$\left.\exp\left(xG(\varepsilon_i t)\right)\exp\left(yH(\varepsilon_i t)\right)\right]$$

$$= t\left[\sum_{i=1}^{r}\left\{\frac{A_i'(t)}{A_i(t)} + x\varepsilon_i G'(\varepsilon_i t) + y\varepsilon_i H'(\varepsilon_i t)\right\}\right.$$
$$\left.A_i(t)\exp\left(xG(\varepsilon_i t)\right)\exp\left(yH(\varepsilon_i t)\right)\right]$$

$$= \sum_{n=1}^{\infty}\sum_{k=0}^{n-1}\sum_{i=1}^{r}\left(\alpha_k^{(i)} + x\varepsilon_i^{k+1}\mu_k + y\varepsilon_i^{k+1}\eta_k\right)p_{n-k-1}(x,y)t^n.$$

Thus we get

$$np_n(x,y) = \sum_{k=0}^{n-1}\sum_{i=1}^{r}\left(\alpha_k^{(i)} + x\varepsilon_i^{k+1}\mu_k + y\varepsilon_i^{k+1}\eta_k\right)p_{n-k-1}(x,y).$$

This gives the proof of statement.

Theorem 2.4: A necessary and sufficient condition that $p_n(x,y)$ be of Sheffer A-type zero, there exists sequence g_k and h_k, independent of x, y and n, such that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)p_n(x,y) = \sum_{k=0}^{n-1}\sum_{i=1}^{r}\left(\varepsilon_i^{k+1}g_k + \varepsilon_i^{k+1}h_k\right)\ p_{n-k-1}(x,y),$$

where $p_n(x,y)$ is symmetric and a class of polynomials in two variables.

Proof: Let (See [2])

$$\sum_{n=0}^{\infty}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)p_n(x,y)\,t^n = \sum_{i=1}^{r}\left\{G(\varepsilon_i t) + H(\varepsilon_i t)\right\}A_i(t)\exp\left(xG(\varepsilon_i t)\right)\exp\left(yH(\varepsilon_i t)\right)$$

$$= \sum_{i=1}^{r} \sum_{k=0}^{\infty} \left(\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k\right)\ t^{k+1} \ \sum_{n=0}^{\infty} p_n(x,y) t^n,$$

$$= \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} \sum_{i=1}^{r} \left(\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k\right)\ p_{n-k-1}(x,y) t^n.$$

Thus

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) p_n\left(x,y\right) = \sum_{k=0}^{n-1} \sum_{i=1}^{r} \left(\varepsilon_i^{k+1} g_k + \varepsilon_i^{k+1} h_k\right)\ p_{n-k-1}(x,y).$$

This is the proof of theorem 2.4.

References:

1. J. W. Brown, On multivariable Sheffer sequences, J. Math. Anal. Appl., Vol. 69, (1979), pp.398-410.

2. A. K. Shukla and S. J. Rapeli, An Extension of Sheffer Polynomials, Proyecciones Journal of Mathematics, Vol. 30, No. 2 (2011), pp. 265-275.