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ON THE NUMERICAL RECONSTRUCTION OF A SPRING-MASS SYSTEM FROM ITS NATURAL FREQUENCIES

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Abstract

Given the sequences of real numbers $(\lambda_i)_1^n$, $(\mu_i)_1^{n-1}$, *satisfying the interlacing property* $\lambda_i < \mu_i < \lambda_{i+1}$, $1 \leq i \leq n-1$, *we present a new numerical procedure to construct a spring-mass system with eigenvalues* $(\lambda_i)_1^n$, *where the interlaced spectrum* $(\mu_i)_1^{n-1}$ *corresponds to the modified system whose mass* m_{r+1}, $1 \leq r \leq n-2$, *is fixed. The method, which is a modification of the fast orthogonal reduction technique, appears to be computationally less expensive than other in the literature.*

KEY WORDS : *Inverse vibration problems, mass matrix, stiffness matrix, Jacobi matrix.*

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1. Introduction

Inverse problems have important applications in physics and engineering, in areas such as circuit, control and vibration theory. The vibration analysis of an structural system is performed as an approximation to the continuous system obtained by discretizing the mechanical properties. For an undamped discrete system with n-degrees of freedom the governing equation is

$$(K_n - \lambda M_n)x_n = 0, \tag{1.1}$$

where the matrices M_n and K_n of order n are the mass matrix and the stiffness matrix, respectively. The eigenvalues λ_i of the equation (1.1) are related with the natural frequencies ω_i ($\lambda_i = \omega_i^2$) and the eigenvectors $x_n^{(i)} = (x_{n,1}^{(i)}, x_{n,2}^{(i)}, \ldots, x_{n,n}^{(i)})^T$, $i = 1, 2, ..., n$, represent the vibration modes.

Consider the spring-mass system consisting of n masses $m_i > 0$ connected by n springs of stiffnesses $k_i > 0$, as shown in figure (1). Here, the mass matrix is $M_n = \text{ diag}\{m_1, m_2, ..., m_n\}$ and the stiffness matrix

$$K_n = \begin{bmatrix} k_1 + k_2 & -k_2 & & & & \\ -k_2 & k_2 + k_3 & -k_3 & & & \\ & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & -k_{n-1} & k_{n-1} + k_n & -k_n & \\ & & & -k_n & k_n & \end{bmatrix}$$

The spring-mass system, denoted by (M_n, K_n), is a real physical system if $m_i > 0$ and $k_i > 0$, $i = 1, 2, ..., n$. These properties imply that M_n and K_n are positive definite matrices. Moreover, as K_n is also symmetric and tridiagonal, the eigenvalues are real, positive and distinct.

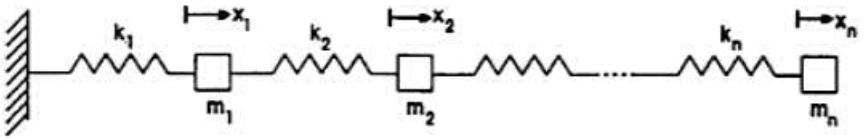

Figure 1: Spring-mass system fixed-free.

The structural properties of the matrices M_n and K_n allow us to reduce the generalized eigenvalue equation (1.1) to the standard form

$$(J_n - \lambda_i I_n)\, v_n^{(i)} = 0,$$

where

$$J_n = B_n^{-1} K_n (B_n)^{-1}, \qquad v_n^{(i)} = B_n x_n^{(i)}, \tag{1.2}$$

and $B_n = \text{diag}\{m_1^{1/2}, m_2^{1/2}, ..., m_n^{1/2}\}$. Thus, the matrix J_n is symmetric tridiagonal positive definite with the same eigenvalues of the system (M_n, K_n). We call J_n a Jacobi matrix. Clearly, the relation (1.2) implies that the diagonal elements of J_n are positive, while the co-diagonal elements are negative. By the similarity transformation $J' = D_n J_n D_n$, where $D_n = \text{diag}\{(-1)^i, i = 1, 2, \ldots, n\}$, the codiagonal elements become positive. Then, without loss of generality we may assume that J_n has the form

$$J_n = \begin{bmatrix} a_1 & b_1 & & & \\ b_1 & a_2 & b_2 & & \\ & \cdot & \cdot & \cdot & \cdot \\ & & b_{n-2} & a_{n-1} & b_{n-1} \\ \cdot & & & b_{n-1} & a_n \end{bmatrix},$$

with $a_i, b_i > 0$, $i = 1, 2, \ldots, n$.

The natural frequencies of the system (M_n, K_n) are usually computed by using the matrices M_n and K_n. The problem here is to identify a system from the natural frequencies. This identification

problem may be solved in two ways. The first way start with a pair of matrices (the mass matrix and the stiffness matrix) and then by using an optimization method, determines the matrices M_n and K_n which identify the specified spectral data. The second strategy start with the spectral information to recover the matrices M_n and K_n, directly.

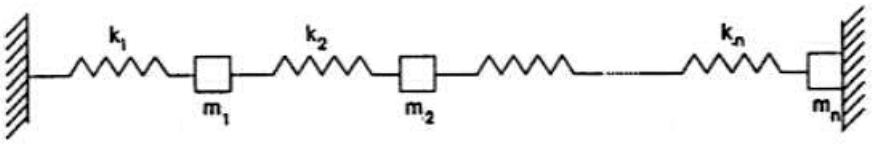

Figure 2:

The second procedure have been studied by Gladwell [4], [5], [6] and others [10], [11], [12]. Krein [8] proved that the matrices M_n and K_n can be uniquely reconstructed if the following information is given:
The eigenvalues $(\lambda_i)_1^n$ of the original system (Fig 1).
The eigenvalues $(\mu_i)_1^{n-1}$ corresponding to the modified system whose last mass is fixed (Fig 2).
An additional factor. For example, the total mass of the system $m_T = \sum_{i=1}^{n} m_i$.

The interlacing property

$$\lambda_1 < \mu_1 < \lambda_2 < \cdots < \mu_{n-1} < \lambda_n$$

is a necessary and sufficient condition for the existence of a real physical system, where the set $(\mu_i)_1^{n-1}$ is called the interlaced spectrum. The construction of the system is performed in two parts. Firstly, the matrix J_n is reconstructed by the adequate use of some standard tridiagonalization method for symmetric matrices, like Householder transformations, Givens rotations, or Lanczos method. Once J_n is obtained and considering the relations in (1.2), the second part consists in to separate the masses and the stiffnesses of J_n. This is possible if we consider the equation

$$K_n \left(1, 1, \ldots, 1\right)^T = B_n J_n B_n \left(1, 1, \ldots, 1\right)^T = \left(k_1, 0, \ldots, 0\right)^T,$$

where J_n was previously determined. This imply that

$$J_n \left(m_1^{1/2}, m_2^{1/2}, \ldots, m_n^{1/2}\right)^T = \left(k_1 m_1^{-1/2}, 0, \ldots, 0\right)^T.$$

Now, if $y_n = (y_{n,1}, y_{n,2}, \cdots, y_{n,n})^T \neq 0$ is the solution of the system

$$J_n y_n = (1, 0, \ldots, 0)^T,$$

then

$$m_i = m_n \left(\frac{y_{n,i}}{y_{n,n}}\right)^2, \qquad i = 1, 2, \ldots, n-1. \tag{1.3}$$

Moreover, if the total mass of the system is known, from (1.3), we have that the last mass is given by

$$m_n = \frac{m_T}{1 + \sum_{i=1}^{n-1} \left(\frac{y_{n,i}}{y_{n,n}}\right)^2}.$$

Hence, the mass matrix M_n is completely determined. Now, we may use the relations (1.2) to compute the stiffness matrix K_n. In this way, we may determine the system (M_n, K_n), completely.

The reconstruction of a spring-mass system may also be performed by using the interlaced spectrum corresponding to a modified system with different bounds conditions (see [4], [10]). The modified system considered in this paper consists in to fix any mass of the system (M_n, K_n), other than the extreme masses. Thus, the problem we study here may be formulated as follows:

Problem 1. *Given the sequences of real numbers $(\lambda_i)_1^n$, $(\mu_i)_1^{n-1}$ satisfying the interlacing property*

$$\lambda_i < \mu_i < \lambda_{i+1}, \qquad 1 \leq i \leq n-1$$

reconstruct a spring-mass system (M_n, K_n) with eigenvalues $(\lambda_i)_1^n$, where the interlaced spectrum $(\mu_i)_1^{n-1}$ corresponds to the modified system whose mass m_{r+1}, $1 \leq r \leq n-2$, is fixed.

It is clear that if we fix the $(r+1)^{th}$ mass, the resulting modified system consists of two spring-mass systems, (M_r, K_r) and (M_p, K_p), with natural frequencies $(\gamma_i)_1^r$ and $(\nu_i)_1^p$, respectively, where $p = n - r - 1$. The structural properties of the matrices related to the systems (M_r, K_r) and (M_p, K_p) allow to partition the matrix J_n in the form:

$$J_n = \begin{bmatrix} J_r & b_r & 0 \\ b_r & a_{r+1} & b_{r+1} \\ 0 & b_{r+1} & J_p \end{bmatrix}, \tag{1.4}$$

where the Jacobi submatrices J_r and J_p are given by $J_r = B_r^{-1} K_r (B_r^T)^{-1}$ and $J_p = B_p^{-1} K_p (B_p^T)^{-1}$, with $B_r = \text{ diag}\{m_1^{1/2}, m_2^{1/2}, ..., m_r^{1/2}\}$, $B_p = \text{diag}\{m_{r+2}^{1/2},$
$m_{r+3}^{1/2}, ..., m_p^{1/2}\}$. As the system (M_n, K_n) can be reconstructed from the matrix J_n, it is enough to reconstruct J_n from the sets $(\lambda_i)_1^n$, $(\gamma_i)_1^r$, and $(\nu_i)_1^p$ to obtain (M_n, K_n). Thus, the Problem 1 is reduced to:

Problem 2. *Given the sequence of real numbers* $(\lambda_i)_1^n$, $(\mu_i)_1^{n-1} = (\gamma_i)_1^r \cup (\nu_i)_1^p$ *satisfying the interlacing property*

$$\lambda_i < \mu_i < \lambda_{i+1}, \qquad 1 \leq i \leq n-1 \tag{1.5}$$

reconstruct the matrix J_n *of (1.4) such that*

$$\sigma(J_n) = (\lambda_i)_1^n, \qquad \sigma(J_r) = (\gamma_i)_1^r, \qquad \sigma(J_p) = (\nu_i)_1^p, \quad p = n - r - 1.$$

The existence and uniqueness of the solution for the Problem 2 have been considered in [1], [3], [7]. Algorithms to compute the solution J_n are proposed in [6] and [13]. For the tridiagonalization process, they use the *Lanczos algorithm and Householder transformations*, respectively. The computational costs in these methods are in general of order On^3.

In [2] and [7] is presented the *fast orthogonal reduction* algorithm, which reconstruct the matrix J_n from $(\lambda_i)_1^n$ and $(\mu_i)_1^{n-1}$, where $(\mu_i)_1^{n-1}$ is the interlaced spectrum corresponding to the submatrix $J_{n\backslash 1}$. In section 2, we modify the *fast orthogonal reduction* algorithm to derive a new direct numerical procedure to solve Problem 2. This new method has only a computational cost of order On^2. In section 3 we present some numerical examples which confirm the efficiency of the method.

2. The algorithm

The basic method in which our proposal is based is the fast orthogonal reduction method presented by Gragg and Harrow in [7]. The idea is firstly to construct a bordered diagonal matrix A_{n+1} of the form

$$A_{n+1} = \begin{bmatrix} a_0 & v_n^T \\ v_n & \Lambda_n \end{bmatrix}, \tag{2.1}$$

where a_0 is a dummy entry, $\Lambda_n = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$, and $v_n = (v_{n,1}^{(1)}, v_{n,1}^{(2)}, \ldots, v_{n,1}^{(n)})^T$ is the first (or last) row of eigenvectors of J_n. Then, we reduce A_{n+1} to the tridiagonal form by applying a sequence of orthogonal plane rotations in a particular order.

This orthogonal reduction process is called *algorithm of Rutishauser*, which reduce the matrix (2.1) to the tridiagonal form

$$J_{n+1} = \begin{bmatrix} a_0 & e_1^T \\ e_1 & J_n \end{bmatrix},$$

where $e_1 = (1, 0, \ldots, 0)^T$ and J_n is the required Jacobi matrix with eigenvalues $(\lambda_i)_1^n$. The *fast orthogonal reduction* method has a computational cost of $O(n^2)$ operations, which is increased to $O(n^3)$ if we use Lanczos or Householder transformations. A complete discussion of the *fast orthogonal reduction* method can be found in [2], [7].

The idea of our proposal is to construct the submatrices J_r, J_p, and the elements b_r, b_{r+1}, and a_{r+1}, where $p = n - r - 1$, by using an adequate modification of the *fast orthogonal reduction* algorithm. Clearly, the diagonal element a_{r+1} can be computed by

$$a_{r+1} = \sum_{i=1}^{n} \lambda_i - \sum_{i=1}^{r} \gamma_i - \sum_{i=1}^{p} \nu_i. \tag{2.2}$$

Now let us define the bordered diagonal matrices

$$A_{r+1} = \begin{bmatrix} \Lambda_r & u_r \\ u_r^T & a_0^r \end{bmatrix}, \quad A_{p+1} = \begin{bmatrix} a_0^p & w_p^T \\ w_p & \Lambda_p \end{bmatrix}, \tag{2.3}$$

where $u_r = (u_{r,r}^{(1)}, u_{r,r}^{(2)}, \ldots, u_{r,r}^{(r)})^T$ and $w_p = (w_{p,1}^{(1)}, w_{p,1}^{(2)}, \ldots, w_{p,1}^{(p)})^T$ are respectively, the last and first row of the matrices of eigenvectors of J_r

and J_p. The submatrices Λ_r and Λ_p are diagonal matrices having as their diagonal entries the eigenvalues of J_r and J_p, respectively. If the eigenvectors u_r and w_p are known, then it is possible to apply the *fast orthogonal reduction* algorithm to orthogonally reduce the matrices A_{r+1} and A_{p+1} to their tridiagonal forms, obtaining in this way the searched matrices J_r and J_p.

Consider the following notation: In general, $A_{j\backslash i}$ will denote the principal submatrix of order $(j-1)$, obtained from A_j, by deleting its i^{th} row and column. $P_n(\lambda)$, $Q_r(\lambda)$, and $S_p(\lambda)$ will denote the characteristic polynomials of J_n, J_r, and J_p, with $p = n-r-1$, respectively, while $Q_{r\backslash r}(\lambda)$ and $S_{p\backslash 1}(\lambda)$ will denote the characteristic polynomials of $J_{r\backslash r}$, and $J_{p\backslash 1}$, respectively.

By expanding $\det(\lambda I_n - J_n)$ along its r^{th} row we find that

$$P_n(\lambda) = (\lambda - a_{r+1})Q_r(\lambda)S_p(\lambda) - b_r^2 Q_{r\backslash r}(\lambda)S_p(\lambda) - b_{r+1}^2 Q_r(\lambda)S_{p\backslash 1}(\lambda). \tag{2.4}$$

We know that

$$P_n(\lambda) = \prod_{i=1}^{n}(\lambda - \lambda_i), \quad Q_r(\lambda) = \prod_{i=1}^{r}(\lambda - \gamma_i), \quad S_p(\lambda) = \prod_{i=1}^{p}(\lambda - \nu_i) \tag{2.5}$$

Setting $\lambda = \gamma_i$ and $\lambda = \nu_i$ en (2.4) we obtain

$$P_n(\gamma_i) = -b_r^2 Q_{r\backslash r}(\gamma_i)S_p(\gamma_i), \quad 1 \le i \le r \tag{2.6}$$

and

$$P_n(\nu_i) = -b_{r+1}^2 S_{p\backslash 1}(\nu_i)Q_r(\nu_i), \quad 1 \le i \le p, \tag{2.7}$$

respectively.

Assume that $U_r = \left[u_r^{(1)}\ u_r^{(2)}\ \cdots\ u_r^{(r)}\right]$ is the orthogonal matrix of eigenvectors of J_r. Then $U_r^T(\lambda I_r - J_r)U_r = \lambda I_r - \Lambda_r$, and we have

$$(\lambda I_r - J_r)^{-1} = U_r(\lambda I_r - \Lambda_r)^{-1}U_r^T. \tag{2.8}$$

The left side of (2.8) is

$$(\lambda I_r - J_r)^{-1} = \frac{1}{Q_r(\lambda)} \cdot \mathrm{adj}(\lambda I_r - J_r) = \frac{1}{Q_r(\lambda)} \cdot \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ \cdot & \cdot & \cdot & \cdot \\ * & * & * & Q_{r\setminus r}(\lambda) \end{bmatrix},$$

while the right side is

$$U_r(\lambda I_r - \Lambda_r)^{-1} U_r^T = \begin{bmatrix} \dfrac{u_r^{(1)}}{\lambda - \gamma_1} & \dfrac{u_r^{(2)}}{\lambda - \gamma_2} \cdots \dfrac{u_r^{(r)}}{\lambda - \gamma_r} \end{bmatrix} U_r^T.$$

Comparing the entries in position (r, r) in both sides of (2.8) we find that

$$\frac{Q_{r\setminus r}(\lambda)}{Q_r(\lambda)} = \sum_{i=1}^{r} \frac{\left[u_{r,r}^{(i)}\right]^2}{\lambda - \gamma_i},$$

Taking the limit when λ tends to γ_i, we obtain

$$\frac{Q_{r\setminus r}(\gamma_i)}{Q_r'(\gamma_i)} = \left[u_{r,r}^{(i)}\right]^2, 1 \le i \le r, \tag{2.9}$$

where $Q_r^{'}(\gamma_i) = \prod_{j=1, j\neq i}^{r} (\gamma_i - \gamma_j)$. From (2.6) and (2.9), we obtain

$$b_r^2 \left[u_{r,r}^{(i)}\right]^2 = -\frac{P_n(\gamma_i)}{S_p(\gamma_i) Q_r'(\gamma_i)}, 1 \le i \le r. \tag{2.10}$$

and

$$\begin{array}{rcl} \sum_{i=1}^{r} \left(-\frac{P_n(\gamma_i)}{S_p(\gamma_i) Q_r'(\gamma_i)} \right) & = & \sum_{i=1}^{r} b_r^2 \left[u_{r,r}^{(i)}\right]^2 \\ & = & b_r^2 \sum_{i=1}^{r} \left[u_{r,r}^{(i)}\right]^2 \\ & = & b_r^2 \end{array} \tag{2.11}$$

The interlacing property of the eigenvalues (1.5) guarantee that the right side of (2.10) is always positive. Thus, the equations (2.5), (2.10) and (2.11) allow us to determine all the elements of the vector u_r and the codiagonal element b_r.

If $W_p = \left[w_p^{(1)}\ w_p^{(2)}\ \cdots\ w_p^{(p)}\right]$ is the matrix of eigenvectors of J_p, the same procedure leads to

$$\frac{S_{p\backslash 1}(\nu_i)}{S_p^{'}(\nu_i)} = \left[w_{p,1}^{(i)}\right]^2, 1 \leq i \leq r, \tag{2.12}$$

where $S_p^{'}(\nu_i) = \prod_{j=1, j\neq i}^{r} (\nu_i - \nu_j)$. From (2.12) and (2.7) we have

$$b_{r+1}^2 \left[w_{p,1}^{(i)}\right]^2 = -\frac{P_n(\nu_i)}{S_p^{'}(\nu_i) Q_r(\nu_i)}, 1 \leq i \leq p, \tag{2.13}$$

and

$$\begin{array}{rcl} \sum_{i=1}^{p} \left(-\frac{P_n(\nu_i)}{S_p(\nu_i) Q_r(\nu_i)}\right) & = & \sum_{i=1}^{p} b_{r+1}^2 \left[w_{p,1}^{(i)}\right]^2 \\ & = & b_{r+1}^2 \sum_{i=1}^{p} \left[w_{p,1}^{(i)}\right]^2 \\ & = & b_{r+1}^2 \end{array} \tag{2.14}$$

Hence, from (2.5), (2.13) and (2.14), the first row of the matrix of eigenvectors of J_p and the codiagonal element b_{r+1} are computed.

The algorithm can be summarized as follows:

Algorithm 1. *Given the sequences of real numbers* $(\lambda_i)_1^n$, $(\mu_i)_1^{n-1} = (\gamma_i)_1^r \cup (\nu_i)_1^p$, $p = n - r - 1$, *satisfying the interlacing property* $\lambda_i < \mu_i < \lambda_{i+1}, 1 \leq i \leq n-1$, *the algorithm produces the matrices* J_n, J_r, *and* J_p *of (1.4) such that* $\sigma(J_n) = (\lambda_i)_1^n, \sigma(J_r) = (\gamma_i)_1^r, \sigma(J_p) = (\nu_i)_1^p$, $p = n - r - 1$*:*

1. *Compute* a_{r+1} *by (2.2).*
2. *Find* b_r *and* b_{r+1} *using equations (2.11) and (2.14), respectively.*
3. *Compute the elements of the vectors* u_r *and* w_p *by (2.10) and (2.13), respectively.*
4. *Use the fast orthogonal reduction algorithm to reduce the matrices* A_{r+1} *and* A_{p+1} *of (2.3) to the tridiagonal form* J_{r+1} *and* J_{p+1}*, respectively, to obtain the Jacobi matrix of (1.4).*

This algorithm compute the elements a_{r+1}, b_r, and b_{r+1}, with approximately $O(n^2)$ operations. In step 4, the algorithm produces the matrices J_r and J_p with about $O(r^2)$ and $O(p^2)$ operations, respectively. Thus, to compute the whole matrix J_n we need $O(n^2)$ operations. Then this procedure is computationally less expensive than the algorithms proposed in [6] and [13], which require about $O(n^3)$ operations. Since it updates only a few entries of the matrix in each step (Householder transformations update all the entries in each step), the rounding errors has less influence on the process.

3. Numerical Results

In this Section we present two numerical examples to show the accuracy of the algorithm given in Section 2. The data used correspond to the eigenvalues of a known Jacobi matrix $\overline{J}_n$, so that the reconstructed matrix can be compared with $\overline{J}_n$. To this end, we define $e_a = \frac{\|\overline{a}-a\|_\infty}{\|\overline{a}\|_\infty}$, and $e_b = \frac{\|\overline{b}-b\|_\infty}{\|\overline{b}\|_\infty}$, where $\overline{a}, \overline{b}$ are the vectors whose components are diagonal and codiagonal elements of $\overline{J}_n$, and a, b are the obtained after the algorithm 3 is applied.

Table 1 shows the numerical results obtained when we recover a Jacobi matrix J_n of order $n = 6$ with diagonal elements $a_i = 2.0$ and codiagonal elements $b_i = 1.0$. The exact eigenvalues of J_n, J_r, and J_p are gives by $\lambda_k = 2 - 2\cos\left(\frac{k\pi}{n+1}\right)$, $\gamma_k = 2 - 2\cos\left(\frac{k\pi}{r+1}\right)$, and $\nu_k = 2 - 2\cos\left(\frac{k\pi}{n-p-1}\right)$, respectively. The same example is considered in Table 2, for $n = 20$. The obtained relative errors confirm a better accuracy than the examples presented in ([13]).

Table 1

n	r	p	a_i	b_i	e_a	e_b
6	2	3	2.00000000	1.00000000	5.0×10^{-7}	1.0×10^{-7}
			2.00000000	1.00000000		
			2.00000100	0.99999990		
			2.00000000	1.00000000		
			2.00000000	1.00000000		
			2.00000000			
6	3	2	2.00000000	1.00000000	5.0×10^{-7}	1.0×10^{-7}
			2.00000000	1.00000000		
			2.00000000	0.99999990		
			2.00000100	1.00000000		
			2.00000000	1.00000000		
			2.00000000			
6	4	1	2.00000000	1.00000000	5.0×10^{-7}	1.0×10^{-7}
			2.00000000	1.00000000		
			2.00000000	1.00000000		
			2.00000000	0.99999990		
			2.00000000	1.00000000		
			2.00000100			

Table 2

n	r	p	a_i	b_i	a_i	b_i	e_a	e_b
20	16	3	2.00000100	1.000000	2.00000000	0.99999990	1.0×10^{-6}	1.0×10^{-6}
			2.00000000	0.99999940	1.99999900	1.00000000		
			1.99999900	1.00000000	2.00000100	1.00000000		
			2.00000000	1.00000000	2.00000000	1.00000000		
			2.00000000	1.00000000	2.00000000	1.00000000		
			2.00000000	0.99999990	2.00000000	0.99999990		
			2.00000100	1.00000100	2.00000000	1.00000000		
			2.00000000	1.00000100	2.00000000	1.00000000		
			2.00000200	0.99999990	2.00000000	1.00000000		
			2.00000000	0.99999960	2.00000000			
20	12	7	2.00000100	1.00000100	2.00000100	1.00000000	1.0×10^{-6}	1.0×10^{-6}
			2.00000000	0.99999990	2.00000000	1.00000000		
			2.00000100	0.99999980	2.00000000	1.00000000		
			2.00000000	1.00000000	2.00000000	1.00000000		
			2.00000000	1.00000000	2.00000000	0.99999990		
			2.00000000	0.99999990	2.00000000	1.00000000		
			2.00000000	0.99999990	2.00000000	1.00000000		
			2.00000000	0.99999970	2.00000000	0.99999990		
			2.00000000	0.99999980	2.00000000	1.00000000		
			1.99999800	1.00000000				
20	10	9	2.00000200	1.00000000	2.00000000	1.00000000	1.0×10^{-6}	1.0×10^{-6}
			2.00000000	0.99999940	2.00000000	0.99999990		
			2.00000000	0.99999970	2.00000000	1.00000000		
			1.99999800	1.00000000	1.99999900	0.99999950		
			2.00000000	1.00000000	2.00000000	0.99999980		
			2.00000000	1.00000000	2.00000000	0.99999960		
			2.00000100	1.00000000	2.00000100	1.00000000		
			2.00000000	0.99999980	2.00000100	1.00000000		
			2.00000000	1.00000000	2.00000100	1.00000100		
			2.00000000	0.99999980	2.00000000			

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