Proyecciones
Vol. 21, N^o 1, pp. 51-63, May 2002.
Universidad Católica del Norte
Antofagasta - Chile
DOI: 10.4067/S0716-09172002000100004

ON FUZZY WEAKLY SEMIOPEN FUNCTIONS

MIGUEL CALDAS
Universidade Federal Fluminense, Brasil

GOVINDAPPA NAVALAGI
G. H. College, India
and
RATNESH SARAF
Govt. K. N. G. College - India

Abstract

In this paper, we introduce and characterize fuzzy weakly semiopen functions between fuzzy topological spaces as natural dual to the fuzzy weakly semicontinuous functions and also study these functions in relation to some other types of already known functions.

2000 Math. Subject Classification : *Primary: 54A40.*

Key Words and Phrases : *Fuzzy semi-open sets, fuzzy weakly open functions, fuzzy extremally disconnected spaces, fuzzy almost compact spaces.*

1. Introduction and Preliminaries

The concept of fuzzy sets was introduced by Prof. L.A. Zadeh in his classical paper[15]. After the discovery of the fuzzy subsets, much attention has been paid to generalize the basic concepts of classical topology in fuzzy setting and thus a modern theory of fuzzy topology is developed. The notion of fuzzy subsets naturally plays a significant role in the study of fuzzy topology which was introduced by C.L. Chang [4] in 1968. In 1980, Ming and Ming [9], introduced the concepts of quasi-coincidence and q-neighbourhoods by which the extensions of functions in fuzzy setting can very interestingly and effectively be carried out. In 1985, D.A. Rose [13] defined weakly open functions in a topological spaces. In 1997 J.H. Park, Y.B. Park and J.S. Park [12] introduced the notion of weakly open functions for a fuzzy topological space. In this paper we introduce and discuss the concept of fuzzy weakly semiopen function which is weaker than fuzzy weakly open and f.a.o.N functions introduced by [12] and Nanda [11] respectively and we obtained several properties and characterizations of these functions comparing with the other functions. Here it is seen that fuzzy semiopenness implies fuzzy weakly semiopenness but not conversely. But under a certain condition the converse is also true.

Throughout this paper by (X, τ) or simply by X we mean a fuzzy topological space (fts, shorty) due to Chang [4]. A point fuzzy in X with support $x \in X$ and value $p (0 < p \leq 1)$ is denoted by x_p. Two fuzzy sets λ and β are said to be quasi-coincident (q-coincident, shorty) denoted by $\lambda q \beta$, if there exists $x \in X$ such that $\lambda(x) + \beta(x) > 1$ [9] and by $\overline{q}$ we denote "is not q-coincident". It is known [9] that $\lambda \leq \beta$ if and only if $\lambda \overline{q}(1 - \beta)$. A fuzzy set λ is said to be q-neighbourhood (q-nbd) of x_p if there is a fuzzy open set μ such that $x_p q \mu$ and $\mu \leq \lambda$ if $\mu(x) \leq \lambda(x)$ for all $x \in X$. The interior, closure and the complement of a fuzzy set λ in X are denoted by $Int(\lambda), Cl(\lambda)$ and $1 - \lambda$ respectively. For definitions and results not explained in this paper, the reader is referred a [1, 4, 5, 9, 12, 14, 15] assuming them to be well known.

Definition 1 A fuzzy set λ in a fts X is called,
(i) Fuzzy semiopen [1] if $\lambda \leq Cl(Int(\lambda))$.
(ii) Fuzzy semiclosed [1] if $Int(Cl(\lambda)) \leq \lambda$.

(iii) Fuzzy preopen [3] if $\lambda \leq Int(Cl(\lambda))$.
(iv) Fuzzy regular open [1] if $\lambda = Int(Cl(\lambda))$.
(v)Fuzzy α-open [3] if $\lambda \leq Int(Cl(Int(\lambda)))$.
(vi)Fuzzy β-open [8]if $\lambda \leq Cl(Int(Cl(\lambda)))$.

Recall that if, λ be a fuzzy set in a fts X then $sCl(\lambda) = \cap\{\beta/\beta \geq \lambda, \quad \beta$ is fuzzy semiclosed$\}$ (resp. $sInt(\lambda) = \cup\{\beta/\lambda \geq \beta, \quad \beta$ is fuzzy semiopen$\}$) is called a fuzzy semiclosure of λ (resp. fuzzy semi interior of λ) [14].

Result: A fuzzy set λ in a fts X is fuzzy semiclosed (resp. fuzzy semiopen) if and only if $\lambda = sCl(\lambda)$ (resp. $\lambda = sInt(\lambda)$) [7].

Definition 2 Let $f : (X, \tau) \to (Y, \sigma)$ be a function from a fts (X, τ) into a fts (Y, σ). The function $f$is called:
(i) fuzzy semiopen [1] if $f(\lambda)$ is a fuzzy semiopen set of Y for each fuzzy open set λ in X.
(ii) fuzzy weakly open [12] if $f(\lambda) \leq Int(f(Cl(\lambda)))$ for each fuzzy open set λ in X.
(iii) fuzzy almost open (written as f.a.o.N) [11] if $f(\lambda)$ is a fuzzy open set of Y for each fuzzy regular open set λ in X.
(iv) fuzzy β-open [8] if $f(\lambda)$ is a fuzzy β-open set of Y for each fuzzy open set λ in X.

2. Fuzzy Weakly Semiopen Functions

Since fuzzy semicontinuity [1] is dual to fuzzy semiopenness [1], we define in this paper the concept of fuzzy weak semiopenness as natural dual to the fuzzy weak semicontinuity [5].

Definition 3 A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy weakly semiopen if $f(\lambda) \leq sInt(f(Cl(\lambda)))$ for each fuzzy open set λ of X.

Clearly, every fuzzy weakly open function is fuzzy weakly semiopen and every fuzzy semiopen function is also fuzzy weakly semiopen.

Example 2.1. Let $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiopen which is neither fuzzy semiopen nor fuzzy weakly open. Let $X = \{x, y, z\}$, $Y = \{a, b, c\}$ and fuzzy sets λ and μ are defined as:
$\lambda(x) = 0, \quad \lambda(y) = 0.3, \quad \lambda(z) = 0.2$;
$\mu(a) = 0, \quad \mu(b) = 0.2, \quad \mu(c) = 0.1$.
Let $\tau = \{0, \lambda , 1\}$ and $\sigma = \{ 0, \mu ,1\}$. Then the mapping f: (X , τ) $\to$ (Y , σ) defined by f(x) = a, f(y) = b and f(z) = c is fuzzy weakly semiopen but neither fuzzy semiopen nor fuzzy weakly open.

Recall that a fuzzy point x_p is said to be a fuzzy θ-cluster point of a fuzzy set λ [10], if and only if for every fuzzy open q-nbd μ of x_p , $Cl(\mu)$ is q-coincident with λ. The set of all fuzzy θ-cluster points of λ is called the fuzzy θ-closure of λ and will be denoted by $Cl_\theta(\lambda)$. A fuzzy λ will be called θ-closed if and only if $\lambda = Cl_\theta(\lambda)$. The complement of a fuzzy θ-closed set is called of fuzzy θ-open and the θ-interior of λ denoted by $Int_\theta(\lambda)$ is defined as $Int_\theta(\lambda) = \{x_p/$ for some fuzzy open q-nbd, β of $x_p, Cl(\beta) \leq \lambda\}$.

Lemma 2.2. [2] Let λ be a fuzzy set in a fts X, then:
1) λ is a fuzzy θ-open if and only if $\lambda = Int_\theta(\lambda)$.
2) $1 - Int_\theta(\lambda) = Cl_\theta(1 - \lambda)$ and $Int_\theta(1 - \lambda) = 1 - Cl_\theta(\lambda)$.
3) $Cl_\theta(\lambda)$ is a fuzzy closed set but not necessarily is a fuzzy θ-closed set.

Theorem 2.3. For a function $f : (X, \tau_1) \to (Y, \tau_2)$, the following conditions are equivalent:
(i) f is fuzzy weakly semiopen;
(ii) $f(Int_\theta(\lambda)) \leq sInt(f(\lambda))$ for every fuzzy subset λ of X;
(iii) $Int_\theta(f^{-1}(\beta)) \leq f^{-1}(sInt(\beta))$ for every fuzzy subset β of Y;
(iv) $f^{-1}(sCl(\beta)) \leq Cl_\theta(f^{-1}(\beta))$ for every fuzzy subset β of Y;
(v) For each fuzzy θ-open set λ in X, $f(\lambda)$ is fuzzy semiopen in Y;
(vi) For any fuzzy set β of $Y$and any fuzzy θ-closed set λ in X containing $f^{-1}(\beta)$, where X is a fuzzy regular space, there exists a fuzzy semiclosed set δin Y containing β such that $f^{-1}(\delta) \leq \lambda$.

Proof. (i)$\to$ (ii): Let λ be any fuzzy subset of X and x_p be a fuzzy point in $Int_\theta(\lambda)$. Then , there exists a fuzzy open q-nbd γ *of* x_p such that $\gamma \leq Cl(\gamma) \leq \lambda$. Then, $f(\gamma) \leq f(Cl(\gamma)) \leq f(\lambda)$. Since

f is fuzzy weakly semiopen, $f(\gamma) \leq sInt(f(Cl(\gamma))) \leq sInt(f(\lambda))$. It implies that $f(x_p)$ is a point in $sInt(f(\lambda))$. This shows that $x_p \in f^{-1}(sInt(f(\lambda)))$. Thus $Int_\theta(\lambda) \leq f^{-1}(sInt(f(\lambda)))$, and so $f(Int_\theta(\lambda)) \leq sInt(f(\lambda))$. (ii)→(i): Let μ be a fuzzy open set in X. As $\mu \leq Int_\theta(Cl(\mu))$ implies , $f(\mu) \leq f(Int_\theta(Cl(\mu))) \leq sInt(f(Cl(\mu)))$. Hence f is fuzzy weakly semiopen. (ii)→(iii): Let β be any fuzzy subset of Y. Then by (ii), $f(Int_\theta(f^{-1}(\beta)) \leq sInt(\beta)$. Therefore $Int_\theta(f^{-1}(\beta)) \leq f^{-1}(sInt(\beta))$. (iii)→(ii): This is obvious. (iii)→(iv): Let β be any fuzzy subset of Y. Using (iii), we have $1-Cl_\theta(f^{-1}(\beta)) = Int_\theta(1-f^{-1}(\beta)) = Int_\theta(f^{-1}(1-\beta)) \leq f^{-1}(sInt(1-\beta)) = f^{-1}(1-sCl(\beta)) = 1-(f^{-1}(sCl(\beta))$. Therefore, we obtain $f^{-1}(sCl(\beta)) \leq Cl_\theta(f^{-1}(\beta))$. (iv)→(iii): Similary we obtain, $1 - f^{-1}(sInt(\beta)) \leq 1 - Int_\theta(f^{-1}(\beta))$, for every fuzzy subset β of Y, i.e., $Int_\theta(f^{-1}(\beta)) \leq f^{-1}(sInt(\beta))$. (iv)→(v): Let λ be a fuzzy θ-open set in X. Then $1 - f(\lambda)$ is a fuzzy set in Y and by (iv), $f^{-1}(sCl(1 - f(\lambda))) \leq Cl_\theta(f^{-1}(1 - f(\lambda)))$. Therefore, $1 - f^{-1}(sInt(f(\lambda))) \leq Cl_\theta(1-\lambda) = 1-\lambda$. Then, we have $\lambda \leq f^{-1}(sInt(f(\lambda)))$ which implies $f(\lambda) \leq sInt(f(\lambda))$. Hence $f(\lambda)$ is fuzzy semiopen in Y. (v)→(vi): Let β be any fuzzy set in Y and λ be a fuzzy θ-closed set in X such that $f^{-1}(\beta) \leq \lambda$. *Since* $1-\lambda$ *is fuzzy* θ-open in X, by (v), $f(1-\lambda)$ is fuzzy semiopen in Y. Let $\delta = 1 - f(1-\lambda)$. Then δ is fuzzy semiclosed and also $\beta \leq \delta$. Now, $f^{-1}(\delta) = f^{-1}(1 - f(1-\lambda)) = 1 - f^{-1}(f(\lambda)) \leq \lambda$. (vi)→(iv): Let β be any fuzzy set in Y. Then by Corollary 3.6 of [10] $\lambda = Cl_\theta(f^{-1}(\beta))$ is fuzzy θ-closed set in X and $f^{-1}(\beta) \leq \lambda$. Then there exists a fuzzy semiclosed set δ in Y containing β such that $f^{-1}(\delta) \leq \lambda$. Since δ is fuzzy semiclosed $f^{-1}(sCl(\beta)) \leq f^{-1}(\delta) \leq Cl_\theta(f^{-1}(\beta))$.

Furthermore, we can prove the following.

Theorem 2.4. Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a function. Then the following statements are equivalent.
(i) f is fuzzy weakly semiopen;
(ii) For each fuzzy point x_p in X and each fuzzy open set μ of X containing x_p, there exists a semiopen set δ containing $f(x_p)$ such that $\delta \leq f(Cl(\mu))$.

Proof (i)→(ii): Let $x_p \in X$ and μ be a fuzzy open set in X containing x_p. Since f is fuzzy weakly semiopen. $f(\mu) \leq sInt(f(Cl(\mu)))$.

Let $\delta = sInt(f(Cl(\mu)))$. Hence $\delta \leq f(Cl(\mu))$, with δ *containing* $f(x_p)$. (ii)→(i): Let μ be a fuzzy open set in X and let $y_p \in f(\mu)$. It following from (ii) $\delta \leq f(Cl(\mu))$ for some δ semiopen in Y containing y_p. Hence we have, $y_p \in \delta \leq sInt(f(Cl(\mu)))$. This shows that $f(\mu) \leq sInt(f(Cl(\mu)))$, i.e., f is a fuzzy weakly semiopen function.

Theorem 2.5. Let $f : (X, \tau_1) \to (Y, \tau_2)$ be a bijective function. Then the following statements are equivalent:
(i) f is fuzzy weakly semiopen.
(ii) $sCl(f(\lambda)) \leq f(Cl(\lambda))$ for each fuzzy open set λ in X.
(iii) $sCl(f(Int(\beta)) \leq f(\beta)$ for each fuzzy closed set β in X.

Proof.
(i)→(iii): Let β be a fuzzy closed set in X. Then we have $f(1-\beta) = 1 - f(\beta) \leq sInt(f(Cl(1-\beta)))$ and so $1 - f(\beta) \leq 1 - sCl(f(Int(\beta))$. Hence $sCl(f(Int(\beta)) \leq f(\beta)$. (iii)→(ii): Let λ be a fuzzy open set in X. Since $Cl(\lambda)$ is a fuzzy closed set and $\lambda \leq Int(Cl(\lambda))$ by (iii) we have $sCl(f(\lambda)) \leq sCl(f(Int(Cl(\lambda))) \leq f(Cl(\lambda))$. (ii)→(iii): Similar to (iii)→(ii). (iii)→(i) : Clear.

For the following theorem , the proof is mostly straightforward and is omitted.

Theorem 2.6.
For a function $f : (X, \tau_1) \to (Y, \tau_2)$ the following conditions are equivalent:
(i) f is fuzzy weakly semiopen;
(ii) For each fuzzy closed subset β of X, $f(Int(\beta)) \leq sInt(f(\beta))$;
(iii) For each fuzzy open subset $\lambda$of X, $f(Int(Cl(\lambda))) \leq sInt(f(Cl(\lambda)))$;
(iv) For each fuzzy regular open subset λ of X, $f(\lambda) \leq sInt(f(Cl(\lambda)))$;
(v) For every fuzzy preopen subset λ of X, $f(\lambda) \leq sInt(f(Cl(\lambda)))$;
(vi) For every fuzzy α-open subset λ of X, $f(\lambda) \leq sInt(f(Cl(\lambda)))$.

We define one additional near fuzzy semiopen condition. This condition when combined with fuzzy weak semiopenness imply fuzzy semiopenness.

Definition 4. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to satisfy the fuzzy weakly semiopen interiority condition if $sInt(f(Cl(\lambda))) \leq f(\lambda)$ for every fuzzy open subset λ of X.

Recall that, a function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy strongly continuous [2], if for every fuzzy subset λ of $X, f(Cl(\lambda)) \leq f(\lambda)$.

Obviously, every fuzzy strongly continuous function satisfies the fuzzy weakly semiopen interiority condition but the converse does not hold as is shown by the following example.

Example 2.7. Fuzzy weakly semiopen interiority condition which is not fuzzy strongly continuous.

Let $X = \{a, b\}$, $Y = \{x, y\}$. Define λ and μ as:

$$\lambda(a) = 0.3, \quad \lambda(b) = 0.4, \quad ;$$

$$\mu(x) = 0.7, \mu(y) = 0.8.$$

Let $\tau = \{0,\ \lambda\ ,\ 1\}$ and $\sigma = \{\ 0,\ \mu\ ,1\}$ .Then the mapping f: (X , τ) $\to$ (Y , σ) defined by $f(a) = x$, $f(b) = y$ satisfies fuzzy weakly semiopen interiority but is not fuzzy strongly continuous.

Theorem 2.8. Every function that satisfies the fuzzy weakly semiopen interiority condition into a fuzzy discrete topological space is fuzzy strongly continuous.

Theorem 2.9. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiopen and satisfies the fuzzy weakly semiopen interiority condition, then $f$is fuzzy semiopen.

Proof. Let λ be a fuzzy open subset of X. Since f is fuzzy weakly semiopen $f(\lambda) \leq sInt(f(Cl(\lambda)))$. However, because f satisfies the fuzzy weakly semiopen interiority condition, $f(\lambda) = sInt(f(Cl(\lambda)))$ and therefore $f(\lambda)$ is fuzzy semiopen.

Corolario 2.10. If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiopen and fuzzy strongly continuous, then f is fuzzy semiopen.

The following example shows that neither of this fuzzy interiority condition yield a decomposition of fuzzy semiopenness.

Example 2.11. f does not satisfy the fuzzy weakly semiopen interiority condition. However f is fuzzy semiopen.

Let $X = \{a, b, c\}$,$Y = \{x, y, z\}$. Define λ and μ as follows :

$$\lambda(a) = 0, \quad \lambda(b) = 0.2, \quad \lambda(c) = 0.7 \quad ;$$

$$\mu(x) = 0, \quad \mu(y) = 0.2, \quad \mu(z) = 0.2 \quad .$$

Let $\tau = \{0, \lambda , 1\}$ and $\sigma = \{ 0, \mu ,1\}$. Then the mapping f: (X, τ) $\to$ (Y, σ) defined by $f(a) = x$, $f(b) = y$ and $f(c) = z$ is fuzzy semiopen but not fuzzy weakly semiopen interiority .

A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be fuzzy contra-open (resp. fuzzy contra-closed) if $f(\lambda)$ is a fuzzy closed set (resp. fuzzy open set) of Y for each fuzzy open (resp. fuzzy closed) set λ in X.

Theorem 2.12 (i) If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy preopen and fuzzy contra-open, then f is a fuzzy weakly semiopen function.
(ii) If $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy contra-closed, then f is a fuzzy weakly semiopen function .

Proof.
(i) Let λ be a fuzzy open subset of X. Since f is fuzzy preopen $f(\lambda) \leq Int(Cl(f(\lambda)))$ and since f is fuzzy contra-open, $f(\lambda)$ is fuzzy closed. Therefore $f(\lambda) \leq Int(Cl(f(\lambda))) = Int(f(\lambda)) \leq Int(f(Cl(\lambda))) \leq sInt(f(Cl(\lambda)))$.
(ii) Let λbe an fuzzy open subset of$X$. Then, we have $f(\lambda) \leq f(Cl(\lambda))) \leq sInt(f(Cl(\lambda)))$.

The converse of Theorem 2.12 does not hold. Can be seen in Example 2.1

Theorem 2.13. Let X be a fuzzy regular space. Then $f:(X,\tau_1)\to(Y,\tau_2)$ is fuzzy weakly semiopen if and only if f is fuzzy semiopen.

Proof. The sufficiency is clear. Necessity. Let λ be a non-null fuzzy open subset of X. For each x_p fuzzy point in λ, let μ_{x_p} be a fuzzy open set such that $x_p \in \mu_{x_p} \leq Cl(\mu_{x_p}) \leq \lambda$. Hence we obtain that $\lambda = \cup\{\mu_{x_p} : x_p \in \lambda\} = \cup\{Cl(\mu_{x_p}) : x_p \in \lambda\}$ and , $f(\lambda) = \cup\{f(\mu_{x_p}) : x_p \in \lambda\} \leq \cup\{sInt(f(Cl(\mu_{x_p})) : x_p \in \lambda\} \leq sInt(f(\cup\{Cl(\mu_{x_p}) : x_p \in \lambda\}) =sInt(f(\lambda))$. Thus f is fuzzy semiopen.

Note that, $f:(X,\tau_1)\to(Y,\tau_2)$ is said to be fuzzy contra-pre-semiclosed provide that $f(\lambda)$ is fuzzy semi-open for each fuzzy semi-closed subset λ of X.

Theorem 2.14. If $f:(X,\tau_1)\to(Y,\tau_2)$ is fuzzy weakly semiopen and Y has the property that union of fuzzy semi-closed sets is fuzzy semi-closed and if for each fuzzy semi-closed subset β of X and each fiber $f^{-1}(y_p)\leq 1-\beta$ there exists a fuzzy open subset μ of X for which $\beta\leq\mu$ and $f^{-1}(y_p)\overline{q}Cl(\mu))$, then f is fuzzy contra-pre-semiclosed.

Proof. Assume β is a fuzzy semi-closed subset of X and let $y_p \in 1-f(\beta)$. Thus $f^{-1}(y_p)\leq 1-\beta$ and hence there exists a fuzzy open subset μ of X for which $\beta\leq\mu$ and $f^{-1}(y_p)\overline{q}Cl(\mu)$. Therefore $y_p \in 1-f(Cl(\mu)) \leq 1-f(\beta)$. Since f is fuzzy weakly semiopen $f(\mu)\leq sInt(f(Cl(\mu)))$. By complement, we obtain $y_p \in sCl(1-f(Cl(\mu)))\leq 1-f(\beta)$. Let $\delta_{y_p} = sCl(1-fCl(\mu)))$. Then δ_{y_p} is a fuzzy semi-closed subset of Ycontaining y_p. Hence $1-f(\beta) = \cup\{\delta_{y_p} : y_p \in 1-f(\beta)\}$ is fuzzy semi-closed and therefore $f(\beta)$ is fuzzy semi-open.

Theorem 2.15. If $f:(X,\tau_1)\to(Y,\tau_2)$ is an f.a.o.N function, then it is fuzzy weakly semiopen. The converse is not generally true.

Proof. Let λ be a fuzzy open set in X. Since f is f.a.o.N and $Int(Cl(\lambda))$ is fuzzy regular open, $f(Int(Cl(\lambda)))$ is fuzzy open in Y and hence $f(\lambda)\leq f(Int(Cl(\lambda))\leq Int(f(Cl(\lambda))) \quad \leq sInt(f(Cl(\lambda)))$. This shows that f is fuzzy weakly semiopen.

Example 2.16. f is not f.a.o.N and $f$is fuzzy weakly semiopen.

The function f defined in Example 2.11 is fuzzy weakly semiopen but not f.a.o.N.

Lemma 2.17. [6] If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy continuous function, then for any fuzzy subset λ of X, $f(Cl(\lambda)) \leq Cl(f(\lambda))$

Theorem 2.18. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy weakly semiopen and fuzzy continuous function, then f is a fuzzy β-open function.

Proof. Let λ be a fuzzy open set in X. Then by fuzzy weak semiopenness of f, $f(\lambda) \leq sInt(f(Cl(\lambda)))$.Since f is fuzzy continuous $f(Cl(\lambda)) \leq Cl(f(\lambda))$ and since for any fuzzy subset β of X, $sInt(Cl(\beta)) \leq \beta \cap Cl(Int(Cl(\beta)))$, we obtain that,
$f(\lambda) \leq sInt(f(Cl(\lambda))) \leq sIn(Cl(f(\lambda))) \leq Cl(Int(Cl(f(\lambda)))$ Thus, $f(\lambda) \leq Cl(Int(Cl(f(\lambda)))$ which shows that $f(\lambda)$ is a fuzzy β-open set in $Y$and hence by Definition 1.2(iv), f is a fuzzy β-open function.

Corolario 2.19. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy weakly semiopen and fuzzy strongly continuous function. Then f is a fuzzy β-open function.

Recall that, two non-null fuzzy sets λ and β in a fuzzy topological spaces X (i.e., neither λ nor β is 0_X) are said to be fuzzy semi-separated [7] if $\lambda \overline{q} sCl(\beta)$ and $\beta \overline{q} sCl(\lambda)$ or equivalently if there exist two fuzzy semi-open sets μ and ν such that $\lambda \leq \mu$, $\beta \leq \nu$, $\lambda \overline{q} \nu$ and $\beta \overline{q} \mu$.

A fuzzy topological space X which can not be expressed as the union of two fuzzy semi-separated sets is said to be a fuzzy semi-connected space.

Theorem 2.20. If $f : (X, \tau_1) \to (Y, \tau_2)$ is a fuzzy weakly semiopen of a space X onto a fuzzy semi-connected space Y, then X is fuzzy connected.

PROOF. If possible, let X be not connected. Then there exist fuzzy separated sets β and γ in X such that $X = \beta \cup \gamma$. Since β and γ

are fuzzy separated, there exist two fuzzy open sets μ and ν such that $\beta \leq \mu$, $\gamma \leq \nu$, $\beta \overline{q} \nu$ and $\gamma \overline{q} \mu$. Hence we have $f(\beta) \leq f(\mu)$, $f(\gamma) \leq f(\nu)$, $f(\beta)\overline{q}f(\nu)$ and $f(\gamma)\overline{q}f(\mu)$. Since f is fuzzy weakly semiopen, we have $f(\mu) \leq sInt(f(Cl(\mu)))$ and $f(\nu) \leq sInt(f(Cl(\nu)))$ and since μ and ν are fuzzy open and also fuzzy closed, we have $f(Cl(\mu)) = f(\mu)$, $f(Cl(\nu)) = f(\nu)$. Hence $f(\mu)$ and $f(\nu)$ are fuzzy semiopen in Y. Therefore , $f(\beta)$ and $f(\gamma)$ are fuzzy semi-separated sets in Y and $Y = f(X) = f(\beta \cup \gamma) = f(\beta) \cup f(\gamma)$. Hence this contrary to the fact that Y is fuzzy semi-connected. Thus X is fuzzy connected.

Definition 5. A space X is said to be fuzzy hyperconnected if every non-null fuzzy open subset of X is fuzzy dense in X

Theorem 2.21. If X is a fuzzy hyperconnected space, then a function $f : (X, \tau_1) \to (Y, \tau_2)$ is fuzzy weakly semiopen if and only if $f(X)$ is fuzzy semi-open in Y.

Proof. The necessity is clear. For the sufficiency observe that for any fuzzy open subset λ of X, $f(\lambda) \leq f(X) = sInt(f(X) = sInt(f(Cl(\lambda)))$.

References

[1] K.K. Azad, On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity, J.Math. Anal. Appl., 82, pp. 14-32, (1981).

[2] R.N. Bhoumik and A. Mukherjee, Fuzzy weakly completely continuous functions, Fuzzy Sets and Systems, 55, pp. 347-354, (1993).

[3] A.S. Bin Shahna, On fuzzy strong semicontinuity and fuzzy precontinuity,Fuzzy Sets and Systems, 44, pp. 303-308, (1991).

[4] C.L.Chang, Fuzzy topological spaces, J. Math. Anal Appl., 24, pp. 182-192, (1968).

[5] D.Coker and A.H.Es, On Fuzzy S-closed spaces (Preprint).

[6] S.Ganguly and S. Saha, A note on semi-open sets in fuzzy topological spaces, Fuzzy Sets and Systems, 18, pp. 83-96, (1986).

[7] B. Ghosh, Semi-continuous and semi-closed mappings and semi-connectedness in fuzzy setting, Fuzzy Sets and Systems, 35, pp. 345-355, (1990).

[8] A.S. Mashhour, M.H. Ghanim and M.A. Fath Alla, On fuzzy non-continuous mappings, Bull. Cal. Math. Soc.,78, pp. 57-69, (1986).

[9] P.P. Ming and L.Y. Ming, Fuzzy topology I. Neighborhood structure of fuzzy point and Moore-Smith convergence, J.Math. Anal. Appl., 76, pp. 571-594, (1980).

[10] M.N. Mukherjee and S.P. Sinha, fuzzy θ -closure operator on fuzzy topological spaces, Internat. J. Math. I& Math. Sci. 14, pp. 309-314, (1991).

[11] S. Nanda, On fuzzy topological spaces, Fuzzy Sets and Systems, 19(1986), pp. 193- 197, (1986).

[12] J. H. Park, Y. B. Park and J. S. Park, Fuzzy weakly open mappings and fuzzy RS-compact sets, Far East J. Math. Sci. Special Vol., Part II, pp. 201-212, (1997).

[13] D. A. Rose, On weak openness and almost openness, Interntt.J.Math. I& Math. Sci., 7, pp. 35-40, (1984).

[14] T. H.Yalvaç, Semi-interior and semi-closure of fuzzy set, J.Math. Anal.Appl., 132, pp. 356-364, (1988).

[15] L. A. Zadeh, Fuzzy sets, Inform. and Control, 8, pp. 338-353, (1965).

Received : December 2001.

M. Caldas
Departamento de Matemática Aplicada
Universidade Federal Fluminense
Rua Mário Santos Braga, s/n
CEP: 24020-140
Niteroi
RJ Brasil
e-mail: gmamccs@vm.uff.br

G. B.Navalagi
Department of Mathematics
KLE Societys
G. H.College, Haveri-581110
Karnataka
India
e-mail: gnavalagi@hotmail.com

and

R. K. Saraf
Department of Mathematics
Govt. K. N. G. College
DAMOH-470661, M.P.
India