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A NEW NOTION OF SP-COMPACT L-FUZZY SETS *

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Abstract

A new notion of SP-compactness is introduced in L-topological spaces by means of semi-preopen L-sets and their inequality, where L is a complete De Morgan algebra. This new notion does not depend on the structure of basis lattice L and L does not require any distributivity. This new notion implies semicompactness, hence it also implies compactness. This new notion is a good extension and it has many characterizations if L is completely distributive De Morgan algebra.

Key Words and Phrases: *L-topology; semi-preopen L-set; SP-compactness.*

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1.Introduction

In general topological spaces, the concepts of semi-preopen sets and semi-preclosed sets were introduced by Andrijevic [1]. Thakur and Singh extended these concepts to [0,1]-topological spaces [11] in the Chang's[4] sense. In [2], we introduced the concept of SP-compactness in L-topological spaces. It preserves many good properties of compactness in general topological spaces. However, the SP-compactness relies on the structure of basis lattice L and L is required to be completely distributive. In [10], a new definition of fuzzy compactness is presented in L-topological spaces by means of open L-sets and their inequality, where L is a complete de Morgan algebra. This new definition doesn't depend on the structure of L.

In this paper, following the lines of [10], we'll introduce a new notion of SP-compactness in L-topological spaces by means of semi-preopen L-sets and their inequality. It is a strong form of semi-compactness[8], hence it is also a strong form of compactness[10]. It can also be characterized by semi-preclosed L-sets and their inequality. It is defined for any L-subset, and it is hereditary for semi-preclosed subsets, finitely additive, and is preserved under SP-irresolute mapping. This new form of SP-compactness is a good extension and it has many characterizations when L is completely distributive De Morgan algebra.

2. Preliminaries

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete De Morgan algebra, X a nonempty set. L^X is the set of all L-fuzzy sets (or L-sets for short) on X. The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$. An element a in L is called prime element if $b \wedge c \leq a$ implies that $b \leq a$ or $c \leq a$. a in L is called a co-prime element if a' is a prime element [6]. The set of nonunit prime elements in L is denoted by $P(L)$. The set of nonzero co-prime elements in L is denoted by $M(L)$.

The binary relation $\prec$ in L is defined as follows: for $a, b \in L, a \prec b$ iff for every subset $D \subseteq L$, the relation $b \leq supD$ always implies the existence of $d \in D$ with $a \leq d$ [5]. In a completely distributive De Morgan algebra L, each element b is a sup of $\{a \in L | a \prec b\}$. $\{a \in L | a \prec b\}$ is called the greatest minimal family of b in the sense of [7,12], in symbol $\beta(b)$. Moreover for $b \in L$, define $\beta^*(b) = \beta(b) \cap M(L)$, $\alpha(b) = \{a \in L | a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$. For $a \in L$ and $A \in L^X$, we denote $A^{(a)} = \{x \in X | A(x) \not\leq a\}$ and $A_{(a)} = \{x \in X | a \in \beta(A(x))\}$. For a subfamily $\psi \subseteq L^X, 2^{(\psi)}$ denotes the set of all finite subfamilies of ψ.

An L-topological space (or L-ts for short) is a pair (X, δ), where δ is a subfamily of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima. δ is called an L-topology on X. Each member of δ is called an open L-set and its quasi-complement is called a closed L-set. The semi-preopen set and semi-preclosed set are defined in [0,1]-topological space in [11]. Analogously we can generalize it to L-subset in L-topological spaces. Let (L^X, δ) be an L-ts. $A \in L^X$ is called semi-preopen if there is a preopen set B such that $B \leq A \leq B^-$, and semi-preclosed if there is a preclosed set B such that $B^o \leq A \leq B$, where B^o and B^- are the interior and closure of B, respectively.

Definition 2.1 ([7,12]). For a topological space (X, τ), let $\omega_L(\tau)$ denote the family of all the lower semi-continuous maps from (X, τ) to L, that is, $\omega_L(\tau) = \{A \in L^X | A^{(a)} \in \tau, a \in L\}$. Then $\omega_L(\tau)$ is an L-topology on X, in this case, $(X, \omega_L(\tau))$ is topologically generated by (X, τ).

Definition 2.2 ([7,12]). An L-ts (X, δ) is weak induced if for all $a \in L$, for all $A \in \delta$, it follows that $A^{(a)} \in [\delta]$, where $[\delta]$ denotes the topology formed by all crisp sets in δ. It is obvious that $(X, \omega_L(\tau))$ is weak induced.

Definition 2.3([8,9]). Let (X, δ) be an L-ts, $a \in L \setminus \{1\}$, and $A \in L^X$. A family $\mu \subseteq L^X$ is called

(1) an a-shading of A if for any $x \in X$, $(A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$.

(2) a strong a-shading (briefly S-a-shading) of A if $\bigwedge_{x \in X}(A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$.

(3) an a-R-neighborhood family (briefly a-R-NF) of A if for any $x \in X$, $(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\geq a$.

(4) a strong a-R-neighborhood family (briefly S-a-R-NF)of A if $\bigvee_{x \in X}(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\geq a$.

It is obvious that an S-a-shading of A is an a-shading of A, an S-a-R-NF of A is an a-R-NF of A, and μ is an S-a-R-NF of A iff μ' is an S-a-shading of A.

Definition 2.4([8]). Let (X, δ) be an L-ts, $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is called

(1)a β_a-cover of A if for any $x \in X$, it follows that $a \in \beta(A'(x) \vee \bigvee_{B \in \mu} B(x))$.

(2) a strong β_a-cover (briefly S-β_a-cover) of A if $a \in \beta(\bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\mu} B(x)))$.

(3) a Q_a-cover of A if for any $x \in X$, it follows that $A'(x) \vee \bigvee_{B\in\mu} B(x) \geq a$.

It is obvious that an S-β_a-cover of A must be a β_a-cover of A, and a β_a-cover of A must be a Q_a-cover of A.

Definition 2.5([8,9]). Let $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is said to have weak a-nonempty intersection in A if $\bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\mu} B(x)) \geq a$. μ is said to have the finite weak a-intersection property in A if every finite subfamily ν of μ has weak a-nonempty intersection in A.

Lemma 2.6 ([8]). Let L be a complete Heyting algebra, $f : X \to Y$ be a map and $f_L^{\rightarrow} : L^X \to L^Y$ is the extension of f , then for any family $\psi \subseteq L^Y$,

$$\bigvee_{y\in Y}(f_L^{\rightarrow}(A)(y) \wedge \bigwedge_{B\in\psi} B(y)) = \bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\psi} f_L^{\rightarrow}(B)(x)).$$

Definition 2.7([2,3,11]). Let (X, δ) and (Y, τ) be two L-ts's. A map $f : (X, \delta) \to (Y, \tau)$ is called

(1) semi-precontinuous if $f_L^{\leftarrow}(B)$ is semi-preopen in (X, δ) for every $B \in \tau$.

(2) semi-preirresolute if $f_L^{\leftarrow}(B)$ is semi-preopen in (X, δ) for every semi-preopen L-set B in (Y, τ).

3. Definitions and properties

Definition 3.1. Let (X, δ) be an L-ts. $A \in L^X$ is called SP-compact if for every family μ of semi-preopen L-sets, it follows that

$$\bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\mu} B(x)) \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X}(A'(x) \vee \bigvee_{B\in\nu} B(x)).$$

(X, δ) is called SP-compact if $\underline{1}$ is SP-compact.

Example 3.2. Let $X = \{x\}$ and $L = \{0, 1/3, 2/3, 1\}$. For each $a \in L$ define $a' = 1-a$. Let $\delta = \{\emptyset, A, X\}$, where $A(x) = 2/3$, then δ is a topology on X. Clearly, any L-set in (X, δ) is SP-compact.

Example 3.3. Let X be an infinite set(or X be a singleton), A and C be two [0, 1]-sets on X defined as $A(x) = 0.5$, for all $x \in X$; $C(x) = 0.6$, for all $x \in X$. Take $\delta = \{\emptyset, A, X\}$, then δ is a topology on X. Obviously, any

[0,1]-set in (X,δ) is semi-preopen, and the set of all semi-open [0,1]-sets in (X,δ) is δ. In this case, we easily obtain that C is not SP-compact, and any [0,1]-set in (X,δ) is semi-compact.

Remark 3.4. Since every semi-open L-set is semi-preopen[2,11], every SP-compact L-set is semi-compact. Example 3.3 shows that semi-compact L-set needn't be SP-compact.

Theorem 3.5. Let (X,δ) be an L-ts. $A \in L^X$ is SP-compact iff for every family μ of semi-preclosed L-sets, it follows that

$$\bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\mu} B(x)) \geq \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X}(A(x) \wedge \bigwedge_{B\in\nu} B(x)).$$

proof. This is immediate from Definition 3.1 and quasi-complement.

Theorem 3.6. Let (X,δ) be an L-ts and $A \in L^X$. Then the following conditions are equivalent.

(1) A is SP-compact.

(2) For any $a \in L \setminus \{1\}$, each semi-preopen S-a-shading μ of A has a finite subfamily which is an S-a-shading of A.

(3) For any $a \in L \setminus \{0\}$, each semi-preclosed S-a-R-NF ψ of A has a finite subfamily which is an S-a-R-NF of A.

(4) For any $a \in L \setminus \{0\}$, each family of semi-preclosed L-sets which has the finite weak a-intersection property in A has weak a-nonempty intersection in A.

proof. This is immediate from Definition 3.1 and Theorem 3.5.

Theorem 3.7. Let L be a complete Heyting algebra. If both C and D are SP-compact, then $C \vee D$ is SP-compact.

Proof. For any family μ of semi-preclosed L-sets, by Theorem 3.5 we have that

$$\begin{aligned}
&\bigvee_{x\in X}((C \vee D)(x) \wedge \bigwedge_{B\in\mu} B(x)) \\
&= \{\bigvee_{x\in X}(C(x) \wedge \bigwedge_{B\in\mu} B(x))\} \vee \{\bigvee_{x\in X}(D(x) \wedge \bigwedge_{B\in\mu} B(x))\} \\
&\geq \{\bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X}(C(x) \wedge \bigwedge_{B\in\nu} B(x))\} \vee \{\bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X}(D(x) \wedge \bigwedge_{B\in\nu} B(x))\} \\
&= \bigwedge_{\nu\in 2^{(\mu)}} \bigvee_{x\in X}((C \vee D)(x) \wedge \bigwedge_{B\in\nu} B(x)).
\end{aligned}$$

This shows that $C \vee D$ is SP-compact.

Theorem 3.8. If C is SP-compact and D is semi-preclosed, then $C\wedge D$ is SP-compact.

Proof. For any family μ of semi-preclosed L-sets, by Theorem 3.5 we have that

$$\begin{aligned}
&\bigvee_{x\in X}((C\wedge D)(x)\wedge\bigwedge_{B\in\mu}B(x))\\
&=\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in\mu\cup\{D\}}B(x))\\
&\geq\bigwedge_{\nu\in 2^{(\mu\cup\{D\})}}\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in\nu}B(x))\\
&=\{\bigwedge_{\nu\in 2^{(\mu)}}\bigvee_{x\in X}(C(x)\wedge\bigwedge_{B\in\nu}B(x))\}\wedge\{\bigwedge_{\nu\in 2^{(\mu)}}\bigvee_{x\in X}(C(x)\wedge D(x)\wedge\bigwedge_{B\in\nu}B(x))\}\\
&=\{\bigwedge_{\nu\in 2^{(\mu)}}\bigvee_{x\in X}(C(x)\wedge D(x)\wedge\bigwedge_{B\in\nu}B(x))\}\\
&=\bigwedge_{\nu\in 2^{(\mu)}}\bigvee_{x\in X}((C\wedge D)(x)\wedge\bigwedge_{B\in\nu}B(x)).
\end{aligned}$$

This shows that $C\wedge D$ is SP-compact.

Corollary 3.9. Let (X,δ) be SP-compact and $D\in L^X$ is semi-preclosed. Then D is SP-compact.

Definition 3.10. Let (X,δ) and (Y,τ) be two L-ts's. A map $f:(X,\delta)\to(Y,\tau)$ is called

(1) strongly semi-precontinuous if $f_L^{\leftarrow}(B)$ is semi-preopen in (X,δ) for every semi-open L-set B in (Y,τ).

(2) strongly semi-preirresolute if $f_L^{\leftarrow}(B)$ is semi-open in (X,δ) for every semi-preopen L-set B in (Y,τ).

Remark 3.11. It is obvious that a strongly semi-precontinuous map is semi-precontinuous, and a strongly semi-preirresolute map is semi-preirresolute. None of the converses need be true.

Example 3.12. Let $X=\{x,y\}, L=[0,1], \forall a\in L, a'=1-a$, and $A,B,C,D\in L^X$ defined as follows:

$A(x)=0.2,\quad A(y)=0.1;$
$B(x)=0.5,\quad B(y)=0.5;$
$C(x)=0.3,\quad C(y)=0.2;$
$D(x)=0.6,\quad D(y)=0.7.$

Then $\delta=\{0,A,B,1\}$ and $\tau=\{0,C,1\}$ are topologies on X. Let $f:(X,\delta)\to(X,\tau)$ be an identity mapping. Obviously, f is semi-precontinuous.

We can easily get that D is a semiopen L-set in (X, τ) and that $f_L^{\leftarrow}(D)$ is not semi-preopen in (X, δ). Thus, f is not strongly semi-precontinuous.

Example 3.13. Let $X = \{x, y\}, L = [0, 1], \forall a \in L, a' = 1 - a$, and $A, B, C \in L^X$ defined as follows:

$A(x) = 0.5, \quad A(y) = 0.5;$
$B(x) = 0.7, \quad B(y) = 0.6;$
$C(x) = 0.7, \quad C(y) = 0.8.$

Then $\delta = \{0, A, 1\}$ and $\tau = \{0, B, 1\}$ are topologies on X. Let $f : (X, \delta) \to (X, \tau)$ be an identity mapping. Obviously, f is semi-preirresolute. We can easily get that C is a semi-preopen L-set in (X, τ) and that $f_L^{\leftarrow}(C)$ is not semiopen in (X, δ). Thus, f is not strongly semi-preirresolute.

Theorem 3.14. Let L be a complete Heyting algebra and $f : (X, \delta) \to (Y, \tau)$ be a semi-preirresolute map. If A is an SP-compact L-set in (X, δ), then so is $f_L^{\rightarrow}(A)$ in (Y, τ).

Proof. Suppose that μ is a family of semi-preclosed L-sets in (Y, τ), by Lemma 2.6 and SPR-compactness of A, we have that

$$\begin{aligned}
&\bigvee_{y \in Y} (f_L^{\rightarrow}(A)(y) \wedge \bigwedge_{B \in \mu} B(y)) \\
&= \bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \mu} f_L^{\leftarrow} B(x)) \\
&\geq \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \nu} f_L^{\leftarrow} B(x)) \\
&= \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{y \in Y} (f_L^{\rightarrow}(A)(y) \wedge \bigwedge_{B \in \nu} B(y)).
\end{aligned}$$

Therefore $f_L^{\rightarrow}(A)$ is SP-compact.

Analogously, we can obtain the following theorems.

Theorem 3.15. Let L be a complete Heyting algebra and $f : (X, \delta) \to (Y, \tau)$ be a semi-precontinuous map. If A is an SP-compact L-set in (X, δ), then $f_L^{\rightarrow}(A)$ is a compact L-set in (Y, τ).

Theorem 3.16. Let L be a complete Heyting algebra and $f : (X, \delta) \to (Y, \tau)$ be a strongly semi-precontinuous map. If A is an SP-compact L-set in (X, δ), then $f_L^{\rightarrow}(A)$ is a semi-compact L-set in (Y, τ).

Theorem 3.17. Let L be a complete Heyting algebra and $f : (X, \delta) \to (Y, \tau)$ be a strongly semi-preirresolute map. If A is a semi-compact L-set in (X, δ), then $f_L^{\rightarrow}(A)$ is an SP-compact L-set in (Y, τ).

4. Further properties and goodness

In this section, we assume that L is a completely distributive de Morgan algebra.

Theorem 4.1. Let (X, δ) be an L-ts and $A \in L^X$. Then the following conditions are equivalent.

(1) A is SP-compact.

(2) For any $a \in L \setminus \{0\}$, each semi-preclosed S-a-R-NF ψ of A has a finite subfamily which is an S-a-R-NF of A.

(3) For any $a \in L \setminus \{0\}$, each semi-preclosed S-a-R-NF ψ of A has a finite subfamily which is an a-R-NF of A.

(4) For any $a \in L \setminus \{0\}$ and any semi-preclosed S-a-R-NF ψ of A, there exist a finite subfamily φ of ψ and $b \in \beta(a)$ such that φ is an S-b-R-NF of A.

(5) For any $a \in L \setminus \{0\}$ and any semi-preclosed S-a-R-NF ψ of A, there exist a finite subfamily φ of ψ and $b \in \beta(a)$ such that φ is a b-R-NF of A.

(6) For any $a \in L \setminus \{1\}$, each semi-preopen S-a-shading μ of A has a finite subfamily which is an S-a-shading of A.

(7) For any $a \in L \setminus \{1\}$, each semi-preopen S-a-shading μ of A has a finite subfamily which is an a-shading of A.

(8) For any $a \in L \setminus \{1\}$ and any semi-preopen S-a-shading μ of A, there exist a finite subfamily ν of μ and $b \in \alpha(a)$ such that ν is an S-b-shading of A.

(9) For any $a \in L \setminus \{1\}$ and any semi-preopen S-a-shading μ of A, there exist a finite subfamily ν of μ and $b \in \alpha(a)$ such that ν is a b-shading of A.

(10) For any $a \in L \setminus \{0\}$, each semi-preopen S-β_a-cover μ of A has a finite subfamily which is an S-β_a-cover of A.

(11) For any $a \in L \setminus \{0\}$, each semi-preopen S-β_a-cover μ of A has a finite subfamily which is a β_a-cover of A.

(12) For any $a \in L \setminus \{0\}$ and any semi-preopen S-β_a-cover μ of A, there exist a finite subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is an S-β_b-cover of A.

(13) For any $a \in L \setminus \{0\}$ and any semi-preopen S-β_a-cover μ of A, there exist a finite subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is a β_b-cover of A.

(14) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each semi-preopen Q_a-cover μ of A has a finite subfamily which is a Q_b-cover of A.

(15) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each semi-preopen Q_a-cover μ of A has a finite subfamily which is a β_b-cover of A.

(16) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each semi-preopen Q_a-cover μ of A has a finite subfamily which is an S-β_b-cover of A.

Proof. This is analogous to the proof of Theorem 5.3 in [8].

Remark 4.2. In Theorem 4.1, $a \in L\setminus\{0\}$ and $b \in \beta(a)$ can be replaced by $a \in M(L)$ and $b \in \beta^*(a)$ respectively. $a \in L \setminus \{1\}$ and $b \in \alpha(a)$ can be replaced by $a \in P(L)$ and $b \in \alpha^*(a)$ respectively. Thus, we can obtain other 15 equivalent conditions about the SP-compactness.

Lemma 4.3 Let $(X, \omega_L(\tau))$ be generated topologically by (X, τ). If A is a preopen L-set in (X, τ), then χ_A is a preopen set in $(X, \omega_L(\tau))$. If B is a preopen L-set in $(X, \omega_L(\tau))$, then $B_{(a)}$ is a preopen set in (X, τ) for every $a \in L$.

Proof. This is analogous to the proof of Theorem 5.7 in [9].

Lemma 4.4 Let $(X, \omega_L(\tau))$ be generated topologically by (X, τ). If A is a semi-preopen L-set in (X, τ), then χ_A is a semi-preopen set in $(X, \omega_L(\tau))$. If B is a semi-preopen L-set in $(X, \omega_L(\tau))$, then $B_{(a)}$ is a semi-preopen set in (X, τ) for every $a \in L$.

Proof. This is analogous to the proof Theorem 5.4 in [8], by Lemma 4.3.

Theorem 4.5. Let (X, τ) be a topological space and $(X, \omega_L(\tau))$ be generated topologically by (X, τ). Then $(X, \omega_L(\tau))$ is SP-compact iff (X, τ) is SP-compact.

Proof. Necessity. Let μ be a semi-preopen cover of (X, τ). Then $\{\chi_A | A \in \mu\}$ is a family of semi-preopen L-sets in $(X, \omega_L(\tau))$ with $\bigwedge_{x\in X}(\bigvee_{A\in\mu} \chi_A(x)) = 1$. From SP-compactness of $(X, \omega_L(\tau))$, we have that

$$1 = \bigwedge_{x\in X} (\bigvee_{A\in\mu} \chi_A(x)) \leq \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X} (\bigvee_{A\in\nu} \chi_A(x)).$$

This implies that there exists $\nu \in 2^{(\mu)}$ such that $\bigwedge_{x\in X}(\bigvee_{A\in\nu} \chi_A(x)) = 1$. Hence, ν is a cover of (X, τ). Therefore (X, τ) is SP-compact.

Sufficiency. Let μ be a family of semi-preopen L-sets in $(X, \omega_L(\tau))$ and $\bigwedge_{x\in X}(\bigvee_{B\in\mu} B(x)) = a$. If $a = 0$, obviously we have that

$$\bigwedge_{x\in X}(\bigvee_{B\in\mu} B(x)) \le \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X}(\bigvee_{B\in\nu} B(x)).$$

Now we suppose that $a \neq 0$. In this case, for any $b \in \beta(a) \setminus \{0\}$, we have that

$$b \in \beta(\bigwedge_{x\in X}(\bigvee_{B\in\mu} B(x))) \subseteq \bigcap_{x\in X} \beta(\bigvee_{B\in\mu} B(x)) = \bigcap_{x\in X} \bigcup_{B\in\mu} \beta(B(x)).$$

From Lemma 4.4, this implies that $\{B_{(b)}|B \in \mu\}$ is a semi-preopen cover of (X, τ). From SP-compactness of (X, τ), we know that there exists $\nu \in 2^{(\mu)}$ such that $\{B_{(b)}|B \in \nu\}$ is a cover of (X, τ). Hence $b \le \bigwedge_{x\in X}(\bigvee_{B\in\nu} B(x))$. Further, we have that

$$b \le \bigwedge_{x\in X}(\bigvee_{B\in\nu} B(x)) \le \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X}(\bigvee_{B\in\nu} B(x)).$$

This implies that

$$\bigwedge_{x\in X}(\bigvee_{B\in\mu} B(x)) = a = \bigvee\{b|b \in \beta(a)\} \le \bigvee_{\nu\in 2^{(\mu)}} \bigwedge_{x\in X}(\bigvee_{B\in\nu} B(x)).$$

Therefore, $(X, \omega_L(\tau))$ is SP-compact.

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