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UNIFORM CONVERGENCE OF MULTIPLIER CONVERGENT SERIES

CHARLES SWARTZ
NEW MEXICO STATE UNIVERSITY

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Abstract

If λ is a sequence K-space and $\sum x_j$ is a series in a topological vector space X, the series is said to be λ-multiplier convergent if the series $\sum_{j=1}^{\infty} t_j x_j$ converges in X for every $t = \{t_j\} \in \lambda$. We show that if λ satisfies a gliding hump condition, called the signed strong gliding hump condition, then the series $\sum_{j=1}^{\infty} t_j x_j$ converge uniformly for $t = \{t_j\}$ belonging to bounded subsets of λ. A similar uniform convergence result is established for a multiplier convergent series version of the Hahn-Schur Theorem.

Let X be a Hausdorff topological vector space and $\sum x_j$ a (formal) series in X. The series $\sum x_j$ is said to be bounded multiplier convergent if the series $\sum_{j=1}^{\infty} t_j x_j$ converges in X for every $t = \{t_j\} \in l^{\infty}$ ([D]). If X is either a locally convex space or a metric linear space and $\sum x_j$ is bounded multiplier convergent, then the series $\sum_{j=1}^{\infty} t_j x_j$ actually converge uniformly for $t = \{t_j\} \in l^{\infty}, \|t\|_{\infty} \leq 1$ ([Sw1], [Sw2]8.2.7). A series $\sum x_j$ in X is subseries convergent if the subseries $\sum x_{n_j}$ converges in X for every subsequence $\{n_j\}$. If $\sigma \subset \mathbf{N}$, let C_{σ} denote the characteristic function of σ and if x is any sequence, let $C_{\sigma}x$ denote the coordinatewise product of C_{σ} and x. Thus, the series $\sum x_j$ is subseries convergent iff $\sum_{j=1}^{\infty} C_{\sigma}(j) x_j = \sum_{j\in\sigma} x_j$ converges for every $\sigma \subset \mathbf{N}$. A similar uniform convergence result holds for subseries convergent series. Namely, if $\sum x_j$ is subseries convergent, then the series $\sum_{j=1}^{\infty} C_{\sigma}(j) x_j$ converge uniformly for $\sigma \subset \mathbf{N}$ ([Sw2]8.1.2). Another uniform convergence result holds for subseries and bounded multiplier convergent series in the subseries and bounded multiplier convergent versions of the Hahn-Schur Theorem ([Sw1],[Sw2]8.1 and 8.2). We consider conditions under which the same conclusions hold if the multiplier space l^{∞} is replaced by more general sequence spaces.

Let λ be a vector space of scalar sequences which contains c_{00}, the space of all sequences which are eventually 0, and which is equipped with a vector Hausdorff topology under which the coordinate functionals $t = \{t_j\} \to t_j$ are continuous for every $j \in \mathbf{N}$ (i.e., λ is a K space ([B]7.2.2)). Let $\Lambda \subset \lambda$. The series $\sum x_j$ in X is $\Lambda -$ *multiplier convergent* if the series $\sum_{j=1}^{\infty} t_j x_j$ converges in X for every $t = \{t_j\} \in \Lambda$ ([FP],[Sw2]8.3); thus, a series is l^{∞}-multiplier convergent iff the series is bounded multiplier convergent and a series is subseries convergent iff the series is m_0 multiplier convergent, where $m_0 = span\{C_{\sigma} : \sigma \subset \mathbf{N}\}$ is the sequence space of all sequences with finite range. It is natural to ask if a uniform convergence result as above for subseries and bounded multiplier convergent series holds for λ-multiplier convergent series ;i.e., if $\sum x_j$ is λ-multiplier convergent, do the series $\sum_{j=1}^{\infty} t_j x_j$ converge uniformly for $t = \{t_j\}$ belonging to certain families of bounded subsets of λ? Example 5 shows that such a result does not hold in general, but we show in Theorem 3 that such a result does hold if certain subsets of the multiplier space λ satisfy a gliding hump property called the signed strong gliding hump property. We also show that a similar uniform convergence result holds for series in a version of the Hahn-Schur Theorem for λ-multiplier convergent series when certain subsets of the multiplier space λ satisfy the signed strong gliding hump property.

We begin by defining the gliding hump property which we will employ. An interval in $\mathbf{N}$ is a set of the form $[m,n]=\{k\in\mathbf{N}: m\leq k\leq n\}$ where $m\leq n$; a sequence of intervals $\{I_k\}$ is increasing if $\sup I_k<\inf I_{k+1}$. A sign is a variable assuming the values $\{\pm 1\}$. Let $\Lambda\subset\lambda$.The subset Λ has the it signed strong gliding it hump property (signed-SGHP) if for every bounded sequence $\{x^k\}\subset\Lambda$ and for every increasing sequence of intervals $\{I_k\}$, there exists a subsequence $\{n_k\}$ and a sequence of signs $\{s_k\}$ such that $x=\sum_{k=1}^{\infty}s_kC_{I_{n_k}}x^{n_k}$ [coordinate sum] belongs to Λ. The subset Λ has the strong gliding hump property (SGHP) if the signs above can all be chosen to equal to1 ; the SGHP has been employed on numerous occasions ([N],[Sw2],[SS]). The idea of multiplying the "humps" C_Ix by signs was introduced by Stuart ([St1],[St2]) and used to treat weak sequential completeness of $\beta-$duals. For example, the space l^{∞} has the SGHP and the methods of [BSS] can be used to construct other spaces with SGHP. Let M_0 be the subset of m_0 consisting of the sequences of 0's and 1's, $M_0=\{C_\sigma:\sigma\subset\mathbf{N}\}$. Then the subset M_0 has SGHP but the space m_0 does not have SGHP. We now show that the space of bounded series bs ([B]) has the signed-SGHP but fails SGHP; Stuart showed that bs has the signed weak gliding hump property but not the weak gliding hump property and we essentially just use his proof ([St1],[St2]). Recall that bs is the space of all sequences $t=\{t_j\}$ such that $\|t\|=\sup_n\left|\sum_{j=1}^{n}t_j\right|<\infty$ equipped with this norm ([B]1.2); an equivalent norm to $\|\|$ is given by $\|t\|'=\sup\{|\sum_{k\in I}t_k|: I\ an\ interval\}$. If $x=\{x_j\}$ and $y=\{y_j\}$ are sequences, we write $x\cdot y=\sum_{j=1}^{\infty}x_jy_j$ for the formal dot product of x and y when the series converges.

Example 1. bs has signed-SGHP. Actually, bs has an even stronger property than signed-SGHP; it is not necessary to pass to a subsequence in the definition of signed-SGHP. Let $\{I_k\}$ be an increasing sequence of intervals and $\{t^k\}\subset bs$ be bounded. Put $M=\sup\{\left|\sum_{j\in I}t_j^k\right|: k\in\mathbf{N}, I$ an interval in $\mathbf{N}\}<\infty$. Define signs inductively by setting $s_1=signC_{I_1}\cdot t^1$ and $s_{n+1}=-[sign\sum_{k=1}^{n}s_kC_{I_k}\cdot t^k][signC_{I_{n+1}}\cdot t^{n+1}]$. Put $y=\sum_{k=1}^{\infty}s_kC_{I_k}t^k$; we show $\|y\|\leq 2M$. We first show by induction that $\left|\sum_{j=1}^{\max I_n}y_j\right|\leq M$ for every n. For $n=1$, $\left|\sum_{j=1}^{\max I_1}y_j\right|=\left|\sum_{j\in I_1}s_1t_j^1\right|\leq M$. Suppose the inequality holds for n. Then $\left|\sum_{j=1}^{\max I_{n+1}}y_j\right|=\left|\sum_{j=1}^{\max I_n}y_j+\sum_{j\in I_{n+1}}y_j\right|\leq M$ since $\left|\sum_{j\in I_{n+1}}y_j\right|=\left|\sum_{j\in I_{n+1}}s_{n+1}t_j^{n+1}\right|\leq M$ and $\left|\sum_{j=1}^{\max I_n}y_j\right|\leq M$ and both of these terms have opposite signs. Now for arbitrary n let $k=k_n$ be the largest integer such that $\max I_k\leq n$. Then $\left|\sum_{j=1}^{n}y_j\right|=$

$\left|\sum_{j=1}^{\max I_k} y_j + \sum_{j=\max I_k+1}^{n} y_j\right| \le \left|\sum_{j=1}^{\max I_n} y_j\right| + \left|\sum_{j=\min I_{k+1}}^{n} s_{k+1} t_j^{k+1}\right| \le 2M$
so $\|y\| \le 2M$ as desired.

Note that bs does not have SGHP [consider $t = \{1, -1, 1, -1, ...\}$ and $I_k = \{2k-1\}$; then $C_{\cup I_k} t = \{1, 0, 1, 0, ...\} \notin bs$ and likewise the same holds for any subsequence of $\{I_k\}$]. Additional spaces with the signed-SGHP can be constructed employing the methods of [BSS].

We next consider the uniform convergence of multiplier convergent series when subsets of the space of multipliers has signed-SGHP. We first establish a lemma.

Lemma 2. Let $\Lambda \subset \lambda$. Let $\sum x_j$ be Λ-multiplier convergent. If the series $\sum_{j=1}^{\infty} t_j x_j$ do not converge uniformly for $t \in B \subset \Lambda$, then there exist a symmetric neighborhood of $0, V$, $t^k \in B$ and an increasing sequence of intervals $\{I_k\}$ such that $\sum_{j \in I_k} t_j^k x_j \notin V$.

Proof : If the series $\sum_{j=1}^{\infty} t_j x_j$ do not converge uniformly for $t \in B$, there exist a symmetric neighborhood of 0, U, such that for every k there exist $t^k \in B$, $m_k \ge k$ such that $\sum_{j=m_k}^{\infty} t_j^k x_j \notin U$. For $k = 1$, let $m_1, t^1 \in B$ satisfy this condition so $\sum_{j=m_1}^{\infty} t_j^1 x_j \notin U$. Pick a symmetric neighborhood of o, V, such that $V + V \subset U$. There exists $n_1 > m_1$ such that $\sum_{j=n_1+1}^{\infty} t_j^1 x_j \in V$. Then $\sum_{j=m_1}^{n_1} t_j^1 x_j = \sum_{j=m_1}^{\infty} t_j^1 x_j - \sum_{j=n_1+1}^{\infty} t_j^1 x_j \notin V$. Put $I_1 = [m_1, n_1]$. Now just continue the construction.

Theorem 3. Let $\Lambda \subset \lambda$ have signed-SGHP and let $\sum x_j$ be Λ-multiplier convergent. Then the series $\sum_{j=1}^{\infty} t_j x_j$ converge uniformly for t belonging to bounded subsets of Λ.

Proof: If $B \subset \Lambda$ is bounded and $\sum_{j=1}^{\infty} t_j x_j$ fails to converge uniformly for $t \in B$, let the notation be as in Lemma 2. Let n_k, s_k be as in the definition of signed-SGHP above and $t = \sum_{j=1}^{\infty} s_j C_{I_{n_j}} t^{n_j} \in \Lambda$. Then $\sum t_j x_j$ does not converge in X since $\sum_{j \in I_{n_k}} t_j x_j = s_k \sum_{j \in I_{n_k}} t_j^{n_k} x_j \notin V$,i.e., $\sum t_j x_j$ doesn't satisfy the Cauchy condition.

Remark 4. If $\Lambda = l^{\infty}$, then Theorem 3 implies that any bounded multiplier convergent series is such that the series $\sum_{j=1}^{\infty} t_j x_j$ converge uniformly for $\|t\|_{\infty} \le 1$. This statement improves the result for bounded multiplier convergent series given in 8.2.2 of [Sw2], removing the locally convex and sequential completeness assumptions. A (vector) version of Theorem 3 is established in [SS] Lemma 22 under the assumption that the multiplier space has SGHP. A result with the same conclusion as Theorem 3 is given in [WLC], Theorem 7, but the assumptions there are quite different, being topological in nature, and difficult to compare.

Remark 5. If $\lambda = m_0$ and $\Lambda = M_0$,then a series is subseries convergent iff the series is m_0 or M_0 multiplier convergent ($spanM_0 = m_0$). Then Theorem 3 implies that any subseries convergent series $\sum x_j$ is such that the series $\sum_{j=1}^{\infty} C_\sigma(j)x_j$ converge uniformly for $\sigma \subset \mathbf{N}$ ([Sw2]8.1.2).

Thus, Theorem 3 gives a generalization of the known results for uniform convergence of subseries and bounded multiplier convergent series.

Example 6. Without some type of assumption on the multiplier space λ, the conclusion of Theorem 3 can fail even when the multiplier space satisfies gliding hump conditions like the weak gliding hump property or the zero gliding hump property ([Sw2]). Let e^j be the sequence with a 1 in the j^{th} coordinate and 0 in the other coordinates. Then $\sum e^j$ is l^p-multiplier covergent in $(l^p, \|\|_p)$ for any $1 \leq p < \infty$, but the series $\sum_{j=1}^{\infty} t_j e^j$ do not converge uniformly for $\|t\|_p \leq 1$ [Take $t^k = e^k$, $\sum_{j=1}^{\infty} t_j^k e^j = e^k$.].

We next consider a multiplier version of the Hahn-Schur Theorem. The classical scalar version of the Hahn-Schur Theorem asserts that if the scalar series $\sum_j x_{ij}$ is absolutely convergent for every i and $\lim_i \sum_{j\in\sigma} x_{ij}$ exists for every $\sigma \subset \mathbf{N}$ and if $x_j = \lim_i x_{ij}$, then $\sum x_j$ is absolutely convergent and $\sum_{j=1}^{\infty} |x_{ij} - x_j| \to 0$ or, equivalently, $\lim_i \sum_{j\in\sigma}(x_{ij} - x_j) = 0$ uniformly for $\sigma \subset \mathbf{N}$ ([Sw2]5.4). By employing the last statement of the conclusion of the classical Hahn-Schur Theorem,versions of the Hahn-Schur Theorem for vector-valued subseries and bounded multiplier convergent series have been given in [Sw2]8.1,8.2 , and an abstract version of the theorem which includes certain multiplier convergent series is given in [Sw2]9.3 . We now present a version of the Hahn-Schur Theorem for multiplier convergent series when the subset Λ of the multiplier space λ satisfies the signed-SGHP. We first establish a special case of the theorem.

Lemma 7. Suppose that $\Lambda \subset \lambda$ has signed-SGHP, $\sum_j x_{ij}$ is Λ-multiplier convergent for every i and $\lim_i \sum_{j=1}^{\infty} t_j x_{ij} = 0$ for every $t \in \Lambda$. If $B \subset \Lambda$ is bounded, then $\lim_i \sum_{j=1}^{\infty} t_j x_{ij} = 0$ uniformly for $t \in B$.

Proof : It suffices to show that $\lim_i \sum_{j=1}^{\infty} t_j^i x_{ij} = 0$ for any sequence $\{t^i\} \subset B$. Let U be a neighborhood of 0 in X and pick a symmetric neighborhood, V, of 0 such that $V+V+V \subset U$. Set $n_1 = 1$ and pick N_1 such that

$\sum_{j=N_1}^{\infty} t_j^{n_1} x_{n_1 j} \in V$. Since $\lim_i x_{ij} = 0$ ($c_{00} \subset \lambda$) for every j and $\{t_j^i : i \in \mathbf{N}\}$ is bounded from the K space assumption on λ, $\lim_i t_j^i x_{ij} = 0$ for every j ([Sw2]8.2.4) so there exist $n_2 > n_1$ such that $\sum_{j=1}^{N_1-1} t_j^i x_{ij} \in V$ for $i \geq n_2$. Pick $N_2 > N_1$ such that $\sum_{j=N_2}^{\infty} t^{n_2} x_{n_2 j} \in V$. Continuing this construction produces increasing sequences $\{n_k\}, \{N_k\}$ such that $\sum_{l=N_j}^{\infty} t_l^{n_j} x_{n_j l} \in V$ and $\sum_{l=1}^{N_j-1} t_l^i x_{il} \in V$ for $i \geq n_j$. Set $I_j = \{l : N_{j-1} \leq l < N_j\}$. Define the matrix $M = [m_{ij}] = [\sum_{l \in I_j} t_l^{n_j} x_{n_i l}]$. We show that M is a signed K-matrix in the terminology of [St1] ,[St2] ,[Sw2]2.2.4 . First the columns of M go to 0 since $\lim_i x_{il} = 0$ for every l. Given an increasing sequence $\{p_j\}$, by the signed-SGHP assumption there is a subsequence $\{q_j\}$ of $\{p_j\}$ and signs $\{s_j\}$ such that $t = \{t_j\} = \sum_{j=1}^{\infty} s_j C_{I_{q_j}} t^{q_j} \in \Lambda$. Then $\sum_{j=1}^{\infty} s_j m_{iq_j} = \sum_{j=1}^{\infty} s_j \sum_{l \in I_{q_j}} t_l^{q_j} x_{n_i l} = \sum_{j=1}^{\infty} t_j x_{n_i j} \to 0$ by hypothesis. Therefore, M is a signed K-matrix and by the signed version of the Antosik-Mikusinski Matrix Theorem the diagonal of M goes to 0 ([St1] ,[St2],[Sw2]2.2.4). Thus, there exists N such that $m_{ii} \in V$ for $i \geq N$. If $i \geq N$, then $\sum_{l=1}^{\infty} t_l^{n_i} x_{n_i l} = \sum_{l=1}^{N_{i-1}-1} t_l^{n_i} x_{n_i l} + \sum_{l \in I_i} t_l^{n_i} x_{n_i l} + \sum_{l=N_i}^{\infty} t_l^{n_i} x_{n_i l} \in V + V + V \subset U$ so $\lim_i \sum_{l=1}^{\infty} t_l^{n_i} x_{n_i l} = 0$. Since the same argument can be applied to any subsequence, it follows that $\lim_i \sum_{j=1}^{\infty} t_j^i x_{ij} = 0$

Theorem 8. Suppose that $\Lambda \subset \lambda$ has signed-SGHP, $\sum_j x_{ij}$ is Λ-multiplier convergent for every i and $\lim_i \sum_{j=1}^{\infty} t_j x_{ij}$ exists for every $t \in \Lambda$. Let $x_j = \lim_i x_{ij}$ for every j. If $B \subset \Lambda$ is bounded, then (i) $\sum x_j$ is $\Lambda-$multiplier convergent, (ii) $\lim_i \sum_{j=1}^{\infty} t_j x_{ij} = \sum_{j=1}^{\infty} t_j x_j$ uniformly for $t \in B$, (iii) the series $\sum_{j=1}^{\infty} t_j x_{ij}$ converge uniformly for $t \in B$.

Proof : Let $t \in \Lambda$. Since the space Λ has the signed weak gliding hump property, it follows from Stuart's weak sequential completeness result that $\sum_{j=1}^{\infty} t_j x_j$ converges and $\lim_i \sum_{j=1}^{\infty} t_j x_{ij} = \sum_{j=1}^{\infty} t_j x_j$.([Sw2] 12.4.1;see also [St1] 3.5); Stuart's result is for the case when Λ is a vector space with signed-WGHP, but his proof is valid for a subset with signed-WGHP.

Since $\lim_i \sum_{j=1}^{\infty} t_j (x_{ij} - x_j) = 0$ for every $t \in \Lambda$, Lemma 7 applies and gives (ii).

Suppose that (iii) fails to hold. Then there exists a closed symmetric neighborhood of 0, U, in X such that for every i there exist $k_i > i$, a finite interval I_i with $\min I_i > i$, $t^i \in B$ such that $\sum_{k \in I_i} t_k^i x_{k_i k} \notin U$. Put $i_1 = 1$. By the above, there exist $k_1 > 1$, I_1 with $\min I_1 > i_1$, $t^1 \in B$ such that $\sum_{k \in I_1} t_k^1 x_{k_1 k} \notin U$. By Theorem 3 there exists j_1 such that $\sum_{k=j}^{\infty} t_k x_{ik} \in U$ for every $t \in B, 1 \leq i \leq k_1, j \geq j_1$. Set $i_2 = \max\{I_1 + 1, j_1\}$. Again, by the above there exist $k_2 > i_2, I_2$ with $\min I_2 > i_2, t^2 \in B$ such that $\sum_{k\in I_2} t_k^2 x_{k_2 k} \notin U$. Note that $k_2 > k_1$ by the definition of i_2.

Continuing this construction produces an increasing sequence $\{k_i\}$, an increasing sequence of intervals $\{I_i\}$ and $t^i \in B$ such that

(1) $\sum_{k\in I_i} t_k^i x_{k_i k} \notin U$.

Define a matrix $M = [m_{ij}] = [\sum_{k\in I_j} t_k^i x_{k_i k}]$. We claim that M is a signed K-matrix. First, the columns of M converge by hypothesis. Next, given any increasing sequence $\{p_j\}$ there is a subsequence $\{q_j\}$ of $\{p_j\}$ and signs $\{s_j\}$ such that $t = \{t_j\} = \sum_{j=1}^{\infty} s_j C_{I_{q_j}} t^{q_j} \in \Lambda$. Then the sequence $\sum_{j=1}^{\infty} s_j m_{iq_j} = \sum_{j=1}^{\infty} s_j \sum_{l\in I_{q_j}} t_l^{q_j} x_{k_i l} = \sum_{j=1}^{\infty} t_j x_{k_i j}$ converges by hypothesis. Hence, M is a signed K-matrix and the diagonal of M converges to 0 by the signed version of the Antosik-Mikusinski Matrix Theorem ([Sw2]2.2.4). But, this contadicts (1).

A subseries version ($M_0 = \Lambda \subset m_0 = \lambda$) of the Hahn-Schur Theorem is given in [Sw2]8.1 and a bounded multiplier version of the Hahn-Schur Theorem is given in [Sw2]8.2. Both versions follow from Theorem 3. A (vector) version of Theorem 7 for spaces with SGHP is given in Theorem 25 and Corollary 27 of [SS].

Without some assumption on the multiplier space λ, the conclusion of Theorem 8 can fail.

Example 9. Let $x_{ij} = e^j$ if $1 \le j \le i$ and $x_{ij} = 0$ if $i < j$. Then $\sum_j x_{ij}$ is l^p-multiplier convergent in l^p for $1 \le p < \infty$ for every i. If $t \in l^p$, $\sum_{j=1}^{\infty} t_j x_{ij} \to \sum_{j=1}^{\infty} t_j e^j$ in l^p. However, the convergence is not uniform for $\|t\|_p \le 1$ [Take $t^k = e^k$, so $\sum_{j=1}^{\infty} t_j^k x_{ij} = \sum_{j=1}^{i} t_j^k e^j = e^k$ if $i \ge k$.].

There is an abstract version of the Hahn-Schur Theorem which covers certain multiplier convergent series given in [Sw2]9.3; however, the vector version given there uses essentially a strong gliding hump type hypothesis. A (vector) version of Theorem 7 for spaces with SGHP is given in [SS] Theorem 25. Useful vector forms of the signed-SGHP seem to be difficult to formulate. There is a version of Theorem 7 given in [WCC],Theorem 7, and in [AP], Theorem 3.1, but the assumptions there are of a topological nature unlike the algebraic signed-SGHP.

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Charles Swartz
Mathematics Department
New Mexico State University
Las Cruces
NM 88003
USA
e-mail : cswartz@nmsu.edu