Proyecciones Journal of Mathematics
Vol. 36, N^o 1, pp. 1-11, March 2017.
Universidad Católica del Norte
Antofagasta - Chile
DOI: 10.4067/S0716-09172017000100001

Odd vertex equitable even labeling of graphs

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Received : January 2016. Accepted : May 2016

Abstract

In this paper, we introduce a new labeling called odd vertex equitable even labeling. Let G be a graph with p vertices and q edges and $A = \{1, 3, \ldots, q\}$ if q is odd or $A = \{1, 3, \ldots, q+1\}$ if q is even. A graph G is said to admit an odd vertex equitable even labeling if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u)+f(v)$ for all edges uv such that for all a and b in $A, |v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2, 4, \ldots, 2q$ where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits odd vertex equitable even labeling is called odd vertex equitable even graph. We investigate the odd vertex equitable even behavior of some standard graphs.

Keywords : *Mean labeling; odd mean labeling; k-equitable labeling; vertex equitable labeling; odd vertex equitable even labeling; odd vertex equitable even graph.*

AMS Subject Classification : *05C78.*

1. Introduction

All graphs considered here are simple, finite, connected and undirected. Let $G(V, E)$ be a graph with p vertices and q edges. We follow the basic notations and terminologies of graph theory as in [3]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling and a detailed survey of graph labeling can be found in [2]. The concept of mean labeling was introduced in [8].

A graph $G(V, E)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0, 1, 2, \ldots, q\}$ such that for each edge uv, labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then the resulting edge labels are distinct. The concept of k-equitable labeling was introduced by Cahit [1]. Let G be a graph. A labeling $f : V(G) \to \{0, 1, \ldots, k-1\}$ is called k-equitable labeling if the condition $|v_f(i) - v_f(j)| \leq 1, |e_{\overline{f}}(i) - e_{\overline{f}}(j)| \leq 1, i \neq j, i, j = 0, 1, \ldots, k-1$ is satisfied, where as before the induced edge labeling is given by $\overline{f}(\{u, v\}) = |f(u) - f(v)|$ and $v_f(x)$ and $e_{\overline{f}}(x), x \in \{0, 1, \ldots, k-1\}$ is the number of vertices and edges of G respectively with label x. The notion of odd mean labeling was due to Manickam and Marudai [6]. Let $G(V, E)$ be a graph with p vertices and q edges. A graph G is said to be odd mean graph if there exists a function $f : V(G) \to \{0, 1, 2, 3, \ldots, 2q-1\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$ is a bijection. The function f is called an odd mean labeling.

The concept of vertex equitable labeling was due to Lourdusamy and Seenivasan in [5]. Let G be a graph with p vertices and q edges and $A = \{0, 1, 2, \ldots, \lceil \frac{q}{2} \rceil\}$. A graph G is said to be vertex equitable if there exists a vertex labeling $f : V(G) \to A$ induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $1, 2, 3, \ldots, q$, where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. The vertex labeling f is known as vertex equitable labeling. Motivated by the concepts of k-equitable labeling [1], odd mean labeling [6] and vertex equitable labeling [5] of graphs, we define a new labeling called odd vertex equitable even labeling.

Let G be a graph with p vertices and q edges and $A = \{1, 3, \ldots, q\}$ if q is odd or $A = \{1, 3, \ldots, q+1\}$ if q is even. A graph G is said to admit odd vertex equitable even labeling if there exists a vertex labeling $f : V(G) \to A$ that induces an edge labeling f^* defined by $f^*(uv) = f(u) + f(v)$ for all edges uv such that for all a and b in A, $|v_f(a) - v_f(b)| \leq 1$ and the induced edge labels are $2, 4, \ldots, 2q$ where $v_f(a)$ be the number of vertices v with $f(v) = a$ for $a \in A$. A graph that admits odd vertex equitable even labeling then G is called odd vertex equitable even graph.

We observe that $K_{1,3}$ and K_3 are vertex equitable graphs but not odd vertex equitable even graphs. We use the following definitions in the subsequent section.

Definition 1.1. *The disjoint union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.*

Definition 1.2. *The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.*

Definition 1.3. [7] *Let G be a graph with n vertices and t edges. A graph H is said to be a super subdivision of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2,m_i} for some integer $m_i, 1 \leq i \leq t$ in such a way that ends of e_i are merged with two vertices of the 2-vertices part of K_{2,m_i} after removing the edge e_i from G. A super subdivision H of a graph G is said to be an arbitrary super subdivision of a graph G if every edge of G is replaced by an arbitrary $K_{2,m}$ (m may vary for each edge arbitrarily).*

Definition 1.4. [4] *Let T be a tree and u_0 and v_0 be the two adjacent vertices in T. Let u and v be the two pendant vertices of T such that the length of the path u_0-u is equal to the length of the path v_0-v. If the edge u_0v_0 is deleted from T and u and v are joined by an edge uv, then such a transformation of T is called an elementary parallel transformation (or an ept) and the edge u_0v_0 is called transformable edge. If by the sequence of epts, T can be reduced to a path, then T is called a T_p-tree (transformed tree) and such sequence regarded as a composition of mappings (epts) denoted by P, is called a parallel transformation of T. The path, the image of T under P is denoted as $P(T)$. A T_p-tree and the sequence of two epts reducing it to a path are illustrated in Figure 1.*

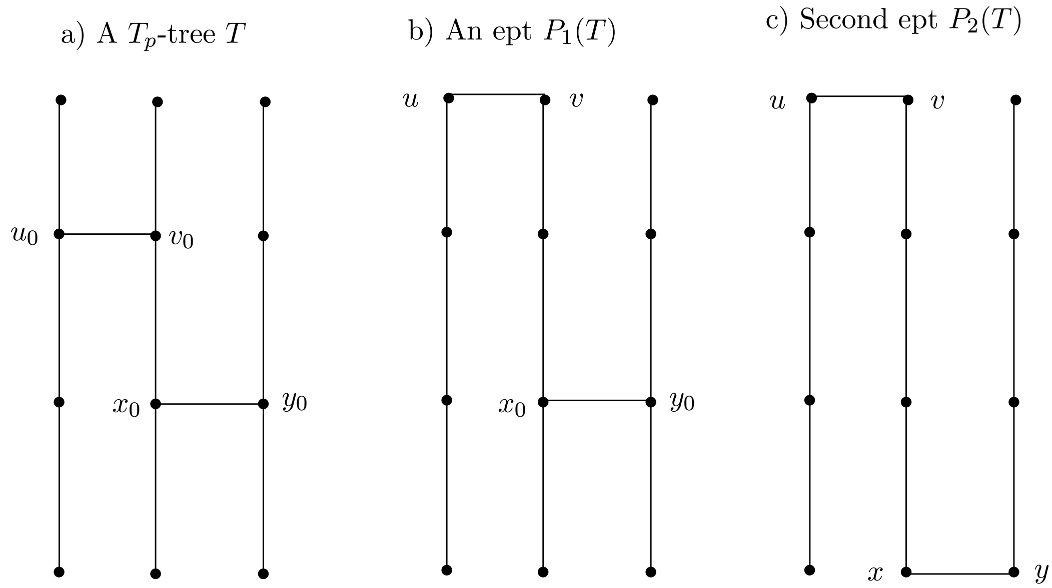

Figure 1

Definition 1.5. *The graph $P_n@P_m$ is obtained by identifying the pendant vertex of a copy of path P_m at each vertex of the path P_n.*

2. Main Results

Theorem 2.1. *Any path is an odd vertex equitable even graph.*

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of the path P_n and it has n vertices and $n-1$ edges. Let $A = \begin{cases} 1, 3, \ldots, n-1 & \text{if } n-1 \text{ is odd} \\ 1, 3, \ldots, n & \text{if } n-1 \text{ is even.} \end{cases}$

Define a vertex labeling $f : V(P_n) \to A$ as follows:

For $1 \leq i \leq n$, $f(u_i) = \begin{cases} i & \text{if } i \text{ is odd} \\ i-1 & \text{if } i \text{ is even.} \end{cases}$

It can be verified that the induced edge labels of P_n are $2, 4, \ldots, 2n-2$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of P_n. Thus P_n is an odd vertex equitable even graph. □

Theorem 2.2. *The graph $P_n@P_m$ is an odd vertex equitable even graph for any $n, m \geq 1$.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices of the path P_n. Let $v_{i1}, v_{i2}, \ldots, v_{im}$ be the vertices on the i^{th} copy of the path P_m so that v_{i_m} is identified with v_i for $1 \leq i \leq n$. Clearly $P_n@P_m$ has mn vertices and $mn-1$ edges.

Let $A = \begin{cases} 1, 3, \ldots, mn-1 & \text{if } mn-1 \text{ is odd} \\ 1, 3, \ldots, mn & \text{if } mn-1 \text{ is even.} \end{cases}$

Define a vertex labeling $f : V(P_n@P_m) \to A$ as follows:

For $1 \leq i \leq n, 1 \leq j \leq m$,

If i is odd, $f(v_{ij}) = \begin{cases} m(i-1)+j & \text{if } j \text{ is odd} \\ m(i-1)+j-1 & \text{if } j \text{ is even.} \end{cases}$

If i is even, $f(v_{ij}) = \begin{cases} mi-j & \text{if } j \text{ is odd} \\ mi-(j-1) & \text{if } j \text{ is even.} \end{cases}$

It can be verified that the induced edge labels of $P_n@P_m$ are $2, 4, \ldots, 2mn-2$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of $P_n@P_m$. Thus, $P_n@P_m$ is an odd vertex equitable even graph. □

Corollary 2.3. *The graph $P_n \odot K_1$ is an odd vertex equitable even graph for any $n \geq 1$.*

Theorem 2.4. *The graph $K_{1,n}$ is an odd vertex equitable even graph if only if $n \leq 2$.*

Proof. Suppose that $n \leq 2$. When $n = 1, K_{1,n} \cong P_2$ and $n = 2, K_{1,n} \cong P_3$. Hence by Theorem 2.1, $K_{1,n}$ is an odd vertex equitable even graph. Suppose that $n \geq 3$ and $K_{1,n}$ is an odd vertex equitable even graph with odd vertex equitable even labeling f. Let $\{V_1, V_2\}$ be the bipartition of $K_{1,n}$ with $V_1 = \{u\}$ and $V_2 = \{u_1, u_2, \ldots, u_n\}$. To get the edge label 2, we have to assign the label 1, to the two adjacent vertices. Thus 1 must be the label of u. Since $n \geq 3$, the maximum value of the edge label is either $n+1$ or $n+2$ according as n is odd or even. Hence, there is no edge with the induced label $2n$. Thus, $K_{1,n}$ is not an odd vertex equitable even graph if $n \geq 3$. □

Theorem 2.5. *The graph $K_{1,n} \cup K_{1,n-2}$ is an odd vertex equitable even graph for any $n \geq 3$.*

Proof. Let u, v be the centre vertices of the two star graphs, $K_{1,n}, K_{1,n-2}$. Assume that $u_1, u_2, \ldots, u_n$ be the vertices incident with u and $v_1, v_2, \ldots, v_{n-2}$ be the vertices incident with v. Hence $K_{1,n} \cup K_{1,n-2}$ has $2n+2$ vertices and $2n-2$ edges. Let $A = \{1, 3, \ldots, 2n-1\}$.

Define a vertex labeling $f : V(K_{1,n} \cup K_{1,n-2}) \rightarrow A$ as follows:
$f(u) = 1, f(v) = 2n-1, f(u_i) = 2i-1$ if $1 \leq i \leq n$ and

$f(v_i) = 2i+1$ if $1 \leq i \leq n-2$.

It can be verified that the induced edge labels of $K_{1,n} \cup K_{1,n-2}$ are $2, 4, \ldots, 4n-4$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of $K_{1,n} \cup K_{1,n-2}$. Thus, $K_{1,n} \cup K_{1,n-2}$ is an odd vertex equitable even graph. □

Theorem 2.6. *The graph $K_{2,n}$ is an odd vertex equitable even graph for all n.*

Proof. Let $\{V_1, V_2\}$ be the bipartition of $K_{2,n}$ with $V_1 = \{u, v\}$ and $V_2 = \{u_1, u_2, \ldots, u_n\}$. It has $n+2$ vertices and $2n$ edges. Let $A = \{1, 3, \ldots, 2n+1\}$.

Define a vertex labeling $f : V(K_{2,n}) \rightarrow A$ as follows:

$f(u) = 1, f(v) = 2n+1$ and $f(u_i) = 2i-1$ if $1 \leq i \leq n$.

It can be verified that the induced edge labels of $K_{2,n}$ are $2, 4, \ldots, 4n$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of $K_{2,n}$. Thus, $K_{2,n}$ is an odd vertex equitable even graph. □

Theorem 2.7. *Let G be a graph with p vertices and q edges and $p \leq \lceil \frac{q}{2} \rceil + 1$ then G is not an odd vertex equitable even graph.*

Proof. Let G be a graph with p vertices and q edges.
Case (i): Let $q = 2m+1$.

Suppose G is an odd vertex equitable even graph. Let $A = \{1, 3, \ldots, 2m+1\}$. To get an edge label 2, there must be two adjacent vertices u and v with label 1. Also to get the edge label $4m+2$, there must be two adjacent vertices x and y with label $2m+1$. Hence, the number of vertices must be greater than or equal to $m+3$. Then G is not an odd vertex equitable even graph.
Case (ii): Let $q = 2m$.

Suppose G is an odd vertex equitable even graph. Let $A = \{1, 3, \ldots, 2m+1\}$. To get the edge label 2, there must be two adjacent vertices u and v each has the label 1. The number of vertices must be greater than or equal to $m+2$. Then G is not an odd vertex equitable even graph. □

Corollary 2.8. *The graph $K_{m,n}$ is not an odd vertex equitable even graph if $m, n \geq 3$.*

Theorem 2.9. *Every T_p-tree is an odd vertex equitable even graph.*

Proof. Let T be a T_p-tree with n vertices. By the definition of a transformed tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \cup E_p$
where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts P used to arrive the path $P(T)$. Clearly, E_d and E_p have the same number of edges.

Now denote the vertices of $P(T)$ successively as $v_1, v_2, \ldots, v_n$ starting from one pendant vertex of $P(T)$ right up to the other.

For $1 \leq i \leq n$, define the labeling f as $f(v_i) = \begin{cases} i & \text{if } i \text{ is odd} \\ i-1 & \text{if } i \text{ is even.} \end{cases}$

Then f is an odd vertex equitable even labeling of the path $P(T)$.

Let v_iv_j be an edge in T for some indices i and j with $1 \leq i < j \leq n$. Let P_1 be the ept that delete the edge v_iv_j and add an edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts.

Since $v_{i+t}v_{j-t}$ is an edge of the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i + 2t + 1$. Therefore i and j are of opposite parity.

The induced label of the edge v_iv_j is given by $f^*(v_iv_j) = f^*(v_iv_{i+2t+1}) = f(v_i)+f(v_{i+2t+1}) = 2(i+t), 1 \leq i \leq n$. Now $f^*(v_{i+t}v_{j-t}) = f^*(v_{i+t}v_{i+t+1}) = f(v_{i+t} + f(v_{i+t+1}) == 2(i+t), 1 \leq i \leq n$. Therefore, we have $f^*(v_iv_j) = f^*(v_{i+t}v_{j-t})$ and hence f is a an odd vertex equitable even labeling of the T_p-tree T. $\square$

Theorem 2.10. *If every edge of a graph G is an edge of a triangle, then G is not an odd vertex equitable even graph.*

Proof. Let G be a graph in which every edge is an edge of a triangle. Suppose G is an odd vertex equitable even graph with odd vertex equitable even labeling f. To get 2 as an edge label, there must be two adjacent vertices u and v such that $f(u) = 1$ and $f(v) = 1$. Let $uvwu$ be a triangle . To get 4 as an edge label, there must be $f(w) = 3$, then uw and vw get the same edge label. This is contradiction to f is an odd vertex equitable even labeling. Hence G is not an odd vertex equitable even graph. $\square$

Corollary 2.11. *The complete graph K_n where $n \geq 3$, the wheel W_n, the triangular snake, double triangular snake, triangular ladder, flower graph FL_n, fan graph $P_n + K_1, n \geq 2$, double fan graph $P_n + K_2, n \geq 2$, friendship graph C_3^n, windmill $K_m^n, m > 3, K_2 + mK_1$, square graph $B_{n,n}^2$, total graph $T(P_n)$ and composition graph $P_n[P_2]$ are not odd vertex equitable even graphs.*

Theorem 2.12. *The cycle C_n is an odd vertex equitable even graph if $n \equiv 0$ of $1(mod\ 4)$.*

Proof. Suppose $n \equiv 0$ or $1(mod\ 4)$. Let $u_1, u_2, \ldots, u_n$ be the vertices of the cycle C_n. Let $A = \begin{cases} 1, 3, \ldots, n & \text{if } n \text{ is odd} \\ 1, 3, \ldots, n+1 & \text{if } n \text{ is even.} \end{cases}$

Define a vertex labeling $f : V(C_n) \to A$ as follows:
$f(u_i) = i$ if i is odd and

$$1 \leq i \leq n,\ f(u_i) = \begin{cases} i-1 & \text{if } i \text{ is even and } 1 \leq i \leq \lfloor \frac{n}{2} \rfloor \\ i+1 & \text{if } i \text{ is even and } \lceil \frac{n}{2} \rceil \leq i \leq n \end{cases}$$

It can be verified that the induced edge labels of cycle are $2, 4, \ldots, 2n$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of cycle. Thus, the cycle C_n is an odd vertex equitable even graph if $n \equiv 0$ or $1(mod\ 4)$. □

Theorem 2.13. *A quadrilateral snake Q_n is an odd vertex equitable even graph.*

Proof. A quadrilateral snake is obtained from a path $u_1, u_2, \ldots, u_n$ by joining u_i, u_{i+1} to the new vertices v_i, w_i respectively and joining v_i and $w_i, 1 \leq i \leq n-1$. It has $3n-2$ vertices and $4n-4$ edges. Let $A = \{1, 3, \ldots, 4n-3\}$.

Define a vertex labeling $f : V(Q_n) \to A$ as follows:

$f(u_i) = 4i - 3$ if $1 \leq i \leq n, f(v_i) = 4i - 3$ and $f(w_i) = 4i - 1$ if $1 \leq i \leq n-1$.

It can be verified that the induced edge labels of quadrilateral snake are $2, 4, \ldots, 8n-8$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of quadrilateral snake. Thus, quadrilateral snake is an odd vertex equitable even graph. □

Theorem 2.14. *The ladder graph L_n is an odd vertex equitable even graph for all n.*

Proof. Let u_i and v_i be the vertices of L_n. Then $E(L_n) = \{u_iu_{i+1} : 1 \leq i \leq n-1\} \cup \{u_iv_i : 1 \leq i \leq n\} \cup \{v_iv_{i+1} : 1 \leq i \leq n-1\}$. Then L_n has $2n$ vertices and $3n-2$ edges.

Let $A = \begin{cases} 1, 3, \ldots, 3n-2 & \text{if } n \text{ is odd} \\ 1, 3, \ldots, 3n-1 & \text{if } n \text{ is even.} \end{cases}$

Define a vertex labeling $f : V(L_n) \to A$ as follows:

$f(u_{2i-1}) = f(v_{2i-1}) = 6i-5$ if $1 \leq i \leq \lceil \frac{n}{2} \rceil$, $f(u_{2i}) = 6i-1$ and $f(v_i) = 6i-3$ if $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$.

It can be verified that the induced edge labels of L_n are $2, 4, \ldots, 6n-4$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is an odd vertex equitable even labeling of L_n. Thus, L_n is an odd vertex equitable even graph. □

Theorem 2.15. *The graph $L_n \odot K_1$ is an odd vertex equitable even graph for all n.*

Proof. Let L_n be the ladder. Let $L_n \odot K_1$ be the graph obtained by joining a pendant edge to each vertex of the ladder. Let u_i and v_i be the vertices of L_n. For $1 \leq i \leq n, u'_i$ and v'_i be the new vertices adjacent with u_i and v_i respectively. Clearly $L_n \odot K_1$ has $4n$ vertices and $5n-2$ edges.

Let $A = \begin{cases} 1, 3, \ldots, 5n-2 & \text{if } n \text{ is odd} \\ 1, 3, \ldots, 5n-1 & \text{if } n \text{ is even.} \end{cases}$

Define a vertex labeling $f : V(L_n \odot K_1) \to A$ as follows:

For $1 \leq i \leq \lceil \frac{n}{2} \rceil$,

$f(u_{2i-1}) = f(u'_{2i-1}) = 10i-9, f(v_{2i-1}) = f(v'_{2i-1}) = 10i-7$.

For $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$,

$f(u_{2i}) = 10i-1, f(v_{2i}) = 10i-5, f(u'_{2i}) = f(v'_{2i}) = 10i-3$.

It can be verified that the induced edge labels of $L_n \odot K_1$ are $2, 4, \ldots, 10n-4$ and $|v_f(i) - v_f(j)| \leq 1$ for all $i, j \in A$. Clearly f is a odd vertex equitable even labeling of $L_n \odot K_1$. Thus, $L_n \odot K_1$ is an odd vertex equitable even graph. □

Theorem 2.16. *The arbitrary super subdivision of any path P_n is an odd vertex equitable even graph.*

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_i = v_iv_{i+1}$ be the edges of the path P_n for $1 \leq i \leq n-1$. Let G be an arbitrary super subdivision of the path P_n. That is, for $1 \leq i \leq n-1$ each edge e_i of the path P_n is replaced by a complete bipartite graph K_{2,m_i} where m_i is any positive integer. Let $V(G) = \{v_i : 1 \leq i \leq n\} \cup \{u_{ij} : 1 \leq j \leq m_i, 1 \leq i \leq n-1\}$.

Clearly G has $m_1+m_2+\ldots+m_{n-1}+n$ vertices and $2(m_1+m_2+\ldots+m_{n-1}$ edges. Let $A=\{1,3,\ldots,2(m_1+m_2+\ldots+m_{n-1})+1\}$.

Define a vertex labeling $f:V(G)\rightarrow A$ as follows:

$f(v_1)=1, f(v_i)=2(m_1+m_2+\ldots+m_{i-1})+1$ if $2\leq i\leq n, f(u_{1j})=2j-1$ if $1\leq j\leq m_1$ and $f(u_{ij})=f(v_i)+2j-2$ if $2\leq i\leq n-1, 1\leq j\leq m_i$.

Therefore the induced edge labels of G are $2,4,\ldots,4(m_1+m_2+\ldots+m_{n-1})$ and $|v_f(i)-v_f(j)|\leq 1$ for all $i,j\in A$. Clearly f is an odd vertex equitable even labeling of G. Thus, arbitrary super subdivision of any path is an odd vertex equitable even graph. $\Box$

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