

ON $\tilde{g}$ -HOMEOMORPHISMS IN TOPOLOGICAL SPACES

M. CALDAS

UNIVERSIDADE FEDERAL FLUMINENSE, BRASIL

S. JAFARI

COLLEGE OF VESTSJAELLAND SOUTH, DENMARK

N. RAJESH

PONNAIYAH RAMAJAYAM COLLEGE, INDIA

and

M. L. THIVAGAR

ARUL ANANDHAR COLLEGE

Received : August 2006. Accepted : November 2008

Abstract

In this paper, we first introduce a new class of closed map called $\tilde{g}$ -closed map. Moreover, we introduce a new class of homeomorphism called $\tilde{g}$ -homeomorphism, which are weaker than homeomorphism. We prove that gc -homeomorphism and $\tilde{g}$ -homeomorphism are independent. We also introduce $\tilde{g}^$ -homeomorphisms and prove that the set of all $\tilde{g}^*$ -homeomorphisms forms a group under the operation of composition of maps.*

2000 Math. Subject Classification : *54A05, 54C08.*

Keywords and phrases : *$\tilde{g}$ -closed set, $\tilde{g}$ -open set, $\tilde{g}$ -continuous function, $\tilde{g}$ -irresolute map.*

1. Introduction

The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map $f : X \rightarrow Y$ when both f and f^{-1} are continuous. It is well known that as Jänich [[5], p.13] says correctly: homeomorphisms play the same role in topology that linear isomorphisms play in linear algebra, or that biholomorphic maps play in function theory, or group isomorphisms in group theory, or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism, Maki et al. [9] introduced g -homeomorphisms and gc -homeomorphisms in topological spaces. Recently, Devi et al. [2] studied semi-generalized homeomorphisms and generalized semi-homeomorphisms.

In this paper, we first introduce $\tilde{g}$ -closed maps in topological spaces and then we introduce and study $\tilde{g}$ -homeomorphisms, which are weaker than homeomorphisms. We prove that gc -homeomorphism and $\tilde{g}$ -homeomorphism are independent. We also introduce $\tilde{g}^*$ -homeomorphisms. It turns out that the set of all $\tilde{g}^*$ -homeomorphisms forms a group under the operation composition of functions.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $cl(A)$, $int(A)$ and A^c denote the closure of A , the interior of A and the complement of A in X , respectively.

We recall the following definitions and some results, which are used in the sequel.

Definition 1. A subset A of a space (X, τ) is called:

- (i) *semi-open* [6] if $A \subset cl(int(A))$,
- (ii) *α -open* [11] if $A \subset int(cl(int(A)))$,
- (iii) *regular open* [17] if $A = int(cl(A))$,
- (iv) *pre-closed* [10] if $cl(int(A)) \subset A$.

The semi-closure [1] (resp. α -closure [11]) of a subset A of X , denoted by $scl_X(A)$ (resp. $\alpha cl_X(A)$) briefly $scl(A)$, (resp. $\alpha cl(A)$) is defined to be the intersection of all semi-closed (resp. α -closed) sets containing A .

Definition 2. A subset A of a space (X, τ) is called a:

- (i) generalized closed (briefly g -closed) set [7] if $cl(A) \subset U$ whenever $A \subset U$ and U is open in (X, τ) .
- (ii) generalized α -closed (briefly $g\alpha$ -closed) set [8] if $\alpha cl(A) \subset U$ whenever $A \subset U$ and U is α -open in (X, τ) .
- (iii) $\hat{g}$ -closed set [19] if $cl(A) \subset U$ whenever $A \subset U$ and U is semi-open in (X, τ) .
- (iv) $*g$ -closed set [20] if $cl(A) \subset U$ whenever $A \subset U$ and U is $\hat{g}$ -open in (X, τ) .
- (v) $\#g$ -semi-closed (briefly $\#gs$ -closed) set [21] if $scl(A) \subset U$ whenever $A \subset U$ and U is $*g$ -open in (X, τ) .
- (vi) $\tilde{g}$ -closed set [4] if $cl(A) \subset U$ whenever $A \subset U$ and U is $\#gs$ -open in (X, τ) .

Where $\hat{g}$ -open (resp. g -open, $g\alpha$ -open, rg -open, $*g$ -open, $\#gs$ -open, $\tilde{g}$ -open), are defined as the complement of $\hat{g}$ -closed (resp. g -closed, $g\alpha$ -closed, rg -closed, $*g$ -closed, $\#gs$ -closed, $\tilde{g}$ -closed).

Definition 3. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called:

- (i) g -continuous [18] if $f^{-1}(V)$ is g -closed in (X, τ) for every closed set V in (Y, σ) .
- (ii) $\hat{g}$ -continuous [19] if $f^{-1}(V)$ is $\hat{g}$ -closed in (X, τ) for every closed set V in (Y, σ) .
- (iii) $\tilde{g}$ -continuous [15] if $f^{-1}(V)$ is $\tilde{g}$ -closed in (X, τ) for every closed set V in (Y, σ) .
- (iv) $\tilde{g}$ -irresolute [13] if $f^{-1}(V)$ is $\tilde{g}$ -closed in (X, τ) for every $\tilde{g}$ -closed set V in (Y, σ) .
- (v) gc -irresolute [18] if $f^{-1}(V)$ is g -closed in (X, τ) for each g -closed set V of (Y, σ) .
- (vi) $\#gs$ -irresolute [21] if $f^{-1}(V)$ is $\#gs$ -closed in (X, τ) for each $\#gs$ -closed set V of (Y, σ) .
- (vii) strongly $\tilde{g}$ -continuous [13] if the inverse image of every $\tilde{g}$ -open set in (Y, σ) is open in (X, τ) .

Definition 4. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called:

- (i) g -open [18] if $f(V)$ is g -open in (Y, σ) for every g -open set V in (X, τ) .
- (ii) $\hat{g}$ -open [19] if $f(V)$ is $\hat{g}$ -open in (Y, σ) for every $\hat{g}$ -open set V in (X, τ) .

Definition 5. A bijective function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a:

- (i) generalized homeomorphism (briefly g -homeomorphism) [9] if f is both

g -continuous and g -open,

(ii) gc -homeomorphism [9] if both f and f^{-1} are gc -irresolute maps,

(iii) $\hat{g}$ -homeomorphism [19] if f is both $\hat{g}$ -continuous and $\hat{g}$ -open.

Definition 6. [14] Let (X, τ) be a topological space and $E \subset X$. We define the $\tilde{g}$ -closure of E (briefly $\tilde{g}\text{-cl}(E)$) to be the intersection of all $\tilde{g}$ -closed sets containing E . i.e.,

$$\tilde{g}\text{-cl}(E) = \cap \{A : E \subset A \text{ and } A \in \tilde{GC}(X, \tau)\}.$$

Proposition 2.1. [13] If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -irresolute, then it is $\tilde{g}$ -continuous.

Proposition 2.2. [14] Let (X, τ) be a topological space and $E \subset X$. The following properties are satisfied:

- (i) $\tilde{g}\text{-cl}(E)$ is the smallest $\tilde{g}$ -closed set containing E and
- (ii) E is $\tilde{g}$ -closed if and only if $\tilde{g}\text{-cl}(E) = E$.

Proposition 2.3. [14] For any two subsets A and B of (X, τ) ,

- (i) If $A \subset B$, then $\tilde{g}\text{-cl}(A) \subset \tilde{g}\text{-cl}(B)$,
- (ii) $\tilde{g}\text{-cl}(A \cap B) \subset \tilde{g}\text{-cl}(A) \cap \tilde{g}\text{-cl}(B)$.

Proposition 2.4. [4] Every $\tilde{g}$ -closed set is $g\alpha$ -closed and hence pre-closed.

Proof. Let A be $\tilde{g}$ -closed in (X, τ) and U be any α -open set containing A . Since every α -open set is $\#gs$ -open [21] and since $\alpha cl(A) \subset cl(A)$ for every subset A of X , we have by hypothesis, $\alpha cl(A) \subset cl(A) \subset U$ and so A is $g\alpha$ -closed in (X, τ) .

In [8], it has been proved that every $g\alpha$ -closed set is pre-closed. Therefore, every $\tilde{g}$ -closed set is pre-closed.

Theorem 2.5. [4] Suppose that $B \subset A \subset X$, B is a $\tilde{g}$ -closed set relative to A and that A is open and $\tilde{g}$ -closed in (X, τ) . Then B is $\tilde{g}$ -closed in (X, τ) .

Corollary 2.6. [4] If A is a $\tilde{g}$ -closed set and F is a closed set, then $A \cap F$ is a $\tilde{g}$ -closed set.

Theorem 2.7. [4] A set A is $\tilde{g}$ -open in (X, τ) if and only if $F \subset \text{int}(A)$ whenever F is $\#gs$ -closed in (X, τ) and $F \subset A$.

Definition 7. [14] Let (X, τ) be a topological space and $E \subset X$. We define the $\tilde{g}$ -interior of E (briefly $\tilde{g}\text{-int}(E)$) to be the union of all $\tilde{g}$ -open sets contained in E .

Lemma 2.8. [14] For any $E \subset X$, $\text{int}(E) \subset \tilde{g}\text{-int}(E) \subset E$.

Proof. Since every open set is $\tilde{g}$ -open, the proof follows immediately.

Definition 8. A topological space (X, τ) is a,

- (i) $T_{1/2}$ space [7] if every g -closed subset of (X, τ) is closed in (X, τ) ,
- (ii) $T_{\tilde{g}}$ space [16] if every $\tilde{g}$ -closed subset of (X, τ) is closed in (X, τ) ,
- (iii) semi- $T_{1/2}$ space [19] if every $\hat{g}$ -closed subset of (X, τ) is closed in (X, τ) .

3. $\tilde{g}$ -closed maps

Malghan [12] introduced the concept of generalized closed maps in topological spaces. In this section, we introduce $\tilde{g}$ -closed maps, $\tilde{g}$ -open maps, $\tilde{g}^*$ -closed maps, $\tilde{g}^*$ -open maps and obtain certain characterizations of these maps.

Definition 9. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $\tilde{g}$ -closed if the image of every closed set in (X, τ) is $\tilde{g}$ -closed in (Y, σ) .

Example 3.1. (a) Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{b\}\}$. Define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = b$, $f(b) = a$ and $f(c) = c$. Then f is a $\tilde{g}$ -closed map.

(b) Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is not a $\tilde{g}$ -closed map.

Proposition 3.2. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed if and only if $\tilde{g}\text{-cl}(f(A)) \subset f(\text{cl}(A))$ for every subset A of (X, τ) .

Proof. Suppose that f is $\tilde{g}$ -closed and $A \subset X$. Then $f(\text{cl}(A))$ is $\tilde{g}$ -closed in (Y, σ) . We have $f(A) \subset f(\text{cl}(A))$ and by Propositions 2.2 and 2.3, $\tilde{g}\text{-cl}(f(A)) \subset \tilde{g}\text{-cl}(f(\text{cl}(A))) = f(\text{cl}(A))$.

Conversely, let A be any closed set in (X, τ) . By hypothesis and Proposition 2.2, we have $A = \text{cl}(A)$ and so $f(A) = f(\text{cl}(A)) \supset \tilde{g}\text{-cl}(f(A))$. Therefore, $f(A) = \tilde{g}\text{-cl}(f(A))$. i.e., $f(A)$ is $\tilde{g}$ -closed and hence f is $\tilde{g}$ -closed.

Proposition 3.3. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}$ -closed mapping, then for each subset A of (X, τ) , $\text{cl}(\text{int}(f(A))) \subset f(\text{cl}(A))$.

Proof. Let f be a $\tilde{g}$ -closed map and $A \subset X$. Then since $\text{cl}(A)$ is a closed set in (X, τ) , we have $f(\text{cl}(A))$ is $\tilde{g}$ -closed and hence pre-closed by Proposition 2.4. Therefore, $\text{cl}(\text{int}(f(\text{cl}(A)))) \subset f(\text{cl}(A))$. i.e., $\text{cl}(\text{int}(f(A))) \subset$

$f(cl(A))$.

The converse of this Proposition need not be true in general as seen from the following example.

Example 3.4. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then for each subset $A \subset X$, we have $cl(int(f(A))) \subset f(cl(A))$, but f is not a $\tilde{g}$ -closed map.

Theorem 3.5. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed if and only if for each subset S of (Y, σ) and for each open set U containing $f^{-1}(S)$ there is a $\tilde{g}$ -open set V of (Y, σ) such that $S \subset V$ and $f^{-1}(V) \subset U$.

Proof. Suppose that f is a $\tilde{g}$ -closed map. Let $S \subset Y$ and U be an open subset of (X, τ) such that $f^{-1}(S) \subset U$. Then $V = (f(U^c))^c$ is a $\tilde{g}$ -open set containing S such that $f^{-1}(V) \subset U$.

For the converse, let S be a closed set of (X, τ) . Then $f^{-1}((f(S))^c) \subset S^c$ and S^c is open. By assumption, there exists a $\tilde{g}$ -open set V of (Y, σ) such that $(f(S))^c \subset V$ and $f^{-1}(V) \subset S^c$ and so $S \subset (f^{-1}(V))^c$. Hence $V^c \subset f(S) \subset f((f^{-1}(V))^c) \subset V^c$ which implies $f(S) = V^c$. Since V^c is $\tilde{g}$ -closed, $f(S)$ is $\tilde{g}$ -closed and therefore f is $\tilde{g}$ -closed.

Proposition 3.6. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\#gs$ -irresolute $\tilde{g}$ -closed and A is a $\tilde{g}$ -closed subset of (X, τ) , then $f(A)$ is $\tilde{g}$ -closed.

Proof. Let U be a $\#gs$ -open set in (Y, σ) such that $f(A) \subset U$. Since f is $\#gs$ -irresolute, $f^{-1}(U)$ is a $\#gs$ -open set containing A . Hence $cl(A) \subset f^{-1}(U)$ as A is $\tilde{g}$ -closed in (X, τ) . Since f is $\tilde{g}$ -closed, $f(cl(A))$ is a $\tilde{g}$ -closed set contained in the $\#gs$ -open set U , which implies that $cl(f(cl(A))) \subset U$ and hence $cl(f(A)) \subset U$. Therefore, $f(A)$ is a $\tilde{g}$ -closed set.

The following example shows that the composition of two $\tilde{g}$ -closed maps need not be $\tilde{g}$ -closed.

Example 3.7. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{a, c\}\}$ and $\gamma = \{\emptyset, Z, \{b\}, \{a, c\}\}$. Define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = f(b) = b$ and $f(c) = a$ and a map $g : (Y, \sigma) \rightarrow (Z, \gamma)$ by $g(a) = c$, $g(b) = b$ and $g(c) = a$. Then both f and g are $\tilde{g}$ -closed maps but their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is not a $\tilde{g}$ -closed map, since for the closed set $\{b, c\}$ in (X, τ) , $(g \circ f)(\{b, c\}) = \{a, b\}$, which is not a $\tilde{g}$ -closed set in (Z, γ) .

Corollary 3.8. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $\tilde{g}$ -closed map and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be $\tilde{g}$ -closed and $\#gs$ -irresolute map, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is $\tilde{g}$ -closed.*

Proof. Let A be a closed set of (X, τ) . Then by hypothesis $f(A)$ is a $\tilde{g}$ -closed set in (Y, σ) . Since g is both $\tilde{g}$ -closed and $\#gs$ -irresolute by Proposition 3.6, $g(f(A)) = (g \circ f)(A)$ is $\tilde{g}$ -closed in (Z, γ) and therefore $g \circ f$ is $\tilde{g}$ -closed.

Proposition 3.9. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ are $\tilde{g}$ -closed maps and (Y, σ) is a $T_{\tilde{g}}$ space, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is a $\tilde{g}$ -closed map.*

Proof. Let A be a closed set of (X, τ) . Then by assumption $f(A)$ is $\tilde{g}$ -closed in (Y, σ) . Since (Y, σ) is a $T_{\tilde{g}}$ space, $f(A)$ is closed in (Y, σ) and again by assumption $g(f(A))$ is $\tilde{g}$ -closed in (Z, γ) . i.e., $(g \circ f)(A)$ is $\tilde{g}$ -closed in (Z, γ) and so $g \circ f$ is $\tilde{g}$ -closed.

Proposition 3.10. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed, $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is g -closed and (Y, σ) is a $T_{\tilde{g}}$ space, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is g -closed.*

Proof. Similar to Proposition 3.9.

Proposition 3.11. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a closed map and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be a $\tilde{g}$ -closed map, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is $\tilde{g}$ -closed.*

Proof. Similar to Proposition 3.9.

Remark 3.12. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ is closed, then their composition need not be a $\tilde{g}$ -closed map as seen from the following example.*

Example 3.13. *Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$, $\sigma = \{\emptyset, Y, \{a, c\}\}$ and $\gamma = \{\emptyset, Z, \{b\}, \{a, c\}\}$. Define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = f(b) = b$ and $f(c) = a$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be the identity map. Then f is a $\tilde{g}$ -closed map and g is a closed map. But their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is not a $\tilde{g}$ -closed map, since for the closed set $\{b, c\}$ in (X, τ) , $(g \circ f)(\{b, c\}) = \{a, b\}$, which is not a $\tilde{g}$ -closed set in (Z, γ) .*

Theorem 3.14. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be two mappings such that their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ be a $\tilde{g}$ -closed mapping. Then the following statements are true if:

- (i) f is continuous and surjective, then g is $\tilde{g}$ -closed.
- (ii) g is $\tilde{g}$ -irresolute and injective, then f is $\tilde{g}$ -closed.
- (iii) f is g -continuous, surjective and (X, τ) is a $T_{1/2}$ space, then g is $\tilde{g}$ -closed.
- (iv) f is $\tilde{g}$ -continuous, surjective and (X, τ) is a semi- $T_{1/2}$ space, then g is $\tilde{g}$ -closed.
- (v) g is strongly $\tilde{g}$ -continuous and injective, then f is closed.

Proof. (i) Let A be a closed set of (Y, σ) . Since f is continuous, $f^{-1}(A)$ is closed in (X, τ) and since $g \circ f$ is $\tilde{g}$ -closed, $(g \circ f)(f^{-1}(A))$ is $\tilde{g}$ -closed in (Z, γ) . i.e., $g(A)$ is $\tilde{g}$ -closed in (Z, γ) , since f is surjective. Therefore, g is a $\tilde{g}$ -closed map.

(ii) Let B be a closed set of (X, τ) . Since $g \circ f$ is $\tilde{g}$ -closed, $(g \circ f)(B)$ is $\tilde{g}$ -closed in (Z, γ) . Since g is $\tilde{g}$ -irresolute, $g^{-1}((g \circ f)(B))$ is $\tilde{g}$ -closed in (Y, σ) . i.e., $f(B)$ is $\tilde{g}$ -closed in (Y, σ) , since g is injective. Thus, f is a $\tilde{g}$ -closed map.

(iii) Let A be a closed set of (Y, σ) . Since f is g -continuous, $f^{-1}(A)$ is g -closed in (X, τ) . Since (X, τ) is a $T_{1/2}$ space, $f^{-1}(A)$ is closed in (X, τ) and so as in (i), g is a $\tilde{g}$ -closed map.

(iv) Let A be a closed set of (Y, σ) . Since f is $\tilde{g}$ -continuous, $f^{-1}(A)$ is $\tilde{g}$ -closed in (X, τ) . Since (X, τ) is a semi- $T_{1/2}$ space, $f^{-1}(A)$ is closed in (X, τ) and so as in (i), g is a $\tilde{g}$ -closed map.

(v) Let D be a closed set of (X, τ) . Since $g \circ f$ is $\tilde{g}$ -closed, $(g \circ f)(D)$ is $\tilde{g}$ -closed in (Z, γ) . Since g is strongly $\tilde{g}$ -continuous, $g^{-1}((g \circ f)(D))$ is closed in (Y, σ) . i.e., $f(D)$ is closed in (Y, σ) , since g is injective. Therefore, f is a closed map.

As for the restriction f_A of a map $f : (X, \tau) \rightarrow (Y, \sigma)$ to a subset A of (X, τ) , we have the following:

Theorem 3.15. Let (X, τ) and (Y, σ) be any topological spaces. Then if:

- (i) $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed and A is a closed subset of (X, τ) , then $f_A : (A, \tau_A) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed.
- (ii) $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\#gs$ -irresolute and $\tilde{g}$ -closed and A is an open subset of (X, τ) , then $f_A : (A, \tau_A) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed.
- (iii) $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed (resp. closed) and $A = f^{-1}(B)$ for

some closed (resp. $\tilde{g}$ -closed) set B of (Y, σ) , then $f_A : (A, \tau_A) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -closed.

Proof. (i) Let B be a closed set of A . Then $B = A \cap F$ for some closed set F of (X, τ) and so B is closed in (X, τ) . By hypothesis, $f(B)$ is $\tilde{g}$ -closed in (Y, σ) . But $f(B) = f_A(B)$ and therefore f_A is a $\tilde{g}$ -closed map.

(ii) Let C be a closed set of A . Then C is g -closed in A . Since A is both open and $\tilde{g}$ -closed, C is $\tilde{g}$ -closed, by Theorem 2.5. Since f is both $\#gs$ -irresolute and $\tilde{g}$ -closed, $f(C)$ is $\tilde{g}$ -closed in (Y, σ) , by Proposition 3.6. Since $f(C) = f_A(C)$, f_A is a $\tilde{g}$ -closed map.

(iii) Let D be a closed set of A . Then $D = A \cap H$ for some closed set H of (X, τ) . Now $f_A(D) = f(D) = f(A \cap H) = f(f^1(B) \cap H) = B \cap f(H)$. Since f is $\tilde{g}$ -closed, $f(H)$ is $\tilde{g}$ -closed and so $B \cap f(H)$ is $\tilde{g}$ -closed in (Y, σ) by Corollary 2.6. Therefore, f_A is a $\tilde{g}$ -closed map.

The next theorem shows that normality is preserved under continuous $\tilde{g}$ -closed maps.

Theorem 3.16. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a continuous, $\tilde{g}$ -closed map from a normal space (X, τ) onto a space (Y, σ) then (Y, σ) is normal.*

Proof. It follows from Theorem 1.12 of [12] and the fact that every $\tilde{g}$ -closed map is g -closed.

Analogous to a $\tilde{g}$ -closed map, we define a $\tilde{g}$ -open map as follows:

Definition 10. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to a $\tilde{g}$ -open map if the image $f(A)$ is $\tilde{g}$ -open in (Y, σ) for each open set A in (X, τ) .*

Proposition 3.17. *For any bijection $f : (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:*

- (i) $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is $\tilde{g}$ -continuous,
- (ii) f is a $\tilde{g}$ -open map and
- (iii) f is a $\tilde{g}$ -closed map.

Proof. (i) $\Rightarrow$ (ii): Let U be an open set of (X, τ) . By assumption $(f^{-1})^{-1}(U) = f(U)$ is $\tilde{g}$ -open in (Y, σ) and so f is $\tilde{g}$ -open.

(ii) $\Rightarrow$ (iii): Let F be a closed set of (X, τ) . Then F^c is open in (X, τ) . By assumption, $f(F^c)$ is $\tilde{g}$ -open in (Y, σ) . i.e., $f(F^c) = (f(F))^c$ is $\tilde{g}$ -open in (Y, σ) and therefore $f(F)$ is $\tilde{g}$ -closed in (Y, σ) . Hence f is $\tilde{g}$ -closed.

(iii) $\Rightarrow$ (i): Let F be a closed set in (X, τ) . By assumption $f(F)$ is $\tilde{g}$ -closed in (Y, σ) . But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is $\tilde{g}$ -continuous on Y .

Definition 11. Let x be a point of (X, τ) and V be a subset of X . Then V is called a $\tilde{g}$ -neighbourhood of x in (X, τ) if there exists a $\tilde{g}$ -open set U of (X, τ) such that $x \in U \subset V$.

In the next two theorems, we obtain various characterizations of $\tilde{g}$ -open maps.

Theorem 3.18. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then the following statements are equivalent:

- (i) f is a $\tilde{g}$ -open mapping.
- (ii) For a subset A of (X, τ) , $f(int(A)) \subset \tilde{g}-int(f(A))$.
- (iii) For each $x \in X$ and for each neighborhood U of x in (X, τ) , there exists a $\tilde{g}$ -neighbourhood W of $f(x)$ in (Y, σ) such that $W \subset f(U)$.

Proof. (i) $\Rightarrow$ (ii): Suppose f is $\tilde{g}$ -open. Let $A \subset X$. Since $int(A)$ is open in (X, τ) , $f(int(A))$ is $\tilde{g}$ -open in (Y, σ) . Hence $f(int(A)) \subset f(A)$ and we have, $f(int(A)) \subset \tilde{g}-int(f(A))$.

(ii) $\Rightarrow$ (iii): Suppose (ii) holds. Let $x \in X$ and U be an arbitrary neighborhood of x in (X, τ) . Then there exists an open set G such that $x \in G \subset U$. By assumption, $f(G) = f(int(G)) \subset \tilde{g}-int(f(G))$. This implies $f(G) = \tilde{g}-int(f(G))$. Therefore, $f(G)$ is $\tilde{g}$ -open in (Y, σ) . Further, $f(x) \in f(G) \subset f(U)$ and so (iii) holds, by taking $W = f(G)$.

(iii) $\Rightarrow$ (i): Suppose (iii) holds. Let U be any open set in (X, τ) , $x \in U$ and $f(x) = y$. Then $x \in U$ and for each $y \in f(U)$, by assumption there exists a $\tilde{g}$ -neighbourhood W_y of y in (Y, σ) such that $W_y \subset f(U)$. Since W_y is a $\tilde{g}$ -neighbourhood of y , there exists a $\tilde{g}$ -open set V_y in (Y, σ) such that $y \in V_y \subset W_y$. Therefore, $f(U) = \cup\{V_y : y \in f(U)\}$. Since the arbitrary union of $\tilde{g}$ -open sets is $\tilde{g}$ -open, $f(U)$ is a $\tilde{g}$ -open set of (Y, σ) . Thus, f is a $\tilde{g}$ -open mapping.

Theorem 3.19. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -open if and only if for any subset B of (Y, σ) and for any closed set S containing $f^{-1}(B)$, there exists a $\tilde{g}$ -closed set A of (Y, σ) containing B such that $f^{-1}(A) \subset S$.

Proof. Similar to Theorem 3.5.

Corollary 3.20. A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}$ -open if and only if $f^{-1}(\tilde{g}-cl(B)) \subset cl(f^{-1}(B))$ for every subset B of (Y, σ) .

Proof. Suppose that f is $\tilde{g}$ -open. Then for any $B \subset Y$, $f^{-1}(B) \subset cl(f^{-1}(B))$. By Theorem 3.19, there exists a $\tilde{g}$ -closed set A of (Y, σ)

such that $B \subset A$ and $f^{-1}(A) \subset cl(f^{-1}(B))$. Therefore, $f^{-1}(\tilde{g}\text{-}cl(B)) \subset f^{-1}(A) \subset cl(f^{-1}(B))$, since A is a $\tilde{g}$ -closed set in (Y, σ) .

Conversely, let S be any subset of (Y, σ) and F be any closed set containing $f^{-1}(S)$. Put $A = \tilde{g}\text{-}cl(S)$. Then A is a $\tilde{g}$ -closed set and $S \subset A$. By assumption, $f^{-1}(A) = f^{-1}(\tilde{g}\text{-}cl(S)) \subset cl(f^{-1}(S)) \subset F$ and therefore by Theorem 3.19, f is $\tilde{g}$ -open.

Finally in this section, we define another new class of maps called $\tilde{g}^*$ -closed maps which are stronger than $\tilde{g}$ -closed maps.

Definition 12. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be a $\tilde{g}^*$ -closed map if the image $f(A)$ is $\tilde{g}$ -closed in (Y, σ) for every $\tilde{g}$ -closed set A in (X, τ) .

For instance the map f in Example 3.1 is a $\tilde{g}^*$ -closed map

Remark 3.21. Since every closed set is a $\tilde{g}$ -closed set we have every $\tilde{g}^*$ -closed map is a $\tilde{g}$ -closed map. The converse is not true in general as seen from the following example.

Example 3.22. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$ and $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is a $\tilde{g}$ -closed map but not a $\tilde{g}^*$ -closed map, since $\{a, c\}$ is a $\tilde{g}$ -closed set in (X, τ) , but its image under f is $\{a, c\}$, which is not $\tilde{g}$ -closed in (Y, σ) .

Proposition 3.23. A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\tilde{g}^*$ -closed if and only if $\tilde{g}\text{-}cl(f(A)) \subset f(\tilde{g}\text{-}cl(A))$ for every subset A of (X, τ) .

Proof. Similar to Proposition 3.2.

Analogous to $\tilde{g}^*$ -closed map we can also define $\tilde{g}^*$ -open map.

Proposition 3.24. For any bijection $f : (X, \tau) \rightarrow (Y, \sigma)$, the following are equivalent:

- (i) $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is $\tilde{g}$ -irresolute,
- (ii) f is a $\tilde{g}^*$ -open and
- (iii) f is a $\tilde{g}^*$ -closed map.

Proof. Similar to Proposition 3.17.

Proposition 3.25. If $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\#gs$ -irresolute and $\tilde{g}$ -closed, then it is a $\tilde{g}^*$ -closed map.

Proof. Follows from Proposition 3.6.

Lemma 3.26. *Let A be a subset of X . Then $p \in \tilde{g}\text{-cl}(A)$ if and only if for any $\tilde{g}$ -neighborhood N of p in X , $A \cap N \neq \emptyset$.*

Definition 13. *Let A be a subset of X . A mapping $r : X \rightarrow A$ is called a $\tilde{g}$ -continuous retraction if r is $\tilde{g}$ -continuous and the restriction r_A is the identity mapping on A .*

Definition 14. *A topological space (X, τ) is called a $\tilde{g}$ -Hausdorff if for each pair x, y of distinct points of X , there exists $\tilde{g}$ -neighborhoods U_1 and U_2 of x and y , respectively, that are disjoint.*

Theorem 3.27. *Let A be a subset of X and $r : X \rightarrow A$ be a $\tilde{g}$ -continuous retraction. If X is $\tilde{g}$ -Hausdorff, then A is a $\tilde{g}$ -closed set of X .*

Proof. Suppose that A is not $\tilde{g}$ -closed. Then there exists a point x in X such that $x \in \tilde{g}\text{-cl}(A)$ but $x \notin A$. It follows that $r(x) \neq x$ because r is $\tilde{g}$ -continuous retraction. Since X is $\tilde{g}$ -Hausdorff, there exists disjoint $\tilde{g}$ -open sets U and V in X such that $x \in U$ and $r(x) \in V$. Now let W be an arbitrary $\tilde{g}$ -neighborhood of x . Then $W \cap U$ is a $\tilde{g}$ -neighborhood of x . Since $x \in \tilde{g}\text{-cl}(A)$, by Lemma 3.26, we have $(W \cap U) \cap A \neq \emptyset$. Therefore there exists a point y in $W \cap U \cap A$. Since $y \in A$, we have $r(y) = y \in U$ and hence $r(y) \notin V$. This implies that $r(W) \not\subset V$ because $y \in W$. This is contrary to the $\tilde{g}$ -continuity of r . Consequently, A is a $\tilde{g}$ -closed set of X .

Theorem 3.28. *Let $\{X_i : i \in I\}$ be any family of topological spaces. If $f : X \rightarrow \prod X_i$ is a $\tilde{g}$ -continuous mapping, then $Pr_i \circ f : X \rightarrow X_i$ is $\tilde{g}$ -continuous for each $i \in I$, where Pr_i is the projection of $\prod X_j$ onto X_i .*

Proof. We shall consider a fixed $i \in I$. Suppose U_i is an arbitrary open set in X_i . Then $Pr_i^{-1}(U_i)$ is open in $\prod X_i$. Since f is $\tilde{g}$ -continuous, we have, $f^{-1}(Pr_i^{-1}(U_i)) = (Pr_i \circ f)^{-1}(U_i)$ $\tilde{g}$ -open in X . Therefore $Pr_i \circ f$ is $\tilde{g}$ -continuous.

4. $\tilde{g}$ -Homeomorphisms

In this section we introduce and study two new homeomorphisms namely $\tilde{g}$ -homeomorphism and $\tilde{g}^*$ -homeomorphism. We prove that gc -homeomorphism and $\tilde{g}$ -homeomorphism are independent and $\tilde{g}^*$ -homeomorphism is an equivalence relation between topological spaces.

Definition 15. A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is called $\tilde{g}$ -homeomorphism if f is both $\tilde{g}$ -continuous and $\tilde{g}$ -open.

Proposition 4.1. Every homeomorphism is a $\tilde{g}$ -homeomorphism but not conversely.

Proof. It follows from definitions.

The converse of the above Proposition 4.1 need not be true as seen from the following example.

Example 4.2. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{a, b\}\}$. The identity map f on X is $\tilde{g}$ -homeomorphism but not a homeomorphism, because it is not continuous.

Thus, the class of $\tilde{g}$ -homeomorphisms properly contains the class of homeomorphisms. Next we show that the class of g -homeomorphisms properly contains the class of $\tilde{g}$ -homeomorphisms.

Proposition 4.3. Every $\tilde{g}$ -homeomorphism is a g -homeomorphism (resp. $\hat{g}$ -homeomorphism) but not conversely.

Proof. Since every $\tilde{g}$ -continuous map is g -continuous (resp. $\hat{g}$ -continuous) and every $\tilde{g}$ -open map is g -open (resp. $\hat{g}$ -open), the proof follows.

The converses of the above Proposition 4.3 need not be true as seen from the following examples.

Example 4.4. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}\}$ and $\sigma = \{\emptyset, Y, \{b\}\}$. Define a map $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = a$ and $f(c) = b$. Then f is a g -homeomorphism. However, f is not a $\tilde{g}$ -homeomorphism.

Example 4.5. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b, c\}\}$ and $\sigma = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is $\hat{g}$ -homeomorphism. However, f is not a $\tilde{g}$ -homeomorphism.

Proposition 4.6. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijection $\hat{g}$ -continuous map. Then the following statements are equivalent:

- (i) f is a $\tilde{g}$ -open map.
- (ii) f is a $\tilde{g}$ -homeomorphism.
- (iii) f is a $\tilde{g}$ -closed map.

Proof. It follows from Proposition 3.17.

The composition of two $\tilde{g}$ -homeomorphism maps need not be a $\tilde{g}$ -homeomorphism as can be seen from the following example.

Example 4.7. Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{\emptyset, Y, \{a, b\}\}$ and $\gamma = \{\emptyset, Z, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ be identity maps. Then both f and g are $\tilde{g}$ -homeomorphisms but their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is not a $\tilde{g}$ -homeomorphism, because for the open set $\{b\}$ in (X, τ) , $(g \circ f)(\{b\}) = \{b\}$, which is not a $\tilde{g}$ -open set in (Z, γ) . Therefore $g \circ f$ is not a $\tilde{g}$ -open map and so $g \circ f$ is not a $\tilde{g}$ -homeomorphism.

We next introduce a new class of maps called $\tilde{g}^*$ -homeomorphisms which forms a sub class of $\tilde{g}$ -homeomorphisms. This class of maps is closed under composition of maps.

Definition 16. A bijection $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $\tilde{g}^*$ -homeomorphism if both f and f^{-1} are $\tilde{g}$ -irresolute.

We denote the family of all $\tilde{g}$ -homeomorphisms (resp. $\tilde{g}^*$ -homeomorphism and homeomorphisms) of a topological space (X, τ) onto itself by $\tilde{g}\text{-}h(X, \tau)$ (resp. $\tilde{g}^*\text{-}h(X, \tau)$ and $h(X, \tau)$).

Proposition 4.8. Every $\tilde{g}^*$ -homeomorphism is a $\tilde{g}$ -homeomorphism but not conversely. i.e., for any space (X, τ) , $\tilde{g}^*\text{-}h(X, \tau) \subset \tilde{g}\text{-}h(X, \tau)$.

Proof. It follows from Proposition 2.1 and the fact that every $\tilde{g}^*$ -open map is $\tilde{g}$ -open.

The function g in Example 4.7 is a $\tilde{g}$ -homeomorphism but not a $\tilde{g}^*$ -homeomorphism, since for the $\tilde{g}$ -closed set $\{a, c\}$ in (Y, σ) , $(g^{-1})^{-1}(\{a, c\}) = g(\{a, c\}) = \{a, c\}$ which is not $\tilde{g}$ -closed in (Z, γ) . Therefore, g^{-1} is not $\tilde{g}$ -irresolute and so g is not a $\tilde{g}^*$ -homeomorphism.

Proposition 4.9. Every $\tilde{g}^*$ -homeomorphism is a g -homeomorphism but not conversely.

Proof. Follows from Propositions 4.8 and 4.3.

The map f in Example 4.4 is a g -homeomorphism but not a $\tilde{g}^*$ -homeomorphism.

Remark 4.10. $\tilde{g}$ -homeomorphism and gc -homeomorphism are independent as can be seen from the following examples.

Example 4.11. Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{a, b\}\}$ and $\sigma = \{\emptyset, Y, \{b\}, \{b, c\}\}$. Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is a gc -homeomorphism but not $\tilde{g}$ -homeomorphism, because f is not a $\tilde{g}$ -continuous map.

The map f in Example 4.4 is a $\tilde{g}$ -homeomorphism but not a gc -homeomorphism, because f^{-1} is not a gc -irresolute map.

Proposition 4.12. If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \gamma)$ are $\tilde{g}^*$ -homeomorphisms, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \gamma)$ is also $\tilde{g}^*$ -homeomorphism.

Proof. Let U be a $\tilde{g}$ -open set in (Z, γ) . Now, $(g \circ f)^{-1}(U) = f^{-1}(g^{-1}(U)) = f^{-1}(V)$, where $V = g^{-1}(U)$. By hypothesis, V is $\tilde{g}$ -open in (Y, σ) and so again by hypothesis, $f^{-1}(V)$ is $\tilde{g}$ -open in (X, τ) . Therefore, $g \circ f$ is $\tilde{g}$ -irresolute. Also for a $\tilde{g}$ -open set G in (X, τ) , we have $(g \circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis $f(G)$ is $\tilde{g}$ -open in (Y, σ) and so again by hypothesis, $g(f(G))$ is $\tilde{g}$ -open in (Z, γ) . i.e., $(g \circ f)(G)$ is $\tilde{g}$ -open in (Z, γ) and therefore $(g \circ f)^{-1}$ is $\tilde{g}$ -irresolute. Hence $g \circ f$ is a $\tilde{g}^*$ -homeomorphism.

Theorem 4.13. The set $\tilde{g}^*-h(X, \tau)$ is a group under the composition of maps.

Proof. Define a binary operation $*$: $\tilde{g}^*-h(X, \tau) \times \tilde{g}^*-h(X, \tau) \rightarrow \tilde{g}^*-h(X, \tau)$ by $f * g = g \circ f$ for all $f, g \in \tilde{g}^*-h(X, \tau)$ and $\circ$ is the usual operation of composition of maps. Then by Proposition 4.12, $g \circ f \in \tilde{g}^*-h(X, \tau)$. We know that the composition of maps is associative and the identity map $I : (X, \tau) \rightarrow (X, \tau)$ belonging to $\tilde{g}^*-h(X, \tau)$ serves as the identity element. If $f \in \tilde{g}^*-h(X, \tau)$, then $f^{-1} \in \tilde{g}^*-h(X, \tau)$ such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of $\tilde{g}^*-h(X, \tau)$. Therefore, $(\tilde{g}^*-h(X, \tau), \circ)$ is a group under the operation of composition of maps.

Theorem 4.14. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a $(\tilde{g}^*$ -homeomorphism. Then f induces an isomorphism from the group $\tilde{g}^*-h(X, \tau)$ onto the group $\tilde{g}^*-h(Y, \sigma)$.

Proof. Using the map f , we define a map $\theta_f : \tilde{g}^*-h(X, \tau) \rightarrow \tilde{g}^*-h(Y, \sigma)$ by $\theta_f(h) = f \circ h \circ f^{-1}$ for every $h \in \tilde{g}^*-h(X, \tau)$. Then θ_f is a bijection. Further, for all $h_1, h_2 \in \tilde{g}^*-h(X, \tau)$, $\theta_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = \theta_f(h_1) \circ \theta_f(h_2)$. Therefore, θ_f is a homeomorphism and so it is an isomorphism induced by f .

Theorem 4.15. *$\tilde{g}^*$ -homeomorphism is an equivalence relation in the collection of all topological spaces.*

Proof. Reflexivity and symmetry are immediate and transitivity follows from Proposition 4.12.

Theorem 4.16. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}^*$ -homeomorphism, then $\tilde{g}\text{-cl}(f^{-1}(B)) = f^{-1}(\tilde{g}\text{-cl}(B))$ for all $B \subset Y$.*

Proof. Since f is a $\tilde{g}^*$ -homeomorphism, f is $\tilde{g}$ -irresolute. Since $\tilde{g}\text{-cl}(f(B))$ is a $\tilde{g}$ -closed set in (Y, σ) , $f^{-1}(\tilde{g}\text{-cl}(f(B)))$ is $\tilde{g}$ -closed in (X, τ) . Now, $f^{-1}(B) \subset f^{-1}(\tilde{g}\text{-cl}(B))$ and so by Proposition 2.3, $\tilde{g}\text{-cl}(f^{-1}(B)) \subset f^{-1}(\tilde{g}\text{-cl}(B))$. Again since f is a $\tilde{g}^*$ -homeomorphism, f^{-1} is $\tilde{g}$ -irresolute. Since $\tilde{g}\text{-cl}(f^{-1}(B))$ is $\tilde{g}$ -closed in (X, τ) , $(f^{-1})^{-1}(\tilde{g}\text{-cl}(f^{-1}(B))) = f(\tilde{g}\text{-cl}(f^{-1}(B)))$ is $\tilde{g}$ -closed in (Y, σ) . Now, $B \subset (f^{-1})^{-1}(\tilde{g}\text{-cl}(f^{-1}(B))) \subset (f^{-1})^{-1}(\tilde{g}\text{-cl}(f^{-1}(B))) = f(\tilde{g}\text{-cl}(f^{-1}(B)))$ and so $\tilde{g}\text{-cl}(B) \subset f(\tilde{g}\text{-cl}(f^{-1}(B)))$. Therefore, $f^{-1}(\tilde{g}\text{-cl}(B)) \subset f^{-1}(f(\tilde{g}\text{-cl}(f^{-1}(B)))) \subset \tilde{g}\text{-cl}(f^{-1}(B))$ and hence the equality holds.

Corollary 4.17. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}^*$ -homeomorphism, then $\tilde{g}\text{-cl}(f(B)) = f(\tilde{g}\text{-cl}(B))$ for all $B \subset X$.*

Proof. Since $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}^*$ -homeomorphism, $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is also a $\tilde{g}^*$ -homeomorphism. Therefore, by Theorem 4.16, $\tilde{g}\text{-cl}((f^{-1})^{-1}(B)) = (f^{-1})^{-1}(\tilde{g}\text{-cl}(B))$ for all $B \subset X$. i.e., $\tilde{g}\text{-cl}(f(B)) = f(\tilde{g}\text{-cl}(B))$.

Corollary 4.18. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}^*$ -homeomorphism, then $f(\tilde{g}\text{-int}(B)) = \tilde{g}\text{-int}(f(B))$ for all $B \subset X$.*

Proof. For any set $B \subset X$, $\tilde{g}\text{-int}(B) = (\tilde{g}\text{-cl}(B^c))^c$. Thus, by utilizing Corollary 4.17, we obtain $f(\tilde{g}\text{-int}(B)) = f((\tilde{g}\text{-cl}(B^c))^c) = (f(\tilde{g}\text{-cl}(B^c)))^c = (\tilde{g}\text{-cl}(f(B^c)))^c = (\tilde{g}\text{-cl}((f(B))^c))^c = \tilde{g}\text{-int}(f(B))$.

Corollary 4.19. *If $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}^*$ -homeomorphism, then $f^{-1}(\tilde{g}\text{-int}(B)) = \tilde{g}\text{-int}(f^{-1}(B))$ for all $B \subset Y$.*

Proof. Since $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is also a $\tilde{g}^*$ -homeomorphism, the proof follows from Corollary 4.18.

Theorem 4.20. *Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is a $\tilde{g}$ -continuous function and $G_g(f) = \{(x, y) \in X \times Y : y = f(x)\}$, where $X \times Y$ is the product topology and $G_g(f)$ is called the $\tilde{g}$ -graph of f . Then the following properties are satisfied:*

- (1) $G_g(f)$, as a subspace of $X \times Y$, is $\tilde{g}$ -homeomorphic to X .
- (2) If Y is $\tilde{g}$ -Hausdorff space, then $G_g(f)$ is $\tilde{g}$ -closed in $X \times Y$.

Proof. (1). The function $g : X \rightarrow G_g(f)$ is defined by $g(x) = (x, f(x))$ for each $x \in X$ is $\tilde{g}$ -continuous and g^{-1} is also $\tilde{g}$ -continuous. It is obvious that g is an injective function. Suppose D (resp. E) is an arbitrary $\tilde{g}$ -neighborhood of $x \in X$ (resp. $g(x) = (x, f(x))$ in $G_g(f)$). So there exist $\tilde{g}$ -open sets U and V in X and Y containing x and $f(x)$ respectively for which $(U \times V) \cap G_g(f) \subset E$ and $U \subset D$ and $f(U) \subset V$. Let $M = (U \times V) \cap G_g(f)$. Then $(x, f(x)) \in M$ and $x \in g^{-1}(M) \subset U \subset D$. It implies that g^{-1} is $\tilde{g}$ -continuous. Moreover, $g(U) \subset U \times f(U) \subset U \times V_1$ and $g(U) \subset G_g(f)$. Therefore, $g(U) \subset (U \times V) \cap G_g(f) \subset E$. Hence g is $\tilde{g}$ -continuous which means that g is a $\tilde{g}$ -homeomorphism.

(2). Suppose that $(x, y) \notin G_g(f)$. Then $y_1 = f(x) \neq y$. By hypothesis, there exist disjoint $\tilde{g}$ -open sets V_1 and V in Y such that $y_1 \in V_1$, $y \in V$. Since f is $\tilde{g}$ -continuous, there exists an open set U in X containing x such that $f(U) \subset V_1$. Then $g(U) \subset U \times V_1$. It follows from this and the fact that $V_1 \cap V = \emptyset$ that $(U \times V) \cap G_g(f) = \emptyset$.

References

- [1] Crossley S.G and Hildebrand S. K, Semi-closure, Texas J. Sci. 22, pp. 99-112, (1971).
- [2] Devi R, Balachandran K and Maki H, Semi-generalized homeomorphisms and generalized semi-homeomorphisms in topological spaces, Indian J. Pure Appl. Math., 26, pp. 271-284, (1995).
- [3] Devi R, Maki H and Balachandran K, Semi-generalized closed maps and generalized semi-closed maps, Mem. Fac. Kochi Univ. Ser. A. Math., 14, pp. 41-54, (1993).

- [4] Jafari S, Noiri T, Rajesh N and Thivagar M. L, Another generalization of closed sets. (Submitted).
- [5] Jänich K, Topologie, Springer-Verlag, Berlin, 1980 (English translation).
- [6] Levine N, Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, 70, pp. 36-41, (1963).
- [7] Levine N, Generalized closed sets in topology, Rend. Circ. Math. Palermo, (2) 19, pp. 89-96; (1970).
- [8] Maki H, Devi R and Balachandran K, Generalized α -closed sets in topology, Bull. Fukuoka Univ. Ed Part III, 42, pp. 13-21, (1993).
- [9] Maki H, Sundram P and Balachandran K, On generalized homeomorphisms in topological spaces, Bull. Fukuoka Univ. Ed. Part III, 40, pp. 13-21, (1991).
- [10] Mashhour A. S, Abd. El-Monsef M. E and El-Deeb S. N, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt, 53, pp. 47-53, (1982).
- [11] Njåstad O, On some classes of nearly open sets, Pacific J. Math. 15, pp. 961-970, (1965).
- [12] Malghan S. R, Generalized closed maps, J. Karnataka Univ. Sci., 27, pp. 82-88, (1982).
- [13] Rajesh N and Ekici E, On a new form of irresoluteness and weak forms of strong continuity (submitted).
- [14] Rajesh N and Ekici E, On $\tilde{g}$ -locally closed sets in topological spaces. Kochi. J of Math. Vol.2 (to appear), (2007).
- [15] Rajesh N and Ekici E, On $\tilde{g}$ -continuous mappings in topological spaces. Proc. Inst. Math. (God. Zb. Inst. Mat.), Skopje (to appear).
- [16] Rajesh N and Ekici E, On a decomposition of $T_{1/2}$ -spaces, Math. Maced. (to appear).
- [17] Stone M, Application of the theory of Boolean rings to general topology, Trans. Amer. Math. Soc. 41, pp. 374-481, (1937).

- [18] Sundaram P, Studies on generalizations of continuous maps in topological spaces, Ph.D. Thesis, Bharathiar University, Coimbatore (1991).
- [19] Veera Kumar M. K. R. S, $\hat{g}$ -closed sets in topological spaces, Bull. Allahabad Math. Soc., 18, pp. 99-112, (2003).
- [20] Veera Kumar M. K. R. S, Between g^* -closed sets and g -closed sets, Antarctica J. Math, Reprint.
- [21] Veera Kumar M. K. R. S, $\#g$ -semi-closed sets in topological spaces, Antarctica J. Math, 2(2), pp. 201-222, (2005).

M. Caldas

Departamento de Matemática Aplicada,
Universidade Federal Fluminense,
Rua Mário Santos Braga, s/n^o
24020-140, Niterói,
RJ BRAZIL
e-mail: gmamccs@vm.uff.br

S. Jafari

College of Vestsjaelland South,
Herrestraede 11
4200 Slagelse, DENMARK
e-mail: jafari@stofanet.dk

N. Rajesh

Department of Mathematics,
Ponnaiyah Ramajayam College,
Thanjavur, TamilNadu,
INDIA
e-mail: nrajesh.topology@yahoo.co.in

and

M. L. Thivagar

Department of Mathematics,
Arul Anandhar College,
Karumathur, Madurai,
TamilNadu,
INDIA
e-mail: mlthivagar@yahoo.co.in