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INCLUSION RELATIONS FOR K-UNIFORMLY STARLIKE FUNCTIONS AND SOME LINEAR OPERATOR

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Abstract

In this paper, we have established the inclusion relations for k-uniformly starlike functions under the $L_q^s(\alpha_1)f(z)$ operator. These results are also extended to k- uniformly convex functions, close to convex and quasi-convex functions.

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1. Introduction

Let A denote the class of functions that are analytic in the open unit disk $U = \{z \in C : |z| < 1\}$

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \quad (n \in N). \tag{1.1}$$

A function $f \in A$ is said to be in $UST(k, \gamma)$, the class of k-uniformly starlike functions of order γ, $\quad 0 \le \gamma < 1$, if f satisfies the condition

$$\Re\left(\frac{zf'(z)}{f(z)}\right) > k\left|\frac{zf'(z)}{f(z)} - 1\right| + \gamma, \quad (k \ge 0). \tag{1.2}$$

A function $f \in A$ is said to be in $UCV(k, \gamma)$, the class of k-uniformly convex functions of order γ, if f satisfies the condition

$$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > k\left|\frac{zf''(z)}{f'(z)}\right| + \gamma, \quad (k \ge 0). \tag{1.3}$$

Uniformly starlike and convex functions were first introduced by Goodman[7] and then studied by various authors. For a wealth of references, see R$\emptyset$nning [15].

Setting

$$\Omega_{k,\gamma} = \{u + \iota v; u > k\sqrt{(u-1)^2 + v^2} + \gamma\}, \tag{1.4}$$

with $p(z) = \frac{zf'(z)}{f(z)}$ or $1 + \frac{zf''(z)}{f'(z)}$ and the considering the functions which map U onto the conic domain $\Omega_{k,\gamma}$, such that $1 \in \Omega_{k,\gamma}$, we may rewrite the conditions (1.2) or (1.3) in the form

$$p(z) \prec q_{k,\gamma}(z). \tag{1.5}$$

We note that the explicit forms of function $q_{k,\gamma}$ for $k = 0$ and $k = 1$ are

$$q_{0,\gamma}(z) = \frac{1 + (1 - 2\gamma)z}{1 - z}, and\; q_{1,\gamma}(z) = 1 + \frac{2(1-\gamma)}{\pi^2}\left(log\frac{1 + \sqrt{z}}{1 - \sqrt{z}}\right)^2.$$

For $0 < k < 1$, we obtain

$$q_{k,\gamma}(z) = \frac{1-\gamma}{1-k^2} cos\{\frac{2}{\pi}(arccos k)\iota log\frac{1 + \sqrt{z}}{1 - \sqrt{z}}\} - \frac{k^2 - \gamma}{1 - k^2},$$

and if $k > 1$, then $q_{k,\gamma}$ has the form

$$q_{k,\gamma}(z) = \frac{1-\gamma}{k^2-1} sin\{\frac{\pi}{2K(k)} \int_0^{\frac{u(z)}{\sqrt{k}}} \frac{dt}{\sqrt{1-t^2\sqrt{1-k^2t^2}}}\} + \frac{k^2-\gamma}{k^2-1},$$

where $u(z) = \frac{z-\sqrt{k}}{1-\sqrt{k}z}$ and K is such that $k = cosh\frac{\pi K'(z)}{4K(z)}$.

By virtue of (1.5) and the properties of the domains $\Omega_{k,\gamma}$ we have

$$\Re(p(z)) > \Re(q_{k,\gamma}(z)) > \frac{k+\gamma}{k+1}. \tag{1.6}$$

We define $UCC(k, \gamma, \beta)$ to be the family of functions $f \in A$ such that

$$\Re\left(\frac{zf'(z)}{g(z)}\right) > k\left|\frac{zf'(z)}{g(z)} - 1\right| + \gamma, \quad (k \geq 0, \quad 0 \leq \gamma < 1),$$

for some $g \in UST(k, \beta)$.

Similarly, we define $UQC(k, \gamma, \beta)$ to be the family of functions $f \in A$ such that

$$\Re\left(\frac{(zf'(z))'}{g'(z)}\right) > k\left|\frac{(zf'(z))'}{g'(z)} - 1\right| + \gamma, \quad (k \geq 0, \quad 0 \leq \gamma < 1),$$

for some $g \in UCV(k, \beta)$.

We note that $UCC(0, \gamma, \beta)$ is the class of close to convex functions of order γ and type β and $UQC(0, \gamma, \beta)$is the class of quasi convex functions of order γ and type β.

The main object of this paper is to study the inclusion properties of the above mentioned classes under the following linear operator which is defined by Dziok [4].

The Fox-Wright psi function is defined by [5,p.50]

$$\cdot_q \psi_s^* \left[\begin{array}{ll} (\alpha_i, A_i)_{1,q} & \\ (\beta_i, \ B_i)_{1,s} & z \end{array} \right] = \cdot_q\psi_s \left[\begin{array}{ll} (\alpha_1, A_1), ..., (\alpha_q, A_q); & \\ (\beta_1, B_1), ..., (\beta_s, B_s); & z \end{array} \right] \tag{1.7}$$

$$= \sum_{n=0}^{\infty} \left(\prod_{i=1}^{q} \Gamma(\alpha_i + A_i n)\right) \left(\prod_{i=1}^{s} \Gamma(\beta_i + B_i n)\right)^{-1} \frac{z^n}{n!},$$

where $\alpha_i \in C(i = 1, ..., q)$, $\beta_i \in C(i = 1, ..., s)$ and the coefficients $A_i \in R_+(i = 1, ..., q)$ and $B_i \in R_+(i = 1, ..., s)$ such that

$$1 + \sum_{i=1}^{s} B_i - \sum_{i=1}^{q} A_i \geq 0, \quad (q, s \in N_0 = N \cup \{0\})$$

The normalized Fox-Wright psi function $._q\psi_s^*(z)$ in series form is represented as

$$._q\,\psi_s^* \left[\begin{array}{ll} (\alpha_i, A_i)_{1,q} & \\ (\beta_i,\ B_i)_{1,s} & z \end{array} \right] = \frac{\Gamma\beta_1...\Gamma\beta_s}{\Gamma\alpha_1...\Gamma\alpha_q}\, ._q\,\psi_s \left[\begin{array}{ll} (\alpha_1, A_1)\,, ..., (\alpha_q, A_q)\,; & \\ (\beta_1, B_1)\,, ..., (\beta_s, B_s)\,; & z \end{array} \right]$$

(1.8)

The $._q\psi_s(z)$ is a special case of Fox's H-function $H_{k,l}^{m,n}(z)$ (see e.g.[5,p.50]) and $._q\psi_s^*(z)$ is a generalization of the familiar generalized hypergeometric function $._qF_s(z)$.

$$_qF_s \left[\begin{array}{ll} (\alpha_i)_{1,q} & \\ (\beta_i)_{1,s} & z \end{array} \right] =_q F_s \left[\begin{array}{ll} (\alpha_1)\,, ..., (\alpha_q)\,; & \\ (\beta_1)\,, ..., (\beta_s)\,; & z \end{array} \right]$$
$$= \sum_{n=0}^{\infty} \frac{(\alpha_1)_n, ..., (\alpha_q)_n}{(\beta_1)_n, ..., (\beta_s)_n} \frac{z^n}{n!},$$

where $(\alpha)_n$ is the Pochhammer symbol, defined in terms of the gamma function by

$$(\alpha)_n = \frac{\Gamma(\alpha + n)}{\Gamma(\alpha)}$$

Corresponding to a function $H_{q,s}(\alpha_1, ..., \alpha_q; A_1, ..., A_q; \beta_1, ..., \beta_s; B_1, ..., B_s; z)$ is defined by

$$H_{q,s}(\alpha_1, ..., \alpha_q; A_1, ..., A_q; \beta_1, ..., \beta_s; B_1, ..., B_s; z) = z ._q\psi_s^*(z)$$

We consider a linear operator

$$Ł_q^s(\alpha_1, ..., \alpha_q; A_1, ..., A_q; \beta_1, ..., \beta_s; B_1, ..., B_s)$$

defined by the convolution

$$Ł_q^s(\alpha_1, ..., \alpha_q; A_1, ..., A_q; \beta_1, ..., \beta_s; B_1, ..., B_s) f(z)$$

$$= H_{q,s}(\alpha_1, ..., \alpha_q; A_1, ..., A_q; \beta_1, ..., \beta_s; B_1, ..., B_s; z) * f(z)$$

For convenience, we write

$$Ł_q^s(\alpha_i) = Ł_q^s(\alpha_1, ..., \alpha_q; A_1, ..., A_q; \beta_1, ..., \beta_s; B_1, ..., B_s) \quad (i = 1, ..., q)$$

Thus, after some calculations, we get

$$z(A_i Ł_q^s(\alpha_i) f(z))' = \alpha_i Ł_q^s(\alpha_i + 1) f(z) - (\alpha_i - A_i) Ł_q^s(\alpha_i) f(z) \quad (i = 1, ..., q)$$

(1.9)

It should be noted that the linear operator $Ł_q^s(\alpha_i)$ $(i = 1, ..., q)$ is a generalization of many operators considered earlier. For a special case of this operator Carlson and Shaffer studied this operator under certain restrictions on parameters[2]. A more general operator was studied by Ponnusamy and R∅nning [24].Also note that special cases of this operator include the Hohlov operator[8], the Ruscheweyh derivative operator [16], the generalized Bernardi-Libera-Livingston linear operator (c.f.[1]) and the Srivastava -Owa fractional derivative operator (c. f. [13], [14]). Many subclasses of analytic functions associated with the operator $Ł_q^s(\alpha_i)$ $(i = 1, ..., q)$ and its many particular cases were investigated recently by Dziok and Srivastava [3,22,23], Liu and Srivastava [10,11] Gangadharan [18], Liu [9] and others (see also [14,19,20,21,25]).

We shall need the following Lemmas in the sequel to prove our theorems:

This lemma is given by Eenigenburg,Miller, Mocanu and Reade [6].

Lemma 1.1. Let β, γ be the complex constants and h be univalently in the unit disk U with $h(0) = c$ and $\Re(\beta h(z) + \gamma) > 0$. Let $g(z) = c + \sum_{n=1}^{\infty} p_n z^n$ be analytic in U. Then

$$g(z) + \frac{zg'(z)}{\beta g(z) + \gamma} \prec h(z) \Rightarrow g(z) \prec h(z).$$

This lemma is given by Miller and Mocanu[17].

Lemma 1.2. Let h be the convex in the unit disk U and let $E \geq 0$. Suppose $B(z)$ is analytic in U with $\Re(B(z)) \geq E$. If g is analytic in U and $g(0) = h(0)$. Then

$$Ez^2 g''(z) + B(z) z g'(z) + g(z) \prec h(z) \Rightarrow g(z) \prec h(z).$$

2. Main Results

Theorem 2.1. Let $\Re(\alpha_1) > A_1\left(\frac{1-\gamma}{k+1}\right)$ and $f \in A$. If $Ł^s_q(\alpha_1+1)f \in UST(k,\gamma)$ then $Ł^s_q(\alpha_1)f \in UST(k,\gamma)$.

Proof. Setting

$$p(z) = \frac{z(A_1 Ł^s_q(\alpha_1)f(z))'}{Ł^s_q(\alpha_1)f(z)}$$

in (1.9), we can write

$$\alpha_1 \frac{Ł^s_q(\alpha_1+1)f(z)}{Ł^s_q(\alpha_1)f(z)} = \frac{z(A_1 Ł^s_q(\alpha_1)f(z))'}{Ł^s_q(\alpha_1)f(z)} + (\alpha_1 - A_1) = p(z) + (\alpha_1 - A_1)$$

(2.1)

Differentiating (2.1) yields

$$\frac{z(A_1 Ł^s_q(\alpha_1+1)f(z))'}{Ł^s_q(\alpha_1+1)f(z)} = p(z) + \frac{zA_1 p'(z)}{p(z)+(\alpha_1 - A_1)} \tag{2.2}$$

From this and argument given in section 1 we may write

$$p(z) + \frac{zA_1 p'(z)}{p(z)+(\alpha_1 - A_1)} \prec q_{k,\gamma}(z)$$

Therefore the theorem follows by Lemma(1.1) and the condition (1.6) since $q_{k,\gamma}$ is univalent and convex in U and $\Re(q_{k,\gamma}) > \left(\frac{k+\gamma}{k+1}\right)$
which proves the required result.

Theorem 2.2. Let $\Re(\alpha_1) > A_1\left(\frac{1-\gamma}{k+1}\right)$ and $f \in A$. If $Ł^s_q(\alpha_1+1)f \in UCV(k,\gamma)$ then $Ł^s_q(\alpha_1)f \in UCV(k,\gamma)$.

Proof. By vitue of (1.2), (1.3) and Theorem 2.1, we have

$$Ł^s_q(\alpha_1+1)f \in UCV(k,\gamma) \Leftrightarrow z(Ł^s_q(\alpha_1+1)f)' \in UST(k,\gamma)$$

$$\Leftrightarrow Ł^s_q(\alpha_1+1)zf' \in UST(k,\gamma)$$

$$\Rightarrow Ł^s_q(\alpha_1)zf' \in UST(k,\gamma)$$

$$\Leftrightarrow Ł_q^s(\alpha_1)f \in UCV(k,\gamma)$$

and the proof is complete.

Theorem 2.3. Let $\Re(\alpha_1) > A_1\left(\frac{1-\gamma}{k+1}\right)$ and $f \in A$. If $Ł_q^s(\alpha_1+1)f \in UCC(k,\gamma,\beta)$ then $Ł_q^s(\alpha_1)f \in UCC(k,\gamma,\beta)$.

Proof. Since $Ł_q^s(\alpha_1+1)f \in UCC(k,\gamma,\beta)$, by definition, we can write

$$\frac{z(A_1Ł_q^s(\alpha_1+1)f)^{'}(z)}{k(z)} \prec q_{k,\gamma}(z)$$

for some $k(z) \in UST(k,\beta)$. For g such that $Ł_q^s(\alpha_1+1)g(z) = k(z)$, we have

$$\frac{z(A_1Ł_q^s(\alpha_1+1)f)^{'}(z)}{Ł_q^s(\alpha_1+1)g(z)} \prec q_{k,\gamma}(z). \tag{2.3}$$

Let $h(z) = \frac{z(A_1Ł_q^s(\alpha_1)f)^{'}(z)}{(Ł_q^s(\alpha_1)g(z))}$ and $H(z) = \frac{z(Ł_q^s(\alpha_1)g)^{'}(z)}{Ł_q^s(\alpha_1)g(z)}$
we observe that h and H are analytic in U and $h(0) = H(0) = 1$. Now by Theorem 2.1, $Ł_q^s(\alpha_1)g \in UST(k,\beta)$ and so $\Re H(z) > \frac{k+\beta}{k+1}$. Also, note that

$$z(A_1Ł_q^s(\alpha_1)f)^{'}(z) = (Ł_q^s(\alpha_1)g(z))h(z). \tag{2.4}$$

Differentiating both sides of (2.4)yields

$$\frac{z(A_1Ł_q^s(\alpha_1)(zf^{'}))^{'}(z)}{Ł_q^s(\alpha_1)g(z)} = \frac{z(Ł_q^s(\alpha_1)g)^{'}(z)}{Ł_q^s(\alpha_1)g(z)}h(z) + zh^{'}(z) = H(z)h(z) + zh^{'}(z).$$

Now using identity (1.9), we obtain

$$\frac{z(A_1Ł_q^s(\alpha_1+1)f)^{'}(z)}{Ł_q^s(\alpha_1+1)g(z)} = \frac{A_1(Ł_q^s(\alpha_1+1)(zf^{'})(z))}{Ł_q^s(\alpha_1+1)g(z)}$$

$$= \frac{A_1[z(A_1Ł_q^s(\alpha_1)(zf^{'}))^{'}(z) + (\alpha_1 - A_1)Ł_q^s(\alpha_1)(zf^{'})(z)]}{z(A_1Ł_q^s(\alpha_1)g)^{\prime}(z) + (\alpha_1 - A_1)Ł_q^s(\alpha_1)g(z)}.$$

$$= \frac{A_1[H(z)h(z) + zh^{'}(z) + \frac{(\alpha_1-A_1)}{A_1}h(z)]}{A_1H(z) + (\alpha_1 - A_1)} = h(z) + \frac{zh^{'}(z)}{A_1H(z) + (\alpha_1 - A_1)}$$

(2.5)

From (2.3),(2.4) and (2.5) we conclude that

$$h(z) + \frac{zh'(z)}{A_1H(z) + (\alpha_1 - A_1)} \prec q_{k,\gamma}(z).$$

Let $E = 0$ and $B(z) = \frac{1}{A_1H(z)+(\alpha_1-A_1)}$, we obtain

$$\Re(B(z)) = \frac{1}{|A_1H(z) + (\alpha_1 - A_1)|^2}\Re(A_1H(z) + (\alpha_1 - A_1)) > 0.$$

The above inequality satisfies the conditions required by Lemma (1.2). Hence $h(z) \prec q_{k,\gamma}(z)$ and so the proof is complete.

Using a similar argument to that in Theorem 2.2 we can prove the following Theorem.

Theorem 2.4. Let $\Re(\alpha_1) > A_1\left(\frac{1-\gamma}{k+1}\right)$ and $f \in A$. If $Ł_q^s(\alpha_1 + 1)f \in UQC(k, \gamma, \beta)$ then $Ł_q^s(\alpha_1)f \in UQC(k, \gamma, \beta)$.

Finally, we examine the closure property of the above classes of functions under the generalized Bernardi-Libera-Livingston operator $L_a(f)$ which is defined by

$$L_a(f) = \frac{a+1}{z^a}\int_0^z t^{a-1}f(t)dt, \qquad (a > -1).$$

Theorem 2.5. Let $a > \left(\frac{-(k+\gamma)}{k+1}\right)$. If $Ł_q^s(\alpha_1)f \in UST(k, \gamma)$ so is $L_a(Ł_q^s(\alpha_1)f)$.

Proof. From definition of $L_a(f)$ and the linearity of the operator $Ł_q^s(\alpha_1)$ we have

$$z(Ł_q^s(\alpha_1)L_a(f))'(z) = (a + 1)Ł_q^s(\alpha_1)f(z) - a(Ł_q^s(\alpha_1)L_a(f))(z). \tag{2.6}$$

Substituting $\frac{z(Ł_q^s(\alpha_1)L_a(f))'(z)}{(Ł_q^s(\alpha_1)L_a(f))(z)} = p(z)$ in (2.6) we may write

$$p(z) = \frac{(a + 1)Ł_q^s(\alpha_1)f(z)}{(Ł_q^s(\alpha_1)L_a(f))(z)} - a. \tag{2.7}$$

Differentiating (2.7) gives

$$\frac{z(Ł_q^s(\alpha_1)(f))'(z)}{(Ł_q^s(\alpha_1)f)(z)} = p(z) + \frac{zp'(z)}{p(z) + a}.$$

Now, the theorem follows by Lemma(1.1), since $\Re(q_{k,\gamma}(z)+a)>0$. A similar argument leads to

Theorem 2.6. Let $a > \left(\frac{-(k+\gamma)}{k+1}\right)$. If $\text{Ł}^s_q(\alpha_1)f \in UCV(k,\gamma)$ so is $L_a(\text{Ł}^s_q(\alpha_1)f)$.

Theorem 2.7. Let $a > \left(\frac{-(k+\gamma)}{k+1}\right)$. If $\text{Ł}^s_q(\alpha_1)f \in UCC(k,\gamma,\beta)$ so is $L_a(\text{Ł}^s_q(\alpha_1)f)$.

Proof. By definition there exists a function $k(z)=(\text{Ł}^s_q(\alpha_1)g)(z) \in UST(k,\beta)$ such that

$$\frac{z(A_1\text{Ł}^s_q(\alpha_1)f)'(z)}{\text{Ł}^s_q(\alpha_1)g(z)} \prec q_{k,\gamma}(z), \quad (z\in U). \tag{2.8}$$

Now from (2.6) we have

$$\frac{z(A_1\text{Ł}^s_q(\alpha_1)f)'(z)}{\text{Ł}^s_q(\alpha_1)g(z)} = \frac{z(A_1\text{Ł}^s_q(\alpha_1)L_a(zf'))'(z)+aA_1\text{Ł}^s_q(\alpha_1)L_a(zf')(z)}{z(\text{Ł}^s_q(\alpha_1)L_a(g(z)))'(z)+a(\text{Ł}^s_q(\alpha_1)L_a(g))(z)}$$

(2.9)

Since $\text{Ł}^s_q(\alpha_1)g \in UST(k,\beta)$, by Theorem 2.5, we have $L_a(\text{Ł}^s_q(\alpha_1)g) \in UST(k,\beta)$.

Let$H(z)=\frac{z(\text{Ł}^s_q(\alpha_1)L_a(g))'(z)}{\text{Ł}^s_q(\alpha_1)L_ag(z)}$ we note that $\Re(H(z)) > \frac{k+\beta}{k+1}$. Now, let h be defined by

$$z(A_1\text{Ł}^s_q(\alpha_1)L_a(f))' = h(z)(\text{Ł}^s_q(\alpha_1)L_a(g))(z).$$

(2.10)

Differentiating both sides of (2.10) yields

$$\frac{z(A_1\text{Ł}^s_q(\alpha_1)(zL_a(f))')'(z)}{(\text{Ł}^s_q(\alpha_1)L_a(g))(z)} = \frac{z(\text{Ł}^s_q(\alpha_1)L_a(g))'(z)}{(\text{Ł}^s_q(\alpha_1)L_a(g))(z)}h(z)+zh'(z) = H(z)h(z)+zh'(z)$$

(2.11)

Therefore from (2.9) and (2.11) we obtain

$$\frac{z(A_1\text{Ł}^s_q(\alpha_1)f)'(z)}{(\text{Ł}^s_q(\alpha_1)(g))(z)} = \frac{zh'(z)+h(z)H(z)+ah(z)}{H(z)+a}$$

This conjunction with (2.8) leads to

$$h(z) + \frac{zh'(z)}{H(z) + a} \prec q_{k,\gamma}(z). \tag{2.12}$$

Assuming $E = 0$ and $B(z) = \frac{1}{H(z)+a}$, we obtain

$\Re(B(z)) > 0$, if $a > \frac{-(k+\beta)}{k+1}$. Now, we conclude that the proof since the required conditions of lemma 1.2 are satisfied. A similar argument yields

Theorem 2.8. Let $a > \left(\frac{-(k+\gamma)}{k+1}\right)$. If $Ł_q^s(\alpha_1)f \in UQC(k, \gamma, \beta)$ so is $L_a(Ł_q^s(\alpha_1)f)$.

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