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θ-GENERALIZED SEMI-OPEN AND θ-GENERALIZED SEMI-CLOSED FUNCTIONS

GOVINDAPPA NAVALAGI
G. H COLLEGE, INDIA
and
Md. HANIF PAGE
B. V. B. COLLEGE OF ENG. AND TECH., INDIA
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Abstract

In this paper, we introduce and study the notions of θ-generalized-semi-open function, θ-generalized- semi-closed function,pre-θ-generalized-semi-open function,pre-θ-generalized-semi-closed function, contra pre-θ-generalized-semi-open,contra pre-θ-generalized-semi-closed function and θ-generlized-sem-homeomorphism in topological spaces and study their properties.

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1. INTRODUCTION

In 1970, Levine [10] first considered the concept of generalized closed (briefly, g-closed) sets were defined and investigated Arya and Nour [1] defined generalized semi-open (briefly, gs-open) sets using semi openness and obtained some characterization of s-normal space. The generalizations of generalized closed and generalized continuity were intensively studied in recent years by Balachandran, Devi ,Maki and Sundaram[2].Recently in [12] the notion of θ-generalized semi closed (briefly, θgs-closed) set was introduced. The aim of this paper is to introduce the notions of θ-generalized semi open (briefly, θgs-open) function, θ-generalized semi closed (briefly, θgs-closed) function, θ-generalized-semi-homeomorphisms and study their simple properties.

2. PRELIMINARIES

Through out this paper(X, τ) and (Y, σ)(or simply X and Y)denote the spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of X, then $Cl(A)$ and $Int(A)$ denote the closure of A and the interior of A in X respectively.

Definition 2.1 : A subset A of a topological space X is called
(i) a semi-open set[9] if $A \subset Cl(Int(A))$,
(ii) a semi-closed set[5] if $A \subset Int(Cl(A))$.

Definition 2.2 : The semi-closure [5] of a subset of X is the intersection of all semiclosed sets that contain A and is denoted by $sCl(A)$.

Definition 2.3 : The θ-closure of a set A is denoted by Cl_θ(A)[16] and is defined by Cl_θ(A) $=\{x \in X\colon Cl(U) \cap A \neq \phi, U \in \tau, x \in U\}$ and a set A is θ-closed if and only if $A = Cl_\theta(A)$.

Definition 2.4 : A point $x \in X$ is called a semi-θ-cluster point of A if $sCl(U) \cap A \neq \phi$, for each semi-open set U containing x.

Definition 2.5 : The set of all semi-θ-cluster point of A is called semi- θ-closure of A and is denoted by $sCl_\theta(A)$. A subset A is called semi-θ-closed set if $sCl_\theta(A) = A$. The complement of semi-θ-closed set is semi-θ-open set.

Definition 2.6 : A subset A of a space (X, τ) is called a generalized closed set (briefly g-closed) [10] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open.

Definition 2.7 : A map $f : X \to Y$ is called
(i) a generalized continuous (briefly g-continuous)[2] if $f^{-1}(F)$ is g-closed in X for every closed set F of Y.
(ii) a gc-irresolute [2] if $f^{-1}(V)$ is g-closed set of X for every g- closed set V of Y.

Definition 2.8 : A function $f : X \to Y$ is called
(i) g-homeomorphism [11] if f is g-continuous and g-open.
(ii) gc-homeomorphism [11] if f and f^{-1} are gc-irresolute.

Definition 2.9 : A subset A of a topological space X is called θ-generalized-semi closed (briefly, θgs-closed) [12] if $sCl_\theta(A) \subset U$ whenever $A \subset U$ and U is open. The complement of θgs-closed set is θgeneralized-semi open (briefly, θgs-open).

We denote the family of θgs-closed sets of X by θGSC(X, τ) and θgs-open sets by θGSO(X, τ) .

Definition 2.10 : For every set $A \subseteq X$, we define θgs-closure of A [12]to be the intersection of all θgs-closed sets containing A and is denoted by θgsCl (A).

In symbol, $\theta gsCl(A) = \{\cap F \colon A \subseteq F where F is \theta gs - closed in X\}$.

Since every θ-g-closed set as well as semi-θ-closed set is θgs-closed set and hence we have, $A \subset \theta gsCl(A) \subset Cl_\theta(A)$ and $A \subset \theta gsCl(A) \subset sCl_\theta(A) \subset Cl_\theta(A)$. A is θgs-closed set, if $A = \theta gsCl(A)$.

Definition 2.11 : A function $f \colon (X, \tau) \to (Y, \sigma)$ is called:
(i) θ-semi-generalized-irresolute (briefly,θ-sg-irresolute)[14] if $f^{-1}(F)$ is θ-sg-closed in (X, τ) for every θ-sg-closed set F of (Y, σ),
(ii) θ-semi-generalized-continuous (briefly,θ-sg-continuous) if $f^{-1}(F)$ is θ-sg-closed in (X, τ) for every closed set F of (Y, σ),

Definition 2.12 : A space X is called $T_{\theta gs}$-space [13] if every θgs-closed set in it is closed set.

Theorem 2.13[13]: Intersection of arbitrary collection of θgs-closed sets is θgs-closed set.

Remark 2.14 :
(i) Any intersection of θgs-closed sets is θgs-closed set. Hence, by complement, any union of θgs-open sets is θgs-open.
(ii) Union of θgs-closed sets may fail to be θgs-closed set.

Definition 2.15[15]: A topological space (X, τ) is called
(i)$\theta gs-$T$_0$ if for any pair of distinct points x and y of X, there exists a θgs-open set containing x but not y or a θgs-open set containing y but not x.
(ii)θ gs-T_1 if for every pair of distinct points $x$and y of X, there exists a θgs-open set containing x but not y and a θgs-open set containing y but not x.
(iii)θ gs-T_2 if for each pair of distinct points x and y of X, there exist disjoint θgs-open sets, one containing x and the other containing y .

3. θgs-OPEN and θgs-CLOSED FUNCTIONS

Definition 3.1 : *A function $f: (X, \tau) \to (Y, \sigma)$ is said to be θgs-open (resp., θgs-closed) if $f(V)$ is θgs-open (resp., θgs-closed) in Y for every open set (resp., closed) V in X.*

Theorem 3.2 : A function $f: X \to Y$ is θgs-closed if and only if for each subset S of Y and for each open set U containing $f^{-1}(S)$ there is a θgs-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof : Assume that f is θgs-closed. Let S be a subset of Y and U be an open set of X such that $S \subseteq f(U)$, that is, $f^{-1}(S) \subseteq U$. Now, U^C is closed set in Y. Then $f(U^C)$ is θgs-closed in X, since f is θgs-closed. So, $Y \setminus f(U^C)$ is θgs-open in. Thus $V = Y \setminus f(U^C)$ is a θgs-open set containing S such that $f^{-1}(V) \subseteq U$.

Conversely, suppose that F is a closed set in X. Then $f^{-1}(Y \setminus f(F)) \subseteq X \setminus F$ and $X \setminus F$ is open. By hypothesis , there is a θgs-open set V of Y such that $Y \setminus f(F) \subseteq V$ and $f^{-1}(V) \subseteq X \setminus F$ and so $F \subseteq X \setminus f^{-1}(V)$. Hence$Y \setminus V \subseteq f(F) \subseteq f(X \setminus f^{-1}(V) \subseteq Y \setminus V$ which implies $f(F) = Y \setminus V$. Since $Y \setminus V$ is θgs-closed, f (F) is θgs-closed and thus f is θgs-closed.

Theorem 3.3 : If $f: X \to Y$ is bijective, θgs-open function. If X is $\theta gs-T_1$ space and $T_{\theta gs} - space$, then Y is $\theta gs - T_1$ space.

Proof : Let y_1 and y_2 be two distinct points of Y. Since f is bijective, there exists distinct points x_1 and x_2 of X such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Since X is a $\theta gs - T_1$ space , there exists θgs-open sets G and H such that $x_1 \in G$ and $x_2 \notin G$ and $x_2 \in H$ and $x_1 \notin H$. Again since X is $T_{\theta gs}$-space, G and H are open sets in X. As f is θgs-open function, $f(G)$ and $f(H)$ are θgs-open sets such that $y_1 = f(x_1) \in f(G)$, $y_2 = f(x_2) \notin f(G)$ and $y_1 = f(x_1) \notin f(H)$, $y_2 = f(x_2) \in f(H)$. Hence Y is $\theta gs - T_1$ space.

Definition 3.4 : A function $f: X \to Y$ *is called pre-θgs-closed (resp.,pre-θgs-open) if f(V)is θgs-closed (resp.,θgs-open)in Y for every θgs-closed (resp.,θgs-open)set V of X.*

Theorem 3.5 : If $f: X \to Y$ is continuous, pre-θgs-closed and A is θgs-closed subset of X, then f(A) is θgs-closed

Proof : Let U be an open set such that $f(A) \subseteq U$. Since f is continuous $f^{-1}(U)$ is an open set containing A, that is $A \subseteq f^{-1}(U)$.Hence $sCl_\theta(A) \subseteq f^{-1}(U)$ as A is θgs-closed set in X. Since f is pre-θgs-closed, f($sCl_\theta(A)$)is θgs-closed set. So, f ($sCl_\theta(A)$) is θgs-closed set contained in an open set U, that is, $f(sCl_\theta(A)) \subseteq U$. Now $sCl_\theta(f(A)) \subseteq sCl_\theta(f(\theta gsCl(A))) \subseteq sCl_\theta(f(sCl_\theta(A))) = f(sCl)_\theta(A)) \subseteq U$. Hence $sCl_\theta(f(A)) \subseteq U$. Therefore f (A) is a θgs-closed set in Y.

Theorem 3.6 : The property of a space being $\theta gs - T_0$ space is preserved under one-one , onto , pre-θgs-open function and hence is a topological property.

Proof : Let X be a $\theta gs - T_0$ space and Y be any other topological space. Let $f: X \to Y$ *be one-one, onto, pre-θgs-open function from X to Y. Let* y_1, $y_2 \in Y$ *with* $y_1 \neq y_2$ *and since f is one-one , onto,there exists distinct points* x_1, $x_2 \in X$ *such that* $f(x_1) = y_1$, $f(x_2) = y_2$. *Since* X *is* $\theta gs - T_0$ *space, there exists a θgs-open set* G *in* X *such that* $x_1 \in G$ *but* $x_2 \notin G$. *Since f is pre-θgs-open,* $f(G)$ *is θgs-open set containing* $f(x_1)$ *but not containing* $f(x_2)$. *Thus there exists a θgs-open set f(G)in Y such that* $(y_1) \in f(G)$ *but* $(y_2) \notin f(G)$ *and hence* Y *is* $\theta gs - T_0$ *space.Again as the property of being* $\theta gs - T_0$ *is preserved under one-one, onto mapping , it is also preserved under homeomorphism and hence is a topological property.*

Theorem 3.7 : *Let* $f: X \to Y$ *be a θgs-closed and* $g: Y \to Z$ pre-θgs-closed and continuous, then their composition $gof: X \to Z$ is θgs-closed.

Proof : Let A be a closed set of X. Then by hypothesis $f(A)$ is θgs-closed set in Y. Since g is pre-θgs-closed and continuous, by Theorem 3.5 $g(f(A) = (gof)(A)$ is θgs-closed in Z. Hence (gof) is θgs-closed.

4. CONTRA PRE-θgs-CLOSED and CONTRA PRE-θgs-OPEN FUNCTIONS

Definition 4.1 : A function $f: X \to Y$ is called contra pre-θgs-open if for every θgs-open set F of X, f (F) is θgs-closed in Y.

Definition 4.2 : A function $f: X \to Y$ is called contra pre-θgs-closed if for every θgs-closed set F of X, f (F) is θgs-open in Y.

The following example shows that contra pre-θgs-closedness and contra pre-θgs-openness are independent.

Example 4.3 : Let $X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ and $\sigma = \{Y, \{a\}, \{b\}, \{a, b\}\}$.We have θgs-open sets in X are $\{X, \{a\}, \{b, c\}\}$ and θgs-open sets in Y are $\{Y, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$. Let $f: X \to Y$ is defined by $f(a) = f(c) = c$, $f(b) = b$ and $g: X \to Y$ is by $g(a) = g(b) = a$,$g(c) = b$. Then f is contra pre-θgs-open but not contra pre-θgs-closed and g is contra pre-θgs-closed but not contra pre-θgs-open.

Remark 4.4 : Contra pre-θgs-closedness and contra pre-θgs-openness are equivalent if the function is bijective.

Theorem 4.5 : For a function $f: X \to Y$ the following properties are equivalent.
(i) f is contra pre-θgs-open.
(ii) For every subset B of Y and every θgs-closed subset F of X with $f^{-1}(B) \subseteq F$, there exists a θgs-open subset O of Y with $B \subseteq O$ and $f^{-1}(O) \subseteq F$.
(iii) For every point $y \in Y$ and every θgs-closed subset F of X with $f^{-1}(y) \subseteq F$, there exists a θgs-open subset O of Y with $y \in O$ and $f^{-1}(O) \subseteq F$.

Proof : $(i) \to (ii)$. Let B be a subset of Y and F be a θgs-closed subset of X with $f^{-1}(B) \subseteq F$. For the case $B \neq \phi$, put $O = [f(F^C]^C$.Then

$f^{-1}(O) = [f^{-1}(F^C)]^C \subseteq F$ and O is a θgs-open subset of Y. We claim that $B \subseteq O$. There are two cases to be considered:

Case 1:$f^{-1}(B) \neq \phi$. Since $f^{-1}(B) \subseteq F$,we have $f(F^C) \subseteq B^C$ and $B \subseteq O$.

Case 2: $f^{-1}(B) = \phi$. Since $f^{-1}(B) = \phi \subseteq F$,we have $f(F^C) \subseteq f(X)$. We have $B \cap f(X) = \phi$, because $f^{-1}(B) = \phi$ and $B \neq \phi$. Thus $B \subseteq [f(X)]^C \subseteq [f(F^C]^C = O$. For the case $B = \phi$, put $O = \phi$. Then the set O is required θgs-open set of Y.

$(ii) \to (iii)$.It suffices to put $B = y$ in (ii)

$(iii) \to (i)$. Let A be a θgs-open subset of X.Then $y \in [f(A)]^C$ and $F = A^C$. First we claim that $f^{-1}(y) \subset F$. For non empty set $f^{-1}(y)$, let $z \in f^{-1}(y)$; $f(z) = y \notin f(A)$ Suppose that $z \notin A$ and so $y = f(z) \in f(A)$. By contradiction, $z \in F$ for any $z \in f^{-1}(y)$, that is $f^{-1}(y) = \phi \subseteq F$. For the case where $f^{-1}(y) = \phi$, we have $f^{-1}(y) = \phi \subseteq F$. For both cases , we can use (iii) and get the following: there exists a θgs-open set $O_y \subseteq Y$ such that $y \in O_y$ and $f^{-1}(O_y) \subseteq F = A^C$. Namely, (*) $f^{-1}(O_y) \cap A = \phi$ holds for each $y \in [f(A)]^C$. Finally we claim that $[f(A)]^C = \cup \left\{ O_y : y \in [f(A)]^C \right\}$. Obviously, we have that $[f(A)]^C \subseteq \cup \left\{ O_y : y \in [f(A)]^C \right\}$.

Conversely, let $z \in \cup \left\{ O_y : y \in [f(A)]^C \right\}$. Then there exists a point $w \in [f(A)]^C$ such that $z \in O_w$. Suppose that $z \notin [f(A)]^C$. Then $z \in f(A)$ and there exists a point $b \in A$ such that $f(b) = z$. Thus we have that $f(b) \in O_w$ and so $b \in f^{-1}(O_w)$. We have a contradiction to (*) above , that is , $b \in f^{-1}(O_w) \cap A$. Hence , we show that $\cup \left\{ O_y : y \in [f(A)]^C \right\} \subseteq [f(A)]^C$ and so $[f(A)]^C = \cup \left\{ O_y : y \in [f(A)]^C \right\}$. Consequently, by Remark 2.12 (i), f(A) is a θgs-closed subset of Y.

Theorem 4.6 :

Let (1) ,(2) and (3) be properties of a function $f\colon X \to Y$ as follows.

1. f is contra pre-θgs-closed.
2. For every set B of Y and every θgs-open subset O of X with $f^{-1}(B) \subseteq$, there exists a θgs-closed subset O of Y with $B \subseteq O$ and $f^{-1}(O) \subseteq F$.
3. For every point $y \in Y$ and every θgs-open subset O of X with $f^{-1}(y) \subseteq O$, there exists a θgs-closed subset F of Y with $y \in F$ and $f^{-1}(F) \subseteq O$. Then

(i) The implications $(1) \to (2) \to (3) hold.$

(ii) Suppose $\theta GSC(Y, \sigma)$ is closed under arbitrary unions (i.e., union of any collection of θgs-closed sets is θgs-closed set). Then the implication $(3) \rightarrow (1) holds.$

Proof : (i)(1) $\rightarrow$ (2). Let B be the subset of Y and O be a θgs-open subset of X with $f^{-1}(B) \subseteq O$. Put $F = [F(O^C)]^C$. Since f is contra pre-θgs-closed , then F is θgs-closed set of Y . By $f^{-1}(B) \subseteq O$, we have $f(O^C) \subseteq B^C$. Moreover, $f^{-1}(F) \subseteq O$.

(2) $\rightarrow$ (1). Let E be a θgs-closed subset of X. Put $B = [f(E^C)]^C \subseteq E^C = O$. Hence $f^{-1}(B) = f^{-1}(f(E))^C = [f^{-1}(f(E))]^C \subseteq E^C = O$. By assumption there exists a θgs-closed set $F \subseteq Y$ for which $B \subseteq F$ and $f^{-1}(F) \subseteq O$. It follows that $B = F$. If $y \in F$ and $y \notin B$, $y \in f(E)$. Therefore $y = f(x)$ for some $x \in E$ and we have that $x \in f^{-1}(F) \subseteq O = E^C$, which is a contrdiction. Since $B = F(i.e, [f(E)^C] = F)$, $f(E)$ is a θgs-open and hence f is contra pre-θgs-closed.

(2) $\rightarrow$ (3). It suffices to put $B = y$ for some $y \in Y$.

(ii)(3) $\rightarrow$ (1). Let A be a θgs-closed subset of X. Let $y \in [f(A)]^C$. Then we have $f^{-1}(y) \subseteq A^C$. By (3) there exists a θgs-closed set $F_y \subseteq Y$ such that $y \in F_y$ and $f^{-1}(F_y) \subseteq A^C$. Namely ,$f^{-1}(y) \cap A = \phi$ holds for each $y \in [f(A)]^C$. By an arrangement similar to that $[f(A)]^C = \cup \left\{ F_y : y \in [f(A)]^C \right\}$ holds. Consequently, using assumption in (iii), f(A) is θgs-open in Y.

Theorem 4.7 : Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are θgs-closed functions and Y be $T_{\theta gs} - space$. Then their composition gof is θgs-closed.

Proof : Let A be a closed set of X. Then by hypothesis f (A) is a θgs-closed set in Y. Since Y is $T_{\theta gs} - space$, f (A) is closed in Y. Since g is θgs-closed , $g(f(A))$ is θgs-closed in Z. But $g(f(A)) = (gof)(A)$. Hence gof is θgs-closed.

Theorem 4.8 : The composition of a closed functions $f : X \rightarrow Y$and $g : Y \rightarrow Z$ is θgs-closed function from X to Z.

Proof : Trivial.

Theorem 4.9 : Let $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ be two functions such that their composition $gof : X \rightarrow Z$ is θgs-closed function. Then the following statements holds;

i) If f is continuous and surjective , then g is θgs-closed.

ii) If g is θgs-irresolute and injective , then f is θgs-closed.

Proof : (i) Let A be a closed set in Y. Then $f^{-1}(A)$ is closed in X as f is continuous. Since gof is θgs-closed and if f is surjective , $(gof)(f^{-1}(A)) = g(f(f^{-1}(A))) = g(A)$ is θgs-closed in Z. Therefore g is a θgs-closed function in X.

(ii) Let H be a closed set of X. Since gof is θgs-closed , (gof) (H) is θgs-closed set in Z. Since g is θgs-irresolute, $g^{-1}(gof)(H)) = g^{-1}(g(f(H))) = f(H)$ is θgs-closed in Y, since g is injective. Thus f is θgs-closed function in X.

Theorem 4.10: For any bijection $f : X \to Y$, the following statements are equivalent (i) Inverse of f is θgs-continuous.
(ii) f is a θgs-open function.
(iii) f is a θgs-closed function.

Proof : (i) $\to$ (ii). Let U be an open set of X. By assumption θgs-continuous, $(f^{-1})^{-1}(U) = f(U)$ is θgs-open in Y and so f is θgs-open function.

(ii) $\to$ (iii). Let F be a closed set of X. Then F^C is open in X. By assumption $f(F^C)$ is θgs-open in Y, that is , $f(F^C) = (f(F))^C$ is θgs-open in Y and therefore $f(F)$ is θgs-closed in Y. Hence f is θgs-closed.

(iii) $\to$ (i). Let F be a closed set in X. By assumption f (F) is θgs-closed in Y. But $f(F) = (f^{-1})^{-1}(F) = f(F)$ is θgs-closed and therefore inverse image of f is θgs-continuous.

5. θgs-HOMEOMORPHISMS

Definition 5.1 : A bijection $f : X \to Y$ is called θ-generalized-semi-homeomorphism (briefly , θgs-homeomorphism) if f is both θgs-continuous and θgs-open.

Remark 5.2 : Every g-homeomorphism is θgs-homeomorphism.

But the converse is not always true by the following example.

Example 5.3 : Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, c\}\}$ and θgs-open sets in Y are $\{Y, \phi, \{b\}, \{c\}, \{b, c\}\}$, $g - open$ sets are $\{Y, \phi, \{b, c\}, \{a, c\}, \{b\}\}$. Let $f : X \to Y$ be a function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. It is easy to verify that f is θgs-homeomorphism but not a g-homeomorphism since b is open in X, while $f(b) = c$ is not g-open in Y.

Theorem 5.4 : Let $f : X \to Y$be a bijective and θgs-continuous function. Then the following statements are equivalent
(i) f is θgs-open
(ii) f is θgs-homeomorphism.
(iii) f is θgs-closed.

Theorem 5.5 : (i) $\to$ (ii). By assumption, f is bijective, θgs-continuous and θgs-open. Then by definition f is a θgs-homeomorphism.

(ii) $\to$ (iii).By assumption f is θgs-open and bijective. By Theorem 4.10, f is θgs-closed.
(iii) $\to$ (iv).By assumption f is bijective and θgs-closed. By Theorem 4.10, f is θgs-open.

Now we introduce the class of functions which included in the class of θgs-homeomorphism.

Definition 5.6 : A bijective $f : X \to Y$ is called θgsc-homeomorphism if f and f^{-1} are θgs-irresolute.

We say that the topological spaces X and Y are θgsc-homeomorphismic, if there exists a θgsc-homeomorphism from X onto Y.

Remark 5.7 : Every θgsc-homeomorphism is θgs-homeomorphism.

However the converse is not true as shown by the following example.

Example 5.8 : Let $X = Y = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. $\sigma = \{Y, \phi, \{a\}, \{b\}, \{a, c\}\}$. We have θgs-closed sets in X are $\{X, \phi, \{a\}, \{b, c\}\}$ and θgs-closed sets in Y are $\{Y, \phi, \{a\}, \{a, b\}, \{a, c\}\}$. Let $f : X \to Y$ be a function defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. It is easy to verify that f is θgs-homeomorphism but not a θgsc-homeomorphism since $\{a, b\}$ is θgs-closed set in Y , while $f^{-1}(\{a, b\}) = \{a, c\}$is not θgs-closed set in X.

Theorem 5.9 : Every homeomorphism is a θgsc-homeomorphism.

Proof : Let $f : X \to Y$ be a homeomorphism. To prove that f is θgs-irresolute, let A be a θgs-closed set of Y. Let U be an open set of X such that $f^{-1}(A) \subseteq U$. Then, $A = fof^{-1}(A) \subseteq f(U)$ and $f(U) \in \sigma$. Since A is θgs-closed in Y , we have $sCl_\theta(A) \subseteq f(U)$. Then $f^{-1}(sCl_\theta(A)) \subseteq f^{-1}of(U) = U$. So, we are done if we show that $sCl_\theta(f^{-1}(A)) \subseteq f^{-1}(sCl_\theta(A))$. Let $x \notin f^{-1}(sCl_\theta(A))$. Since f is bijective, then $f(x) \notin sCl_\theta(A)$. Hnece there exists an open set containing $f(x)$ such that $sCl(W) \cap A = \phi$. Then $f^{-1}(sCl(W) \cap A) = f^{-1}(sCl(W)) \cap f^{-1}(A) = \phi$, since f is homeomorphism. Since $f^{-1}(W)$ is an open set containing x, we obtain that $x \notin sCl_\theta(f^{-1}(A))$. Thus $sCl_\theta(f^{-1}(A)) \subseteq f^{-1}(sCl_\theta(A)) \subseteq U$. Hence $f^{-1}(A)$ is θgs-closed in X , that is , f is θgs-irresolute. Since f^{-1} is a homeomorphism , the θgs-irresoluteness of f^{-1} is similarly proved as above. Thus f is θgsc-homeomorphism.

However, the converse of the above Theorem need not be true as seen from the following example.

Example 5.10 : Let $X = Y = \{a, b, c\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{b, c\}\}$.We have θgs-closed sets in $X$are $\{X, \phi, \{a\}, \{b, c\}\}$ and θgs-closed sets in Y are $\{Y, \phi, \{a\}, \{b, c\}\}$. Let $f : X \to Y$ be an identity function. It is easy to verify that f is θgsc-homeomorphism but not a homeomorphism .

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Govindappa Navalagi
Department of Mathematics
KLE Society's
G. H. College,
Haveri-581 110
Karnataka,
INDIA
e-mail : gnavalagi@hotmail.com

and

Md. Hanif Page
Department of Mathematics
B. V. B. College of Eng. and Tech.
Hubli-580031, Karnataka,
INDIA
e-mail : hanif01@yahoo.com