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FUZZY PARA - LINDELOF SPACES

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Abstract

In this paper we introduce the concept of Para-Lindelof spaces in L-topological spaces by means of locally countable families of L-fuzzy sets. Further some characterizations of fuzzy para-Lindelofness and flintily para-Lindelofness in the weakly induced L-topological spaces are also obtained. More over the behavior of fuzzy para-Lindelof spaces under various types of maps such as fuzzy closed maps, fuzzy perfect maps are also investigated.

Keywords : *L-Topology, Fuzzy para-Lindelofness, Flintily para-Lindelofness, locally countable family.*

2000 AMS Classification : *54D 20, 54A 40.*

1. Introduction

As a generalization of a set, the concept of fuzzy set was introduced by Zadeh [18]. Fuzzy topology comes as the generalization of general topology using the concept of a fuzzy set. In 1968 Chang [6] introduced the concept of fuzzy topology and Lowen [12] introduced a more natural definition of fuzzy topology.

Compactness and metrizability are the heart and soul of general topology. In 1944 J. Dieudonne [7] defined paracompactness as a natural generalization of compactness. Later several other covering properties such as meta-compactness, sub para-compactness, sub meta-compactness, para-Lindelofness etc. have naturally evolved from para compactness. The concept of para-Lindelof spaces was introduced by J. Greever [9] in 1968 and further studies were conducted by Burke ([4, 5]), Fleissner-Reed [8].

The concept of paracompactness in fuzzy topology was introduced by Luo [13]. Authors have introduced the concept and studied some properties regarding metacompactness, subparacompactness, and submetacompactness in L-topological spaces in [14], [3], [2] respectively. In this paper we define locally countable families and introduce the concept of para-Lindelof spaces in L-topological spaces. Besides getting some characterization for para-Lindelof and flintily para-Lindelof in the weakly induced L-topological spaces, it is also seen that these properties are closed hereditary. Further the invariance of these properties under perfect maps is also proved.

Let L be a complete lattice. Its universal bounds are denoted by $\perp$ and $\top$. We presume that L is consistent. i.e., $\perp$ is distinct from $\top$. Thus $\perp \leq \alpha \leq \top$ for all $\alpha \in L$. We note $\vee\phi = \perp$ and $\wedge\phi = \top$. The two point lattice $\{\perp, \top\}$ is denoted by 2. A unary operation $'$ on L is a quasi-complementation. It is an involution (ie., $\alpha'' = \alpha$ for all $\alpha \in L$) that inverts the ordering. (ie., $\alpha \leq \beta$ implies $\beta' \leq \alpha'$). In $(L,')$ the DeMorgan laws hold: $(\vee A)' = \wedge\{\alpha' : \alpha \in A\}$ and $(\wedge A)' = \vee\{\alpha' : \alpha \in A\}$ for every $A \subset L$. Moreover, in particular, $\perp' = \top$ and $\top' = \perp$.

A molecule or co-prime element in a lattice L is a join irreducible element in L and the set of all non zero co-prime elements of L is denoted by $M(L)$ and prime elements by $pr(L)$. A complete lattice L is completely distributive if it satisfies either of the logically equivalent CD1 or CD2 below: CD1: $\wedge_{i\in I}\left(\vee_{j\in J_i} a_{i,j}\right) = \vee_{\phi\in\Pi J_i i\in I}\left(\wedge_{i\in I} a_{i,\phi(i)}\right)$
CD2: $\vee_{i\in I}\left(\wedge_{j\in J_i} a_{i,j}\right) = \wedge_{\phi\in\Pi J_i i\in I}\left(\vee_{i\in I} a_{i,\phi(i)}\right)$
for all $\{\{a_{ij} : j \in J_i\} : i \in I\} \subset P(L)\backslash\{\phi\}$,

If L is a complete lattice, then for a set X, L^X is the complete lattice of all maps from X into L, called L-sets or L-subsets of X. Under point-wise ordering, $a \leq b$ in L^X if and only if $a(x) \leq b(x)$ in L for all $x \in X$. If $A \subset X$, $1_A \in 2^X \subset L^X$ is the characteristic function of A. The constant member of L^X with value α is denoted by α itself. Usually we will not distinguish between a crisp set and its characteristic function. Wang [15] proved that a complete lattice is completely distributive if and only if for each $\alpha \in L$, there exists $B \subseteq L$ such that (i) $a = \vee A$ and (ii) if $A \subseteq L$ and $a \leq \vee B$, then for each $b \in B$, there exists $c \in A$ such that $b \leq c$. B is called the minimal set of a and $\beta(a)$ denote the union of all minimal sets of a. Again $\beta^*(a) = \beta(a) \cap M(L)$. Clearly $\beta(a)$ and $\beta^*(a)$ are minimal sets of a.

For $\alpha \in L$ and $A \in L^X$, we use the following notations.
$A_{[\alpha]} = \{x \in X : A(x) \geq \alpha\}$;
$A^{[\alpha]} = \{x \in X : A(x) \leq \alpha\}$;
$A^{(\alpha)} = \{x \in X : A(x) \not\geq \alpha\}$;
$A_{(\alpha)} = \{x \in X : A(x) \not\leq \alpha\}$.

Clearly L^X has a quasi complementation $'$ defined point-wisely $\alpha'(x) = \alpha(x)'$ for all $\alpha \in L$ and $x \in X$. Thus the DeMorgan laws are inherited by $(L^X,')$.

Let $(L,')$ be a complete lattice equipped with an order reversing involution and X be any non empty set. A subfamily $\tau \subset L^X$ which is closed under the formation of sups and finite infs (both formed in L^X) is called an L-topology on X and its members are called open L-sets. The pair (X, τ) is called an L-topological space (L-ts). The category of all L-topological spaces, together with L-continuous mappings and the composition and identities of set is denoted by L-Top. Quasi complements of open L-sets are called closed L-sets.

We know that the set of all non zero co-prime elements in a completely distributive lattice is $\vee$-generating. Moreover for a continuous lattice L and a topological space (X, T), $T = i_L\omega_L(T)$ is not true in general. By proposition 3.5 in Kubiak [11] we know that one sufficient condition for $T = i_L\omega_L(T)$ is that L is completely distributive.

In [16] Wang extended the Lowen functor ω for completely distributive lattices as follows: For a topological space (X, T), $(X, \omega(T))$ is called the induced space of (X, T) where $\omega(T) = \{A \in L^X : \forall \alpha \in M(L),\ A^{(\alpha')} \in T\}$. In 1992 Kubiak also extended the Lowen functor ω_L for a complete lattice L. In fact when L is completely distributive, $\omega_L = \omega$.

An L-topological space (X, τ) is called weakly induced space if $\forall \alpha \in M(L)$, $\forall A \in \tau$ it is true that $A^{(\alpha')} \in [\tau]$ where $[\tau]$ is the set of all crisp open sets in τ.

Based on these facts, in this paper we use a complete, completely distributive lattice L in L^X. For a standardized basic fixed-basis terminology, we follow Hohle and Rodabaugh [10].

2. Preliminaries and Basic Definitions

2.1. Definition

[17] Let (X, τ) be an L-ts. A fuzzy point x_α is quasi coincident with $D \in L^X$ (and write $x_\alpha \prec D$) if $x_\alpha \nleq D'$. Also D quasi coincides with E at x (D q E at x) if $D(x) \nleq E'(x)$. We say D quasi coincident with E and write D q E if D q E at x for some $x \in X$. Further $D \neg q$ E means D not quasi coincides with E. We say $U \in \tau$ is quasi coincident *nbd* of x_α (Q- *nbd*) if $x_\alpha \prec U$. The family of all Q- *nbds* of x_α is denoted by $Q_\tau(x_\alpha)$ or $Q(x_\alpha)$.

2.2. Definition

[17] Let (X, τ) be an L-ts, $A \in L^X$. $\Phi \subset L^X$ is called a Q-cover of A if for every $x \in$ $Supp(A)$, there exist $U \in \Phi$ such that $x_{A(x)} \prec U$. Φ is a Q-cover of (X, τ) if Φ is a Qcover of $\top$. If $\alpha \in M(L)$, then $\mathbf{C} \in \tau$ is an α-Q-*nbd* of A if $\mathbf{C} \in Q(x_\alpha)$ for every $x_\alpha \leq A$. Φ is called an α-Q-cover of A, if for each $x_\alpha \leq A$, there exists $U \in \Phi$ such that $x_\alpha \prec U$. Φ is called an open α-Q-cover of A if $\Phi \subset \tau$ and Φ is an α-Q-cover of A. $\Phi_0 \subset L^X$ is called a sub α-Q-cover of A if $\Phi_0 \subset \Phi$ and Φ_0 is also an α-Q-cover of A. Φ is called an α^--Q cover of A, if there exists $\gamma \in \beta^*(\alpha)$ such that Φ is γ-Q-cover of A.

2.3. Definition

[17] Let (X, τ) be an L-ts, $D \in L^X$. D is called N-compact if for every $\alpha \in M(L)$, every open α-Q cover of D has a finite sub family which is an α^--Q cover of D. (X, τ) is called N-compact if $\top$ is N-compact.

2.4. Definition

[**?**] Let (X, τ) be an L-ts, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called locally finite at x_λ, if there exist $U \in Q(x_\lambda)$ and a finite subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow A_t \neg q$ U. And $\mathbf{A}$ is called *-locally finite at x_λ if there exist $U \in Q(x_\lambda)$ and a finite subset T_0 of T such that

$t \in T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q\, U$. **A** is called locally finite (*-locally finite) for short, if **A** is locally finite(*-locally finite) at every molecule $x_\lambda \in M(L^X)$.

2.5. Definition

[14] Let (X, τ) be an L-ts. $\mathbf{A} = \{A_t : t \in T\} \subset L^X$, $x_\lambda \in M(L^X)$. **A** is called point finite at x_λ if $x_\lambda \prec A_t$ for at most finitely many $t \in T$. And **A** is *-point finite at x_λ if there exists at most finitely many $t \in T$ such that $x_\lambda \prec \chi_{(A_t)_{(\perp)}}$. **A** is called point finite (resp. *-point finite) for short, if **A** is point finite (resp. *-point finite) at every molecule x_λ of L^X.

2.6. Definition

Let (X, τ) be an L-ts, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. **A** is called locally countable at x_λ, if there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow A_t \neg q\, U$. And **A** is called *-locally countable at x_λ if there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q\, U$. **A** is called locally countable (*-locally countable) for short, if **A** is locally countable (*-locally countable) at every molecule $x_\lambda \in M(L^X)$.

The previous notions "locally countable family" is defined for L-ts. They can be also defined for L-subsets:

2.7. Definition

Let (X, τ) be an L-ts. $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subset L^X$, $x_\lambda \in M(L^X)$. **A** is called locally countable in A, if **A** is locally countable at every molecule $x_\lambda \in M(\downarrow A)$.

2.8. Definition

Let (X, τ) be an L-ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $B \in L^X$.

A is called σ-locally countable in B if **A** is the countable union of sub families which are locally countable in B. **A** is called σ-locally countable for short, if **A** is σ-locally countable in $\top$.

2.9. Definition

[17] Let (X, τ) be an L-ts. Then by $[\tau]$ we denote the family of support sets of all crisp subsets in τ. $(X, [\tau])$ is a topology and it is the background

space. (X,τ) is weakly induced if $U \in \tau$ is a lower semi continuous function from the background space $(X,[\tau])$ to L.

2.10. Definition

[17] Let (X,τ) be an L-ts. (X,τ) is called weakly α-induced if $U_{(\alpha)} \in [\tau]$ for every $U \in \tau$.

2.11. Proposition

[17] Let (X,τ) be an L-ts. Then the following conditions are equivalent.
(i) (X,τ) is weakly induced.
(ii) (X,τ) is weakly γ-induced for every $\gamma \in pr(L)$.
(iii) (X,τ) is weakly α-induced for every $\alpha \in L$.

2.12. Definition

[17] For a property P of ordinary topological space, a property P^* of L-ts is called a good L-extension of P, if for every ordinary topological space (X,T), (X,T) has the property P if and only if $(X,\omega_L(T))$ has property P^*. In particular when $L=[0,1]$ we say P^* is a good extension of P. Where $\omega_L(T)$ is the family of all lower semi continuous function from (X,T) to L.

2.13. Definition

[17] A collection $\mathbf{A}$ refines a collection $\mathbf{B}(\mathbf{A} < \mathbf{B})$ if for every $A \in \mathbf{A}$, there exists $B \in \mathbf{B}$ such that $A \leq B$.

2.14. Definition

[17] Let (X,τ) be an L-ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$ is a closure preserving collection if for every subfamily $\mathbf{A}_0$ of $\mathbf{A}$, $cl[\vee \mathbf{A}_0] = \vee[cl\mathbf{A}_0]$.

2.15. Proposition

[17] Let (X,τ) be an L-ts. $\mathbf{A} \subset L^X$ is closure preserving. Then for every sub family $\mathbf{A}_0 = \{A_t : t \in T\} \subset \mathbf{A}$, $\vee_{t\in T}\, cl\ A_t$ is a closed subset.

2.16. Theorem

Every locally countable family of subsets is closure preserving.

Proof. Let $\mathbf{A} \subset L^X$ is locally countable, $\mathbf{A}_0 = \{A_t : t \in T\} \subset \mathbf{A}$, then $\mathbf{A}_0$ is locally countable. Since $\vee(cl\mathbf{A}_0) \leq cl(\vee\mathbf{A}_0)$ is clear it is sufficient to prove that $cl(\vee\mathbf{A}_0) \leq \vee(cl\mathbf{A}_0)$. Suppose $x_\alpha \in M(\downarrow\ cl(\vee\mathbf{A}_0))$. Since $\mathbf{A}_0$ is locally countable, there exist $U \in Q(x_\alpha)$ such that $\Rightarrow A_t \neg q\ U$ for every $t \in T \setminus T_0$ where T_0 is a countable subset of T. This implies that $A_t \leq U'$ for every $t \in T \setminus T_0$. If $x_\alpha \nleq \vee(cl\mathbf{A}_0)$, then $x_\alpha \nleq cl\,\mathbf{A}_t$ for every $t \in T_0$ and hence there exist $U_t \in Q(x_\alpha)$ such that $A_t \leq U_t'$. Since T_0 is countable, $V = U \wedge (\vee_{t\in T_0} U_t) \in Q(x_\alpha)$ and $A_t \leq V'$ for every $t \in T$. So $\vee_{t\in T} A_t \leq V'$ and hence $x_\alpha \leq cl(\vee\mathbf{A}_0) = cl(\vee_{t\in T} A_t) \leq cl(V') = V'$. That is x_α is not quasi coincidence with V, which is a contradiction that $V \in Q(x_\alpha)$. Therefore $x_\alpha \in \vee(cl\mathbf{A}_0)$ and thus $cl(\vee\mathbf{A}_0) = \vee(cl\mathbf{A}_0)$. □

2.17. Definition

[14] A collection $\mathbf{U}$ of fuzzy subsets of an L-topological space (X, τ) is said to be well monotone if the subset relation '$<$' is a well order on $\mathbf{U}$.

2.18. Definition

[14] A collection $\mathbf{U}$ of fuzzy subsets of an L-topological space (X, τ) is said to be directed if $U, V \in \mathbf{U}$ implies there exists $W \in \mathbf{U}$ such that $U \vee V < W$.

2.19. Definition

Let (X, τ) be an L-ts, $A \in L^X$, $\mathbf{B} \subset L^X$. Then $st(A, \mathbf{B}) = \vee\{B \in \mathbf{B} : B\ q\ A\}$ is defined as the star of $\mathbf{B}$ about A. If $x_\lambda \in M(L^X)$, then $st(\{x_\lambda\}, \mathbf{B})$ is denoted by $st(x_\lambda, \mathbf{B})$.

2.20. Definition

Let (X, τ) be an L-ts. $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$ is a interior preserving collection if for every subfamily $\mathbf{A}_0$ of $\mathbf{A}$, $int[\wedge\mathbf{A}_0] = \wedge[int\mathbf{A}_0]$.

3. Para-Lindelof Spaces

3.1. Definition

[17] Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$. A is called α-Lindelof if every open α-Q-cover of A has a countable subfamily which is also an α-Q-cover of A. A is Lindelof if A is α- Lindelof for every $\alpha \in M(L)$. And (X, τ) is Lindelof if $\top$ is Lindelof.

3.2. Definition

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$. A is called α-para-Lindelof (α^*-para-Lindelof) if for every open α-Q-cover Φ of A, there exist an open refinement Ψ of Φ which is locally countable (*-locally countable) in A and Ψ is also an α-Q-cover of A. A is para-Lindelof (*-para-Lindelof) if A is α-para-Lindelof (α^*-para-Lindelof) for every $\alpha \in M(L)$. (X, τ) is para-Lindelof (*-para-Lindelof) if $\top$ is para-Lindelof (*-para-Lindelof).

3.3. Definition

Let (X, τ) be an L-ts, $\alpha \in M(L)$. (X, τ) is called σ-para-Lindelof if for every open α-Q-cover Φ of X, there exist an open refinement Ψ of Φ which is σ-locally countable in X and also an α-Q-cover of X.

3.4. Proposition

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$. Then
(i) A is α^*-para-Lindelof $\Rightarrow$ A is α-para-Lindelof.
(ii) A is $*$-para-Lindelof $\Rightarrow$ A is para-Lindelof.

Para-Lindelof and *-Para-Lindelof are hereditary with respect to closed subsets.

3.5. Theorem

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$, $B \in \tau'$. Then
(i) A is α-para-Lindelof $\Rightarrow$ $A \wedge B$ is α-para-Lindelof.
(ii) A is para-Lindelof $\Rightarrow$ $A \wedge B$ is para-Lindelof.

Proof. We need to prove only (i). Suppose that $\mathbf{U}$ is an open α-Q-cover of $A \wedge B$. Take $\mathbf{V} = \mathbf{U} \cup \{B'\}$. Now clearly $\mathbf{V}$ is an open α-Q-cover of A. Since A is α-para-Lindelof, $\mathbf{V}$ has an open refinement $\mathbf{W}$ such that $\mathbf{W}$ is locally countable in A and is also an α-Q-cover of A. Take $\mathbf{W}_0 = \{W \in \mathbf{W} : \exists\ U \in \mathbf{U}, W \leq U\}$. Now we show that $\mathbf{W}_0$ is the required locally countable refinement of $\mathbf{V}$ which is also an α-Q-cover of $A \wedge B$. Clearly $\mathbf{W}_0$ is a locally countable refinement. Let $x_\alpha \leq A \wedge B \leq A$, since $\mathbf{W}$ is an α-Q-cover of A, there exist $W \in \mathbf{W}$ such that $x_\alpha \prec W$. Since $x_\alpha \leq B$, $B \not\leq B'$, i.e. $W \not\leq B'$. Since $\mathbf{W}$ is a refinement of $\mathbf{V} = \mathbf{U} \cup \{B'\}$, $\exists\ U \in \mathbf{U}$ such that $W \leq U$. Thus $W \in \mathbf{W}_0$ and hence $x_\alpha \prec W \in \mathbf{W}_0$. □

A similar theorem holds for α^*-para-Lindelof and *-para-Lindelof spaces also.

3.6. Theorem

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$, $B \in \tau'$. Then
(i) A is α^*-para-Lindelof $\Rightarrow A \wedge B$ is α^*-para-Lindelof.
(ii) A is *-para-Lindelof $\Rightarrow A \wedge B$ is *-para-Lindelof.

3.7. Theorem

Let (X, τ) be a weakly induced L-ts. Then the following conditions are equivalent
(i) (X, τ) is para-Lindelof;
(ii) There exist $\alpha \in M(L)$ such that (X, τ) is α-para-Lindelof;
(iii) $(X, [\tau])$ is para-Lindelof.

Proof. (i) $\Rightarrow$ (ii) is obvious.
(ii) $\Rightarrow$ (iii): Let $\mathbf{U} \subset [\tau]$ be an open cover of X. Now $\mathbf{U}^* = \{\chi_U : U \in \mathbf{U}\}$ is an open α-Q-cover of $\top$ and it has a locally countable refinement $\mathbf{V}$ which is also an α-Q-cover of $\top$.
Let $\mathbf{W} = \{V_{(\alpha')} : V \in \mathbf{V}\}$. Clearly $\mathbf{W}$ is both a refinement of $\mathbf{U}$ and a cover of X. Since (X, τ) is weakly induced, we have $\mathbf{W} \subset [\tau]$. Now we want to prove that $\mathbf{W}$ is locally countable. Let $x \in X$. Since (X, τ) is α-para-Lindelof, there exist $B \in Q(x_\alpha)$ such that B only quasi coincides with a countable number of members $V_0, V_1, V_2, \cdots$ of $\mathbf{V}$. Let $O = B_{(\perp)}$. By the weakly induced property of (X, τ), $O \in [\tau]$. For every $V \in \mathbf{V}$, if $O \cap V_{(\alpha')} \neq \phi$, then there exist an ordinary point $y \in O \cap V_{(\alpha')}$, and hence $B(y) \not\leq \perp$, $V(y) \not\leq \alpha'$. Therefore $V(y)' < \alpha$ and it follows that $B(y) \not\leq V(y)'$ and thus $B \; q \; V$. So $V \in \{V_0, V_1, V_2, \cdots\}$ and O intersects only a countable number of members $V_{0(\alpha')}, V_{1(\alpha')}, V_{2(\alpha')}, \cdots$ of $\mathbf{W}$. Hence $(X, [\tau])$ is para-Lindelof.

(iii) $\Rightarrow$ (i): Suppose that $\alpha \in M(L)$ and $\mathbf{U} \subset \tau$ be an open α-Q-cover of $\top$. Since (X, τ) is weakly induced $\mathbf{U}^* = \{U_{(\alpha')} : U \in \mathbf{U}\}$ is an open cover of $(X, [\tau])$. Since $(X, [\tau])$ is para-Lindelof, there exist a refinement $\mathbf{V}$ of $\mathbf{U}^*$ which is also a locally countable cover of X. For every $V \in \mathbf{V}$, let $U_V \in \mathbf{U}$ such that $V \subset U_{V(\alpha')}$. Let $\mathbf{W} = \{\chi_V \wedge U_V : V \in \mathbf{V}\}$. Now clearly $\mathbf{W}$ is both a refinement of $\mathbf{U}$ and an α-Q-cover of $\top$. Now we will prove that $\mathbf{W}$ is locally countable. Let $x_\alpha \in M(L^X)$. Then since $\mathbf{V}$ is locally countable, there exist a neighbourhood B of x such that B intersects with V_i for countably many $V_i \in \mathbf{V}$. Now we have $\chi_B \in Q(x_\alpha)$. We will show that $\chi_B \; q \; \chi_{V_i} \wedge U_{V_i}$ for at most countably many i. For if possible $\chi_B \; q \; \chi_V \wedge U_V$ for uncountably many $V \in \mathbf{V}$. Then $\chi_B \; q \; \chi_V$ or χ_B

$q\ U_V$ for uncountably many $V \in \mathbf{V}$. In both cases B intersects with V for uncountably many $V \in \mathbf{V}$, which is a contradiction and hence W is locally countable. Therefore (X, τ) is α-para-Lindelof. This completes the proof. □

3.8. Theorem

Let (X, τ) be a weakly induced L-ts. Then the following conditions are equivalent
(i) (X, τ) is *-para-Lindelof;
(ii) There exist $\alpha \in M(L)$ such that (X, τ) is α^*-para-Lindelof;
(iii) $(X, [\tau])$ is para-Lindelof.

Proof. (i) $\Rightarrow$ (ii) is obvious.
(ii) $\Rightarrow$ (iii): Let $\mathbf{U} \subset [\tau]$ be an open cover of X. Now $\mathbf{U}^* = \{\chi_U : U \in \mathbf{U}\}$ is an open α-Q-cover of $\top$ and it has a locally countable refinement $\mathbf{V}$ which is also an α-Q-cover of $\top$.
Take $\mathbf{W} = \{V_{(\alpha')} : V \in \mathbf{V}\}$ then $\mathbf{W}$ is both a refinement of $\mathbf{U}$ and a cover of X. Since (X, τ) is weakly induced, we have $\mathbf{W} \subset [\tau]$. Now we want to prove that $\mathbf{W}$ is locally countable. Let $x \in X$. Since (X, τ) is α^*-para-Lindelof, there exist $B \in Q(x_\alpha)$ such that $\chi_{B(\perp)}$ only quasi coincides with a countable number of members $V_0, V_1, V_2, \cdots$ of $\mathbf{V}$. Then $x \in B_{(\perp)}$. By the weakly induced property of (X, τ), $B_{[\perp]} \in [\tau]$, so $B_{(\perp)}$ is a neighbourhood of x. For every $V \in \mathbf{V}$, if $B_{(\perp)} \cap V_{(\alpha')} \neq \phi$, then there exist an ordinary point $y \in B_{(\perp)} \cap V_{(\alpha')}$, $V(y) \not\leq \alpha'$, $V(y) > \perp$, $V(y)' < \perp$. So $\chi_{B(\perp)}(y) = \top \not\leq V(y)'$, $\chi_{B(\perp)}\, q\ V$, $V \in \{V_0, V_1, V_2, \cdots\}$. Therefore the neighbourhood $B_{(\perp)}$ is of x intersects a countable number of members $V_{0(\alpha')}, V_{1(\alpha')}, V_{2(\alpha')}, \cdots$ of $\mathbf{W}$, thus $\mathbf{W}$ is locally countable in X. Hence $(X, [\tau])$ is para-Lindelof.

(iii) $\Rightarrow$ (i): Suppose that $\alpha \in M(L)$ and $\mathbf{U} \subset \tau$ be an open α-Q-cover of $\top$. Since (X, τ) is weakly induced $\mathbf{U}^* = \{U_{(\alpha')} : U \in \mathbf{U}\}$ is an open cover of $(X, [\tau])$. Since $(X, [\tau])$ is para-Lindelof, there exist a locally countable and open refinement $\mathbf{V}$ of $\mathbf{U}^*$ which is also a cover of X. For every $V \in \mathbf{V}$, let $U_V \in \mathbf{U}$ such that $V \subset U_{V(\alpha')}$. Let $\mathbf{W} = \{\chi_V \wedge U_V : V \in \mathbf{V}\}$. Then $\mathbf{W} \subset \tau$ is clearly a refinement of $\mathbf{U}$ and an α-Q-cover of $\top$. Now we will prove that $\mathbf{W}$ is *-locally countable. Let $x_\alpha \in M(L^X)$ and $B \in Q(x_\alpha)$. If possible let $\chi(\chi_V \wedge U_V)_{(\perp)}\ q\ B$ for uncountably many $V \in \mathbf{V}$. That is $\chi_V \wedge \chi_{UV(\perp)}\ q\ B$ for uncountably many $V \in \mathbf{V}$. And hence $\chi_V\ q\ B$ or $\chi_{UV(\perp)}\ q\ B$ for uncountably many $V \in \mathbf{V}$. In both cases V intersects with the neighbourhood of x for uncountably many $V \in \mathbf{V}$ which is a contradiction that $\mathbf{V}$ is locally countable. Hence W is *-locally countable and this completes the proof. □

3.9. Theorem

Let (X, τ) be an L-ts. Then the following are equivalent
(i) (X, τ) is para-Lindelof;
(ii) For every open α-Q-cover $\mathbf{A}$ of (X, τ), there is a locally countable refinement $\mathbf{B}$ such that if $x_\alpha \in M(L^X)$ then $x_\alpha \in int(st(x_\alpha, \mathbf{B}))$.

Proof. (i) $\Rightarrow$ (ii) is obvious.
(ii) $\Rightarrow$ (i): Suppose $\mathbf{A} = \{A_t : t \in T\}$ is an open α-Q-cover of $\top$. Let $\mathbf{B} = \{B_t : t \in T\}$ be a locally countable refinement as given in (ii). Let $\mathbf{C}$ be an open α-Q-cover of $\top$ such that every element of $\mathbf{C}$ intersects at most countably many elements of $\mathbf{B}$. Then for every $x_\alpha \in M(L^X)$, there is a locally countable refinement $\mathbf{D}$ of $\mathbf{C}$ such that $x_\alpha \in int(st(x_\alpha, \mathbf{D}))$.

For each $B \in \mathbf{B}$, take $A_B \in \mathbf{A}$ such that $B \leq A_B$ and let $G_B = \text{int}(st(B, \mathbf{D})) \wedge A_B$. Then clearly $\mathbf{G} = \{G_B : B \in \mathbf{B}\}$ is an α-Q-cover of $\top$ and hence is an open refinement of $\mathbf{A}$. To show $\mathbf{G}$ is locally countable, let $x_\alpha \in M(L^X)$ and $W \in Q(x_\alpha)$ such that W intersects only countably many elements of $\mathbf{D}$. Now since each $D \in \mathbf{D}$ intersects only countably many elements of $\mathbf{B}$, it follows that W intersects only countably many elements of $\{st(B, \mathbf{D}) : B \in \mathbf{B}\}$. Hence $\mathbf{G}$ is locally countable and the theorem is proved. □

Similar to Theorem 3.9 we can prove the following result:

3.10. Theorem

Let (X, τ) be an L-ts. Then the following are equivalent
(i) (X, τ) is σ-para-Lindelof;
(ii) For any open α-Q-cover $\mathbf{A}$ of (X, τ), there is a σ-locally countable refinement $\mathbf{B} = \cup \mathbf{B}_i$ such that if $x_\alpha \in M(L^X)$ then $x_\alpha \in int(st(x_\alpha, \mathbf{B}_k))$ for some $k \in \mathbf{N}$.

4. Flintily Para-Lindelof Spaces

4.1. Definition

Let (X, τ) be an L-ts. $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. $\mathbf{A}$ is called flintily locally countable at x_λ if there exist $U \in Q(x_\lambda) \cap crs(\tau)$

and a countable subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow A_t \neg q\ U$. And **A** is called flintily locally countable in A, if **A** is flintily locally countable at every molecule $x_\lambda \in M(\downarrow A)$. **A** is called flintily locally countable for short, if **A** is flintily locally countable in $\top$.

4.2. Theorem

In L-ts the following implications hold
Flintily local countable $\Rightarrow$ *-local countable $\Rightarrow$ local countable

4.3. Proposition

Let (X, τ) be an L-ts, $\{A_t : t \in T\} \subseteq L^X$, $x_\lambda \in M(L^X)$. Then
(i) $\{A_t : t \in T\}$ is *-locally countable at $x_\lambda \Rightarrow \{\chi_{(A_t)_{(\perp)}} : t \in T\}$ is *-locally countable at x_λ.
(ii) $\{A_t : t \in T\}$ is flintily locally countable at $x_\lambda \Rightarrow \{\chi_{(A_t)_{(\perp)}} : t \in T\}$ is flintily locally countable at x_λ.

4.4. Theorem

Let (X, τ) be an L-ts, $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$. If **A** is flintily locally countable in A, then $cl\mathbf{A}$ is flintily locally countable in A.

4.5. Remark

Clearly flintily local countability is strictly stronger than *-local countability. But in weakly $\perp$-induced L-ts they are coincident with each other.

4.6. Theorem

Let (X, τ) be a weakly $\perp$-induced L-ts, $A \in L^X$, $\mathbf{A} = \{A_t : t \in T\} \subseteq L^X$. Then **A** is flintily locally countable in A, if and only if **A** is *-locally countable in A.

Proof. By Theorem 4.2, it is enough to prove that *-local countability implies flinty local countability. Suppose **A** is *-local countable in A. Let $x_\lambda \in M(\downarrow A)$. Then there exist $U \in Q(x_\lambda)$ and a countable subset T_0 of T such that $t \in T \setminus T_0 \Rightarrow \chi_{(A_t)_{(\perp)}} \neg q\ U$ is satisfied. Since (X, τ) is weakly $\perp$-induced, $U_{(\perp)} \in [\tau]$. Let $t \in T \setminus T_0$, $y \in A_{t(\perp)}$, then $U'(y) \geq \chi_{(A_t)_{(\perp)}}(y) = \top$. So $y \in U'_{[\top]} = X \setminus U_{(\perp)}$ and hence $(\chi_{U_{(\perp)}})'(y) = \top = \chi_{(A_t)_{(\perp)}}(y)$.

That is to say $\chi_{(A_t)_{(\perp)}} \leq (\chi_{U_{(\perp)}})'$, $\chi_{(A_t)_{(\perp)}} \neg\, q(\chi_{U_{(\perp)}})$. Since $\chi_{U_{(\perp)}} \in \tau$, $\chi_{U_{(\perp)}} \in Q(x_\lambda) \cap crs(\tau)$. Hence $\mathbf{A}$ is flintily locally countable. □

4.7. Definition

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$. A is called flintily α-para-Lindelof if for every open α-Q-cover Φ of A, there exist an open refinement Ψ of Φ which is flintily locally countable in A and Ψ is also an α-Q-cover of A. A is called flintily para-Lindelof if A is flintily α-para-Lindelof for every $\alpha \in M(L)$. And (X, τ) is flintily para-Lindelof if $\top$ is flintily para-Lindelof.

By Theorem 4.2, the following implications hold:

4.8. Theorem

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$, then
(i) A is flintily α-para-Lindelof $\Rightarrow$ A is α^*-para-Lindelof $\Rightarrow$ is α-para-Lindelof.
(ii) A is flintily para-Lindelof $\Rightarrow$ A is *-para-Lindelof $\Rightarrow$ A is para-Lindelof.

Similar to Theorem 3.5 we can prove that flintily para-Lindelofness is hereditary with respect to closed subsets.

4.9. Theorem

Let (X, τ) be an L-ts, $A \in L^X$, $\alpha \in M(L)$, $B \in \tau'$. Then
(i) A is flintily α-para-Lindelof $\Rightarrow$ $A \wedge B$ is flintily α-para-Lindelof.
(ii) A is flintily para-Lindelof $\Rightarrow$ $A \wedge B$ is flintily para-Lindelof.

4.10. Theorem

In a weakly induced L-ts (X, τ), the following are equivalent
(i) (X, τ) is flintily para-Lindelof.
(ii) There exist $\alpha \in M(L)$ such that (X, τ) is flintily α-para-Lindelof;
(iii) $(X, [\tau])$ is para-Lindelof.

Proof. (i) $\Rightarrow$ (ii) is obvious.
(ii) $\Rightarrow$ (iii): Let $\mathbf{U}$ be an open cover of $(X, [\tau])$. Then $\Phi = \{\chi_U : U \in \mathbf{U}\}$ is an open α-Q-cover of $\top$ and by (ii) it has an open and flintily locally countable refinement $\Psi = \{A_t : t \in T\}$ such that Ψ is an α-Q-cover of $\top$. For every $t \in T$, take $V_t = A_{t(\alpha')}$ and $\mathbf{V} = \{V_t : t \in T\}$. Then by the weakly induced property of (X, τ), $\mathbf{V}$ is an open cover of $(X, [\tau])$. Now we will prove $\mathbf{V}$ is a locally countable refinement of $\mathbf{U}$. Let $V_t \in \mathbf{V}$. Since

is Ψ a refinement of Φ, there exist $U \in \mathbf{U}$ such that $A_t \leq \chi_U$. Suppose $x \in V_t$, then $A_t(x) \not\leq \alpha'$, so $\chi_U(x) \neq \perp$, $x \in U$, $V_t \subset U$. Therefore $\mathbf{V}$ is a refinement of $\mathbf{U}$.

Let $x \in X$. Since Ψ is flintily locally countable, there exist $B \in Q(x_\alpha) \cap crs(\tau)$ such that $A_t\, q\, B$ for only a countable number of members A_ts in ψ. Since $B \in Q(x_\alpha)$ is crisp, $B_{(\perp)}$ is the neighbourhood of x. For every $t \in T$ if $A_t \neg q\, B$, then $V_t \cap B_{(\perp)} = \Phi$. So $B_{(\perp)}$ intersects with only a countable members of $\mathbf{V}$, thus $\mathbf{V}$ is locally countable. Hence $(X, [\tau])$ is paraLindelof.
(iii) $\Rightarrow$ (i) suppose $\alpha \in M(L)$, $\mathbf{A} = \{A_t : t \in T\}$ is an open α-Q-cover of $\top$. For every $t \in T$ take $U_t = A_{t(\alpha')}$ and $\mathbf{U} = \{U_t : t \in T\}$. Since $\mathbf{A}$ is an open α-Q-cover of $\top$ and (X, τ) is weakly induced, $\mathbf{U}$ is an open cover of $(X, [\tau])$. Therefore by (iii), there exists an open and locally countable refinement $\mathbf{V} = \{V_s : s \in S\}$ of $\mathbf{U}$ which is also a cover of $(X, [\tau])$. For every $s \in S$ take $t(s) \in T$ such that $V_s \subset U_{t(s)}$, let $W_s = A_{t(s)} \wedge \chi\gamma_s$ then Ws is an open L-set and $Ws \leq A_{t(s)}$ for every $s \in S$. Therefore $\mathbf{W} = \{Ws : s \in S\}$ is an open refinement of $\mathbf{A}$. Now we will show that $\mathbf{W}$ is an open α-Q-cover of $\top$. Let $x_\alpha \in M(L^X)$ take $s \in S$ such that $x \in V_s$ and hence $x \in U_{t(s)}$. So $A_{t(s)}(x) \not\leq \alpha'$, $\alpha \not\leq A_{t(s)}(x)'$. Since $x \in V_s$, $\chi_{Vs} \in Q(x_\alpha)$, we have $Ws = A_{t(s)} \wedge \chi_{Vs} \in Q(x_\alpha)$. Hence $\mathbf{W}$ is an open α-Q-cover of $\top$.

Suppose $x_\alpha \in M(L^X)$, then since $\mathbf{V}$ being locally countable in $(X, [\tau])$, there exist a neighbourhood B of x in $(X, [\tau])$ such that B intersects with only countably many members of $\mathbf{V}$ say $V_{s_0}, V_{s_1}, V_{s_2}, \cdots$. Then for every $s \in S \setminus \{s_0, s_1, s_2 \ldots\}$, $Vs \cap B = \Phi$, $B \subset Vs'$ and thus $\chi_B \leq \chi_{V's} \leq A'_{t(s)} \vee \chi_{V's} = Ws'$. That is $\chi_B \neg\, q\, Ws$. Hence $\mathbf{W}$ is flintily locally countable. This completes the proof. $\Box$

5. Invariant Theorems

In this section we study the behaviour of para-Lindelof spaces under various types of fuzzy mappings.

5.1. Definition

[17] Let (X, τ), (Y, μ) be L-topological spaces, $f : X \to Y$ be an ordinary mapping. Based on this we define the L-fuzzy mapping $f^{\to} : L^X \to L^Y$ and its L-fuzzy reverse mapping $f^{\leftarrow} : L^Y \to L^X$ by
$f^{\to} : L^X \to L^Y$, $f^{\to}(A)(y) = \vee\{A(x) : x \in X, f(x) = y\} \forall A \in L^X, \forall y \in Y$.
$f^{\leftarrow} : L^Y \to L^X$, $f^{\leftarrow}(B)(x) = B(f(x)), \forall B \in L^Y, \forall x \in X$.

5.2. Definition

[17] Let (X, τ), (Y, μ) be L-topological spaces,
$f^{\rightarrow} : L^X \rightarrow L^Y$ an L-fuzzy mapping. We say $f^{\rightarrow}$ is an L-fuzzy continuous mapping from (X, τ) to (Y, μ) if its L-fuzzy reverse mapping $f^{\leftarrow} : L^Y \rightarrow L^X$ maps every open subset in (Y, μ) as an open one in (X, τ). i.e., $\forall V \in \mu$, $f^{\leftarrow}(V) \in \tau$.

5.3. Definition

[17] Let (X, τ), (Y, μ) be L-topological spaces,
$f^{\rightarrow} : L^X \rightarrow L^Y$ an L-fuzzy mapping. We say $f^{\rightarrow}$ is open if it maps every open subset in (X, τ) as an open one in (Y, μ). i.e., $\forall U \in \tau$, $f^{\rightarrow}(U) \in \mu$.

5.4. Definition

[17] Let (X, τ), (Y, μ) be L-topological spaces,
$f^{\rightarrow} : L^X \rightarrow L^Y$ an L-fuzzy mapping. We say $f^{\rightarrow}$ is closed if it maps every closed subset in (X, τ) as an closed one in (Y, μ). i.e., $\forall F \in \tau'$, $f^{\rightarrow}(F) \in \mu'$.

5.5. Definition

[1] Let (X, τ), (Y, μ) be L-ts's, $f^{\rightarrow} : L^X \rightarrow L^Y$ an L-fuzzy mapping. Then $f^{\rightarrow}$ is perfect if it is continuous, closed and $f^{\leftarrow}(y)$ is N-compact for every $y \in Y$.

5.6. Result

[17] If (X, τ), (Y, μ) are two weakly induced L-topological spaces, then
(i) If the map $f^{\rightarrow} : L^X \rightarrow L^Y$ is L-fuzzy continuous, then $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is continuous;
(ii) If the map $f^{\rightarrow} : L^X \rightarrow L^Y$ is L-fuzzy closed, then $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is closed;
(iii) If the map $f^{\rightarrow} : L^X \rightarrow L^Y$ is L-fuzzy open, then $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is open.

5.7. Theorem

Let (X, τ), (Y, μ) are two weakly induced L-topological spaces. Then if $f^{\rightarrow} : L^X \rightarrow L^Y$ is perfect, then so is $f : (X, [\tau]) \rightarrow (Y, [\mu])$.

Proof. Let $y_\alpha \in M(L^Y)$. Since $f^{\rightarrow} : L^X \rightarrow L^Y$ is perfect, $f^{\leftarrow}(y_\alpha)$ is N-compact. Now to prove $f : (X, [\tau]) \rightarrow (Y, [\mu])$ is perfect, it is enough to prove that $f^{\leftarrow}(y_\alpha)$ is compact for every $y \in Y$. Now let $\mathbf{U} \in [\tau]$ be an open cover of $f^{-1}(y)$. Consider $\mathbf{U}* = \{\chi_U : U \in \mathbf{U}\}$. This is an open α-Q-cover of $f^{\leftarrow}(y_\alpha)$. For, let $x_\alpha \leq f^{\leftarrow}(y_\alpha)$. i.e., $f^{\leftarrow}(y_\alpha)(x) = y_\alpha(f(x)) \geq \alpha$. Now let $U \in \mathbf{U}$ be such that $x \in U$. This is possible since U is a cover of $f^{-1}(y)$. Then $\chi_U(x) \geq y_\alpha \geq \alpha$. i.e., $\chi_U(x) \geq \alpha$ or $x_\alpha \leq \chi_U$. Hence clearly $x_\alpha \, q \, \chi_U$. Hence $\{\chi_U : U \in \mathbf{U}\}$ is an open α-Q-cover of $f^{\leftarrow}(y_\alpha)$.
Again $f^{\leftarrow}(y_\alpha)$ being N-compact, there exists a finite sub collection $\mathbf{U}^*_s$ of $\mathbf{U}^*$ which is also an α^--Q cover of $f^{\leftarrow}(y_\alpha)$. Let $\mathbf{U}^*_s = \{\chi_{U1}, \chi_{U2}, \cdots, \chi_{Uk}\}$. Then clearly $\{U_1, U_2, \cdots, U_k\}$ will be a finite sub cover of $f^{-1}(y)$. This completes the proof. □

5.8. Theorem

(X, τ), (Y, μ) are two weakly induced L-tss. If (X, τ) is para-Lindelof and $f^{\rightarrow} : L^X \rightarrow L^Y$ be a closed map with $f^{\leftarrow}(y_\alpha)$ Lindelof for each $y_\alpha \in M(L^Y)$, then (Y, μ) is para-Lindelof.

Proof. Let $\mathbf{U}$ be an open α-Q-cover of Y and let $\mathbf{W} = \{W_t : t \in T\}$ be a locally countable open α-Q-cover refinement of $\{f^{\leftarrow}(U) : U \in \mathbf{U}\}$. Now for any $y_\alpha \in M(L^Y)$, $f^{\leftarrow}(y_\alpha)$ is Lindelof so there is an open set $G_{y\alpha}$ in L^X such that $f^{\leftarrow}(y_\alpha) \leq G_{y\alpha}$ and $G_{y\alpha} \leq W_t$ for countably many $t \in T$. Take $V_{y\alpha}$ as the saturated part of $G_{y\alpha}$. Then $f^{\rightarrow}(V_{y\alpha})$ is an open set about y_α. Consider $\mathbf{H} = \{f^{\rightarrow}(W_t) : W_t \in \mathbf{W}\}$. Now $f^{\rightarrow}(V_{y\alpha})$ meeting only countably many elements of $\mathbf{H}$. Hence $\mathbf{H}$ is locally countable and it is clear that $y_\alpha \in int(st(y_\alpha, \mathbf{H}))$ for every $y_\alpha \in L^Y$. Since $\mathbf{H}$ is a refinement of $\mathbf{U}$, it follows from Theorem 3.9 that (Y, μ) is para-Lindelof. □

Now by Theorem 5.7 we readily have

5.9. Theorem

(X, τ), (Y, μ) are two weakly induced L-tss and $f^{\rightarrow} : L^X \rightarrow L^Y$ be a perfect map. Then (X, μ) is para-Lindelof if and only if (Y, μ) is para-Lindelof.

A similar result we can obtain for flintily para-Lindelof space also:

5.10. Theorem

(X, τ), (Y, μ) are two weakly induced L-tss and $f^{\rightarrow} : L^X \rightarrow L^Y$ be a perfect map. Then (X, μ) is flintily para-Lindelof if and only if (Y, μ) is flintily para-Lindelof.

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