Proyecciones Journal of Mathematics
Vol. 35, N^o 4, pp. 437-455, December 2016.
Universidad Católica del Norte
Antofagasta - Chile
DOI: 10.4067/S0716-09172016000400006

Sum divisor cordial labeling for star and ladder related graphs

A. Lourdusamy
St. Xavier's College (autonomous), India
and
F. Patrick
St. Xavier's College (autonomous), India
Received : April 2016. Accepted : October 2016

Abstract

A sum divisor cordial labeling of a graph G with vertex set V is a bijection f from V to $\{1, 2, \cdots, |V(G)|\}$ such that an edge uv is assigned the label 1 if 2 divides $f(u) + f(v)$ and 0 otherwise; and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a sum divisor cordial labeling is called a sum divisor cordial graph. In this paper, we prove that $D_2(K_{1,n})$, $S^{'}(K_{1,n})$, $D_2(B_{n,n})$, $DS(B_{n,n})$, $S^{'}(B_{n,n})$, $S(B_{n,n})$, $< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >$, $S(L_n)$, $L_n \odot K_1$, SL_n, TL_n, $TL_n \odot K_1$ and CH_n are sum divisor cordial graphs.

AMS Subject Classification 2010 : *05C78.*

Keywords : *Divisor cordial, sum divisor cordial.*

1. Introduction

All graphs considered here are simple, finite, connected and undirected. For all other standard terminology and notations we follow Harary [2]. A labeling of a graph is a map that carries the graph elements to the set of numbers, usually to the set of non-negative or positive integers. If the domain is the set of vertices the labeling is called vertex labeling. If the domain is the set of edges then the labeling is called edge labeling. If the labels are assigned to both vertices and edges then the labeling is called total labeling. For all detailed survey of graph labeling we refer Gallian [1]. A. Lourdusamy and F. Patrick introduced the concept of sum divisor cordial labeling in [3]. In [3, 4], sum divisor cordial labeling behaviour of several graphs like path, complete bipartite graph, bistar and some standard graphs have been investigated. In this paper, we investigate the sum divisor cordial labeling behavior of $D_2(K_{1,n})$, $S^{'}(K_{1,n})$, $D_2(B_{n,n})$, $DS(B_{n,n})$, $S^{'}(B_{n,n})$, $S(B_{n,n})$, $< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >$, $S(L_n)$, $L_n \odot K_1$, SL_n, TL_n, $TL_n \odot K_1$ and CH_n.

Definition 1.1. *Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \to \{1, 2, \cdots, |V(G)|\}$ be a bijection. For each edge uv, assign the label 1 if $2|(f(u) + f(v))$ and the label 0 otherwise. The function f is called a sum divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits a sum divisor cordial labeling is called a sum divisor cordial graph.*

Definition 1.2. *[9] For every vertex $v \in V(G)$, the open neighbourhood set $N(v)$ is the set of all vertices adjacent to v in G.*

Definition 1.3. *[8] For a graph G the splitting graph $S^{'}(G)$ of a graph G is obtained by adding a new vertex $v^{'}$ corresponding to each vertex v of G such that $N(v) = N(v^{'})$.*

Definition 1.4. *[8] The shadow graph $D_2(G)$ of a connected graph G is obtained by taking two copies of G, say $G^{'}$ and $G^{''}$. Join each vertex $u^{'}$ in $G^{'}$ to the neighbours of corresponding vertex $u^{''}$ in $G^{''}$.*

Definition 1.5. *[8] Duplication of a vertex v_k by a new edge $e = v_k^{'}v_k^{''}$ in a graph G produces a new graph $G^{'}$ such that $N(v_k^{'}) \cap N(v_k^{''}) = v_k$.*

Definition 1.6. *[8] Let G be the a graph with $V = S_1 \cup S_2 \cup S_3 \cup \cdots S_i \cup T$ where each S_i is a set of vertices having at least two vertices of the same degree and $T = V(G) \setminus \cup_{i=1}^{t} S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, ..., w_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.*

Definition 1.7. *[6] The subdivision graph $S(G)$ is obtained from G by subdividing each edge of G with a vertex.*

Definition 1.8. *[7] Consider two copies of graph G(wheel, star, fan and friendship) namely G_1 and G_2. Then the graph $G^{'} =< G_1 \Delta G_2 >$ is the graph obtained by joining the apex vertices of G_1 and G_2 by an edge as well as to a new vertex $v^{'}$.*

Definition 1.9. *[6] The slanting ladder SL_n is a graph obtained from two paths $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ by joining each u_i with v_{i+1} for $(1 \leq i \leq n-1)$.*

Definition 1.10. *[5] The triangular ladder TL_n is a graph obtained from L_n by adding the edges $u_i v_{i+1}, 1 \leq i \leq n-1$, where $u_i$and $v_i, 1 \leq i \leq n$are the vertices of L_n such that $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ are two paths of length n in the graph L_n.*

Definition 1.11. *[6] The corona $G_1 \odot G_2$ of two graphs $G_1(p_1, q_1)$ and $G_2(p_2, q_2)$ is defined as the graph obtained by taking one copy of G_1 and p_1 copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2.*

Definition 1.12. *[7] The closed helm CH_n is the graph obtained from helm H_n by joining each pendant vertex to form a cycle.*

2. Main results

Theorem 2.1. *The graph $S^{'}(K_{1,n})$ is sum divisor cordial graph.*

Proof. Let $v_1, v_2, \cdots, v_n$ be the pendant vertices and v be the apex vertex of $K_{1,n}$ and $u, u_1, u_2, \cdots, u_n$ are added vertices corresponding to $v, v_1, v_2, \cdots, v_n$ to obtain $S^{'}(K_{1,n})$. Let $G = S^{'}(K_{1,n})$. Then, G is of order $2n+2$ and size $3n$. Define $f : V(G) \to \{1, 2, \cdots, 2n+2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$
$$f(v_i) = \{ 2\,i + 1 \text{if } i \text{ is odd}$$
$2i + 2$if i is even
$$f(u_i) = \{ 2\,i + 2 \text{if } i \text{ is odd}$$
$2i + 1$if i is even

Then, the induced edge labels are

$f^*(vu_i) = \{ 1$ if i is odd

0if i is even

$f^*(vv_i) = \{ 0$ if i is odd

1if i is even

$f^*(uv_i) = \{ 1$ if i is odd

0if i is even

We observe that, $e_f(0) = \left\lfloor \frac{3n}{2} \right\rfloor$ and $e_f(1) = \left\lceil \frac{3n}{2} \right\rceil$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, $S^{'}(K_{1,n})$ is sum divisor cordial graph. □

Example 2.2. *A sum divisor cordial labeling of* $S^{'}(K_{1,5})$ *is shown in Figure* 2.1

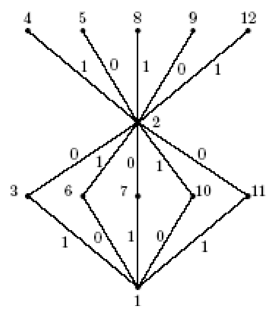

Figure 2.1

Theorem 2.3. *The graph* $D_2(K_{1,n})$ *is sum divisor cordial graph.*

Proof. Let $u, u_1, u_2, \cdots, u_n$ and $v, v_1, v_2, \cdots, v_n$ be the vertices of two copies of $K_{1,n}$. Let $G = D_2(K_{1,n})$. Then
$V(G) = \{u, v, u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n\}$ and $E(G) = \{uu_i, vv_i, uv_i, vu_i : 1 \leq i \leq n\}$. Also, G is of order $2n + 2$ and size $4n$. Define $f : V(G) \to \{1, 2, \cdots, 2n + 2\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 2;$$

$$f(u_i) = \begin{cases} 2i+1 & \text{if } i \text{ is odd} \\ 2i+2 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 2i+2 & \text{if } i \text{ is odd} \\ 2i+1 & \text{if } i \text{ is even} \end{cases}$$

Then, the induced edge labels are

$$f^*(uu_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$$

$$f^*(vu_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f^*(vv_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$$

$$f^*(uv_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n$.

Thus,$|e_f(1) - e_f(0)| \leq 1$.

Hence, $D_2(K_{1,n})$ is sum divisor cordial graph. □

Example 2.4. *A sum divisor cordial labeling of $D_2(K_{1,4})$ is shown in Figure* 2.2

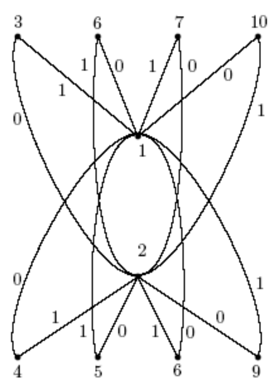

Figure 2.2

Theorem 2.5. *The graph $S'(B_{n,n})$ is sum divisor cordial graph.*

Proof. Let $u, v, u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n$ be the vertices of $B_{n,n}$. Let $u^{'}, v^{'}, u_i^{'}, v_i^{'}$ are added vertices corresponding to u, v, u_i, v_i to obtain $S^{'}(B_{n,n})$. Let $G = S^{'}(B_{n,n})$ Then, G is of order $4n+4$ and size $6n+3$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 4n+4\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 3;$$
$$f(u^{'}) = 2;$$
$$f(v^{'}) = 4;$$
$$f(u_i) = 4i+2; 1 \leq i \leq n$$
$$f(u_i^{'}) = 4i+1; 1 \leq i \leq n$$
$$f(v_i) = 4i+3; 1 \leq i \leq n$$
$$f(v_i^{'}) = 4i+4; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(uv) = 1;$$
$$f^*(uv^{'}) = 0;$$
$$f^*(vu^{'}) = 0;$$
$$f^*(u^{'}u_i) = 1; 1 \leq i \leq n$$
$$f^*(uu_i) = 0; 1 \leq i \leq n$$
$$f^*(uu_i^{'}) = 1; 1 \leq i \leq n$$
$$f^*(v^{'}v_i) = 0; 1 \leq i \leq n$$
$$f^*(vv_i) = 1; 1 \leq i \leq n$$
$$f^*(vv_i^{'}) = 0; 1 \leq i \leq n$$

We observe that, $e_f(0) = 3n+2$ and $e_f(1) = 3n+1$.
Thus,$|e_f(1) - e_f(0)| \leq 1$.
Hence, $S^{'}(B_{n,n})$ is sum divisor cordial graph. □

Example 2.6. *A sum divisor cordial labeling of $S^{'}(B_{5,5})$ is shown in Figure* 2.3

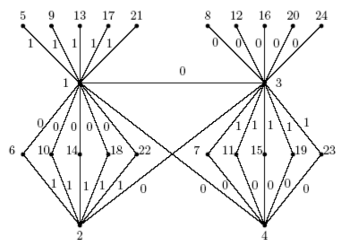

Figure 2.3

Theorem 2.7. *The graph $D_2(B_{n,n})$ is sum divisor cordial graph.*

Proof. Let $u, v, u_1, u_2, \cdots, u_n, v_1, v_2, \cdots, v_n$ be the vertices of $B_{n,n}$. Let $G = D_2(B_{n,n})$. Then $V(G) = \{u, v, u', v'\} \bigcup \{u_i, v_i, u_i', v_i' : 1 \leq i \leq n\}$ and $E(G) = \{uv, u'v, uv', u'v'\} \bigcup \{uu_i, uu_i', u'u_i, u'u_i', vv_i, vv_i', v'v_i, v'v_i' : 1 \leq i \leq n\}$. Also, G is of order $4n + 4$ and size $8n + 4$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 4n + 4\}$ as follows:

$$f(u) = 1;$$
$$f(v) = 3;$$
$$f(u') = 2;$$
$$f(v') = 4;$$
$$f(u_i) = 4i + 1; 1 \leq i \leq n$$
$$f(u_i') = 4i + 2; 1 \leq i \leq n$$
$$f(v_i) = 4i + 4; 1 \leq i \leq n$$
$$f(v_i') = 4i + 3; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(uv) = 1;$$
$$f^*(uv') = 0;$$
$$f^*(vu') = 0;$$
$$f^*(v'u') = 1;$$
$$f^*(uu_i) = 1; 1 \leq i \leq n$$
$$f^*(u'u_i) = 0; 1 \leq i \leq n$$
$$f^*(u'u_i') = 1; 1 \leq i \leq n$$

$$f^*(uu_i^{'}) = 0; 1 \leq i \leq n$$
$$f^*(vv_i) = 0; 1 \leq i \leq n$$
$$f^*(v^{'}v_i) = 1; 1 \leq i \leq n$$
$$f^*(v^{'}v_i^{'}) = 0; 1 \leq i \leq n$$
$$f^*(vv_i^{'}) = 1; 1 \leq i \leq n$$

We observe that, $e_f(0) = 4n + 2$ and $e_f(1) = 4n + 2$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, $D_2(B_{n,n})$ is sum divisor cordial graph. □

Example 2.8. *A sum divisor cordial labeling of* $D_2(B_{4,4})$ *is shown in Figure* 2.4

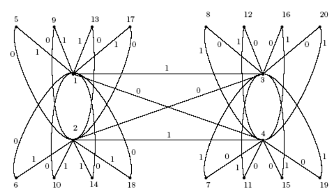

Figure 2.4

Theorem 2.9. *The graph* $DS(B_{n,n})$ *is sum divisor cordial graph.*

Proof. Let $G = DS(B_{n,n})$. Let $V(G) = \{u, v, w_1, w_2\} \bigcup \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{uv, uw_2, vw_2\} \bigcup \{uu_i, vv_i, u_iw_1, v_iw_1 : 1 \leq i \leq n\}$.Then, G is of order $2n + 4$ and size $4n + 3$. Define $f : V(G) \to \{1, 2, \cdots, 2n + 4\}$ as follows:

$$f(u) = 4;$$
$$f(v) = 2;$$
$$f(w_1) = 1;$$
$$f(w_2) = 3;$$
$$f(u_i) = 2i + 4; 1 \leq i \leq n$$
$$f(v_i) = 2i + 3; 1 \leq i \leq n$$

Then, the induced edge labels are

$f^*(uv) = 1;$
$f^*(uw_2) = 0;$
$f^*(vw_2) = 0;$
$f^*(uu_i) = 1; 1 \leq i \leq n$
$f^*(u_i w_1) = 0; 1 \leq i \leq n$
$f^*(vv_i) = 0; 1 \leq i \leq n$
$f^*(v_i w_1) = 1; 1 \leq i \leq n$

We observe that, $e_f(0) = 2n + 2$ and $e_f(1) = 2n + 1$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, $DS(B_{n,n})$ is sum divisor cordial graph. □

Example 2.10. *A sum divisor cordial labeling of* $DS(B_{3,3})$ *is shown in Figure* 2.5

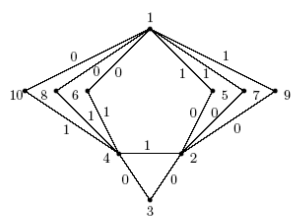

Figure 2.5

Theorem 2.11. *The graph* $S(B_{n,n})$ *is sum divisor cordial graph.*

Proof. Let $G = S(B_{n,n})$. Let $V(G) = \{u, v, w\} \bigcup \{u_i, v_i, u_i^{'}, v_i^{'} : 1 \leq i \leq n\}$ and $E(G) = \{uw, wv\} \bigcup \{uu_i^{'}, u_i^{'}u_i, vv_i^{'}, v_i^{'}v_i : 1 \leq i \leq n\}$. Then, G is of order $4n + 3$ and size $4n + 2$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 4n + 3\}$ as follows:

$f(u) = 1;$
$f(v) = 2;$
$f(w) = 3;$
$f(u_i) = 4i; 1 \leq i \leq n$
$f(u_i^{'}) = 4i + 1; 1 \leq i \leq n$

$$f(v_i) = 4i + 3; 1 \leq i \leq n$$
$$f(v_i^{'}) = 4i + 2; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(uw) = 1;$$
$$f^*(wv) = 0;$$
$$f^*(uu_i^{'}) = 1; 1 \leq i \leq n$$
$$f^*(u_i^{'}u_i) = 0; 1 \leq i \leq n$$
$$f^*(vv_i^{'}) = 1; 1 \leq i \leq n$$
$$f^*(v_i^{'}v_i) = 0; 1 \leq i \leq n$$

We observe that, $e_f(0) = 2n + 1$ and $e_f(1) = 2n + 1$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, $S(B_{n,n})$ is sum divisor cordial graph. $\square$

Example 2.12. *A sum divisor cordial labeling of* $S(B_{3,3})$ *is shown in Figure* 2.6

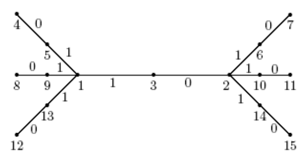

Figure 2.6

Theorem 2.13. *The graph* $< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >$ *is sum divisor cordial graph.*

Proof. Let $v_1^{(1)}, v_2^{(1)}, \cdots, v_n^{(1)}$ be the pendant vertices of $K_{1,n}^{(1)}$ and $v_1^{(2)}, v_2^{(2)}, \cdots, v_n^{(2)}$ be the pendant vertices of $K_{1,n}^{(2)}$. Let u and v be the apex vertices of $K_{1,n}^{(1)}$ and $K_{1,n}^{(2)}$ respectively and u, v are adjacent to a new common vertex x. Let $G =< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >$. Then, G is of order $2n + 3$ and size $2n + 3$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 2n + 3\}$ as follows:

$$f(x) = 2;$$

$f(u) = 1;$
$f(v) = 3;$
$f(u_{2i-1}) = 4i; 1 \leq i \leq \lceil \frac{n}{2} \rceil$
$f(u_{2i}) = 4i + 3; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$
$f(v_{2i-1}) = 4i + 1; 1 \leq i \leq \lceil \frac{n}{2} \rceil$
$f(v_{2i}) = 4i + 2; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

Then, the induced edge labels are

$f^*(ux) = 0;$
$f^*(vx) = 0;$
$f^*(uv) = 1;$
$f^*(uu_{2i-1}) = 0; 1 \leq i \leq \lceil \frac{n}{2} \rceil$
$f^*(uu_{2i}) = 1; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$
$f^*(vv_{2i-1}) = 1; 1 \leq i \leq \lceil \frac{n}{2} \rceil$
$f^*(vv_{2i}) = 0; 1 \leq i \leq \lfloor \frac{n}{2} \rfloor$

We observe that, $e_f(0) = n + 2$ and $e_f(1) = n + 1$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, $< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >$ is sum divisor cordial graph. □

Example 2.14. *A sum divisor cordial labeling of* $< K_{1,5}^{(1)} \Delta K_{1,5}^{(2)} >$ *is shown in Figure* 2.7

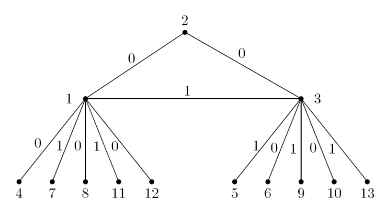

Figure 2.7

Theorem 2.15. *The graph* $L_n \odot K_1$ *is sum divisor cordial graph.*

Proof. Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of L_n. Let x_i be the vertex which is attached to u_i and y_i be the vertex which is attached to v_i. Let $G = L_n \odot K_1$. Then, G is of order $4n$ and size $5n-2$. Define $f : V(G) \to \{1, 2, \cdots, 4n\}$ as follows:

$$f(u_{2i-1}) = 8i-5; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f(u_{2i}) = 8i-3; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f(x_{2i-1}) = 8i-7; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f(x_{2i}) = 8i-2; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f(v_{2i-1}) = 8i-6; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f(v_{2i}) = 8i-1; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f(y_{2i-1}) = 8i-4; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f(y_{2i}) = 8i; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$

Then, the induced edge labels are

$$f^*(u_{2i-1}x_{2i-1}) = 1; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f^*(u_{2i}x_{2i}) = 0; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(u_iu_{i+1}) = 1; 1 \le i \le n-1$$
$$f^*(v_iv_{i+1}) = 0; 1 \le i \le n-1$$
$$f^*(u_{2i-1}v_{2i-1}) = 0; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f^*(u_{2i}v_{2i}) = 1; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$
$$f^*(v_{2i-1}y_{2i-1}) = 1; 1 \le i \le \lceil \tfrac{n}{2} \rceil$$
$$f^*(v_{2i}y_{2i}) = 0; 1 \le i \le \lfloor \tfrac{n}{2} \rfloor$$

We observe that, $e_f(0) = \left\lfloor \frac{5n-2}{2} \right\rfloor$ and $e_f(1) = \left\lceil \frac{5n-2}{2} \right\rceil$.
Thus, $|e_f(1) - e_f(0)| \le 1$.
Hence, $L_n \odot K_1$ is sum divisor cordial graph. □

Example 2.16. *A sum divisor cordial labeling of $L_6 \odot K_1$ is shown in Figure* 2.8

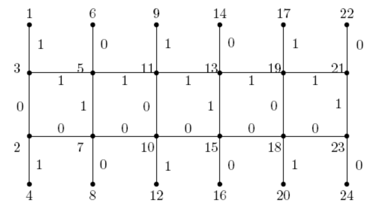

Figure 2.8

Theorem 2.17. *The graph $S(L_n)$ is sum divisor cordial graph.*

Proof. Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of L_n. Let $v_i^{'}$ be the newly added vertex between v_i and v_{i+1}. Let $u_i^{'}$ be the newly added vertex between u_i and u_{i+1}. Let w_i be the newly added vertex between v_i and u_i. Let $G = S(L_n)$. Then, G is of order $5n-2$ and size $6n-4$. Define $f : V(G) \to \{1, 2, \cdots, 5n-2\}$ as follows:

$$f(v_1) = 1;$$
$$f(v_{i+1}) = 5i; 1 \leq i \leq n-1$$
$$f(u_1) = 2;$$
$$f(u_{i+1}) = 5i+3; 1 \leq i \leq n-1$$
$$f(w_1) = 3;$$
$$f(w_{i+1}) = 5i+2; 1 \leq i \leq n-1$$
$$f(v_i^{'}) = 5i-1; 1 \leq i \leq n-1$$
$$f(u_i^{'}) = 5i+1; 1 \leq i \leq n-1$$

Then, the induced edge labels are

$$f^*(v_1v_1^{'}) = 0;$$
$$f^*(v_iv_i^{'}) = 1; 2 \leq i \leq n-1$$
$$f^*(v_i^{'}v_{i+1}) = 0; 1 \leq i \leq n-1$$
$$f^*(u_1u_1^{'}) = 1;$$
$$f^*(u_iu_i^{'}) = 0; 2 \leq i \leq n-1$$
$$f^*(u_i^{'}u_{i+1}) = 1; 1 \leq i \leq n-1$$
$$f^*(v_iw_i) = 1; 1 \leq i \leq n$$

$$f^*(u_iw_i) = 0; 1 \leq i \leq n$$

We observe that, $e_f(0) = 3n - 2$ and $e_f(1) = 3n - 2$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, $S(L_n)$ is sum divisor cordial graph. □

Example 2.18. *A sum divisor cordial labeling of $S(L_5)$ is shown in Figure 2.9*

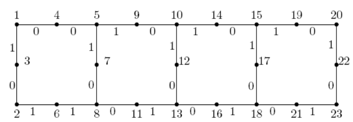

Figure 2.9

Theorem 2.19. *The slanting ladder SL_n is sum divisor cordial graph.*

Proof. Let $G = SL_n$. Let $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{v_iv_{i+1}, u_iu_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\}$. Then, G is of order $2n$ and size $3n - 3$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(u_i) = \{\, 2\,i\text{if } i \equiv 1, 0(mod\ 4)$$

$2i - 1\text{if } i \equiv 2, 3(mod\ 4)$

$$f(v_i) = \{\, 2\,i - 1\text{if } i \equiv 1, 0(mod\ 4)$$

$2i\text{if } i \equiv 2, 3(mod\ 4)$

Then, the induced edge labels are

$$f^*(u_iu_{i+1}) = \{\, 0 \text{ if } i \text{ is odd}$$

$1\text{if } i$ is even

$$f^*(v_iv_{i+1}) = \{\, 0 \text{ if } i \text{ is odd}$$

$1\text{if } i$ is even

$$f^*(u_iv_{i+1}) = \{\, 1 \text{ if } i \text{ is odd}$$

$0\text{if } i$ is even

We observe that, $e_f(0) = \left\lceil \frac{3n-3}{2} \right\rceil$ and $e_f(1) = \left\lfloor \frac{3n-3}{2} \right\rfloor$.

Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, SL_n is sum divisor cordial graph. □

Example 2.20. *A sum divisor cordial labeling of SL_5 is shown in Figure 2.10*

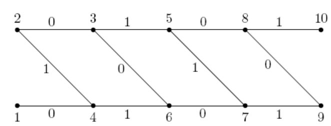

Figure 2.10

Theorem 2.21. *The triangular ladder TL_n is sum divisor cordial graph.*

Proof. Let $G = TL_n$. Let $V(G) = \{u_i, v_i : 1 \leq i \leq n\}$ and $E(G) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_iv_i : 1 \leq i \leq n\}$. Then, G is of order $2n$ and size $4n-3$. Define $f : V(G) \to \{1, 2, \cdots, 2n\}$ as follows:

$$f(u_i) = 2i - 1; 1 \leq i \leq n$$
$$f(v_i) = 2i; 1 \leq i \leq n$$

Then, the induced edge labels are

$$f^*(u_iu_{i+1}) = 1; 1 \leq i \leq n-1$$
$$f^*(u_iv_i) = 0; 1 \leq i \leq n$$
$$f^*(v_iv_{i+1}) = 1; 1 \leq i \leq n-1$$
$$f^*(u_iv_{i+1}) = 0; 1 \leq i \leq n-1$$

We observe that, $e_f(0) = 2n - 1$ and $e_f(1) = 2n - 2$.
Thus, $|e_f(0) - e_f(1)| \leq 1$.
Hence, TL_n is sum divisor cordial graph. □

Example 2.22. *A sum divisor cordial labeling of TL_5 is shown in Figure 2.11*

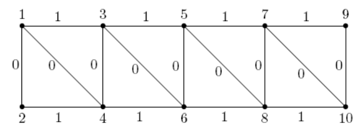

Figure 2.11

Theorem 2.23. *The graph $TL_n \odot K_1$ is sum divisor cordial graph.*

Proof. Let $u_1, u_2, \cdots, u_n$ and $v_1, v_2, \cdots, v_n$ be the vertices of TL_n. Let $G = TL_n \odot K_1$. Then $V(G) = \{u_i, v_i, x_i, y_i : 1 \leq i \leq n\}$ and $E(G) = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1} : 1 \leq i \leq n-1\} \bigcup \{u_ix_i, v_iy_i, u_iv_i : 1 \leq i \leq n\}$. Then, G is of order $4n$ and size $6n-3$. Define $f : V(G) \rightarrow \{1, 2, \cdots, 4n\}$ as follows:

$$f(u_i) = \{ 4\,i - 1 \text{ if } \ i \text{ is odd}$$
$4i - 3$ if i is even
$$f(x_i) = \{ 4\,i - 3 \text{ if } \ i \text{ is odd}$$
$4i - 2$ if i is even
$$f(v_i) = \{ 4\,i - 2 \text{ if } \ i \text{ is odd}$$
$4i - 1$ if i is even
$$f(y_i) = 4i; 1 \leq i \leq n$$
Then, the induced edge labels are
$$f^*(u_ix_i) = \{ 1 \ \text{ if } \ i \text{ is odd}$$
0 if i is even
$$f^*(v_iy_i) = \{ 1 \ \text{ if } \ i \text{ is odd}$$
0 if i is even
$$f^*(u_iv_i) = \{ 0 \ \text{ if } \ i \text{ is odd}$$
1 if i is even
$$f^*(u_iv_{i+1}) = \{ 1 \ \text{ if } \ i \text{ is odd}$$
0 if i is even

$$f^*(u_iu_{i+1}) = 1; 1 \le i \le n-1$$
$$f^*(v_iv_{i+1}) = 0; 1 \le i \le n-1$$

We observe that, $e_f(0) = 3n-2$ and $e_f(1) = 3n-1$.
Thus, $|e_f(1) - e_f(0)| \le 1$.
Hence, $TL_n \odot K_1$ is sum divisor cordial graph. □

Example 2.24. *A sum divisor cordial labeling of $TL_6 \odot K_1$ is shown in Figure* 2.12

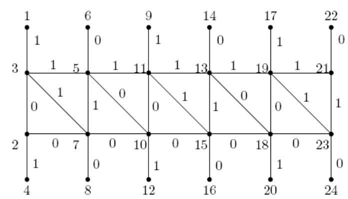

Figure 2.12

Theorem 2.25. *The closed helm graph CH_n is sum divisor cordial graph.*

Proof. Let $G = CH_n$. Let $V(G) = \{v, v_1, v_2, \cdots, v_n, u_1, u_2, \cdots, u_n\}$ and $E(G) = \{vv_i, u_iv_i : 1 \le i \le n\} \bigcup \{v_iv_{i+1}, u_iu_{i+1} : 1 \le i \le n-1\} \bigcup \{v_1v_n, u_1u_n\}$. Then, G is of order $2n+1$ and size $4n$. Define $f : V(G) \to \{1, 2, \cdots, 2n+1\}$ as follows:

$$f(v) = 1;$$
$$f(v_i) = 2i; 1 \le i \le n$$
$$f(u_i) = 2i+1; 1 \le i \le n$$

Then, the induced edge labels are

$$f^*(vv_i) = 0; 1 \le i \le n$$
$$f^*(v_iv_{i+1}) = 1; 1 \le i \le n-1$$
$$f^*(v_1v_n) = 1;$$
$$f^*(u_iv_i) = 0; 1 \le i \le n$$
$$f^*(u_iu_{i+1}) = 1; 1 \le i \le n-1$$
$$f^*(u_1u_n) = 1;$$

We observe that, $e_f(0) = 2n$ and $e_f(1) = 2n$.
Thus, $|e_f(1) - e_f(0)| \leq 1$.
Hence, CH_n is sum divisor cordial graph. $\square$

Example 2.26. *A sum divisor cordial labeling of* CH_4 *is shown in Figure* 2.13

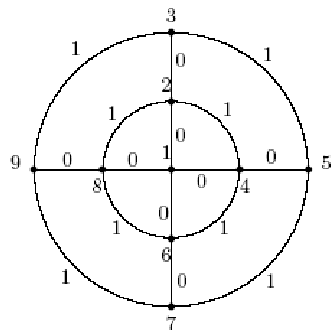

Figure 2.13

3. conclusion

It is very interesting and challenging as well as to investigate graph families which admit sum divisor cordial labeling. Here we have proved $D_2(K_{1,n})$, $S^{'}(K_{1,n})$, $D_2(B_{n,n})$, $DS(B_{n,n})$, $S^{'}(B_{n,n})$, $S(B_{n,n})$, $< K_{1,n}^{(1)} \Delta K_{1,n}^{(2)} >$, $S(L_n)$, $L_n \odot K_1$, SL_n, TL_n, $TL_n \odot K_1$ and CH_n are sum divisor cordial graphs.

References

[1] J. A. Gallian, A Dyamic Survey of Graph Labeling, The Electronic J. Combin., 18 (2015) # DS6.

[2] F. Harary, Graph Theory, Addison-wesley, Reading, Mass, (1972).

[3] A. Lourdusamy and F. Patrick, Sum Divisor Cordial Graphs, Proyecciones Journal of Mathematics, 35(1), pp. 115-132, (2016).

[4] A. Lourdusamy and F. Patrick, Sum Divisor Cordial Labeling for Path and Cycle Related Graphs (submitted for publication)

[5] S. S. Sandhya, S. Somasundaram and S. Anusa Some New Results on Root Square Mean Labeling, International Journal of Mathematical Archive, 5(12), pp. 130-135, (2014).

[6] M. Seenivasan Some New Labeling Concepts, PhD thesis, Manonmaniam Sundaranar University, India, (2013).

[7] S. K. Vaidya and C. M. Barasara, Product Cordial Graphs in the Context of Some Graph Operations, International Journal of Mathematics and Scientific Computing, 1(2), pp. 122-130, (2011).

[8] S. K. Vaidya and N. H. Shah, Some Star and Bistar Related Cordial Graphs, Annals of Pure and Applied Mathematics, 3(1), pp. 67-77, (2013).

[9] S. K. Vaidya and N. J. Kothari, Line Gracefulness of Some Path Related Graphs, International Journal of Mathematics and Scientific Computing, 4(1), pp. 15-18, (2014).

A. Lourdusamy
Department of Mathematics,
St. Xavier's College (autonomous),
Palayamkottai-627002,
Tamilnadu,
India
e-mail : lourdusamy15@gmail.com

and

F. Patrick
Department of Mathematics,
St. Xavier's College (autonomous),
Palayamkottai-627002,
Tamilnadu,
India
e-mail : patrick881990@gmail.com