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Some results on skolem odd difference mean labeling

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Abstract

Let $G = (V, E)$ be a graph with p vertices and q edges. A graph G is said to be skolem odd difference mean if there exists a function $f : V(G) \to \{0, 1, 2, 3, ..., p+3q-3\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, ..., 2q-1\}$ defined by $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits skolem odd difference mean labeling is called skolem odd difference mean graph. We call a skolem odd difference mean labeling as skolem even vertex odd difference mean labeling if all vertex labels are even. A graph that admits skolem even vertex odd difference mean labeling is called skolem even vertex odd difference mean graph.

In this paper we prove that graphs $B(m, n) : P_w$, $\langle P_m \tilde{o} S_n \rangle$, mP_n, $mP_n \cup tP_s$ and $mK_{1,n} \cup tK_{1,s}$ admit skolem odd difference mean labeling. If $G(p, q)$ is a skolem odd differences mean graph then $p \geq q$. Also, we prove that wheel, umbrella, B_n and L_n are not skolem odd difference mean graph.

Keywords *: Skolem difference mean labeling, skolem odd difference mean labeling, skolem odd difference mean graph, skolem even vertex odd difference mean labeling, skolem even vertex odd difference mean graph.*

AMS Subject Classification: *05C78.*

1. Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary [1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling. An excellent survey of graph labeling is available in [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [8]. A graph $G(V, E)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0, 1, 2, ..., q\}$ such that for each uv, labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u) + f(v)$ is odd. Then the resulting edge labels are distinct. The notion of odd mean labeling was due to Manickam and Marudai [3]. Let $G(V, E)$ be a graph with p vertices and q edges. A graph G is said to be odd mean graph if there exists a function $f : V(G) \to \{0, 1, 2, 3, ..., 2q - 1\}$ satisfying f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, ..., 2q - 1\}$ defined by $f^*(uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$ is a bijection. The function f is called an odd mean labeling. In [3], they studied the mean labeling of some standard graphs.

Murugan and Subramaniam [4] introduced the concept of skolem difference mean labeling and some standard results on skolem difference mean labeling were proved in [5] and [6]. A graph $G(V, E)$ with p vertices and q edges is said to have skolem diffference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, ..., p + q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, ..., q$. A graph that admits skolem difference mean labeling is called a skolem difference mean graph. Motivated by the concept of skolem difference mean labeling Ramya et al. [7] introduced the concept of skolem odd difference mean labeling. A graph G is said to have skolem odd difference mean labeling if there exists a function $f : V(G) \to \{0, 1, 2, 3, ..., p + 3q - 3\}$ satisfying that f is 1-1 and the induced map $f^* : E(G) \to \{1, 3, 5, ..., 2q - 1\}$ defined by $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ is a bijection. A graph that admits skolem odd difference mean labeling is called skolem odd difference mean graph. This is an extension work on skolem odd difference mean labeling. We use the following definitions in the subsequent section.

Definition 1.1. *Let $P_w : v_1, v_2, ..., v_w$ be a path. The graph $B(m, n) : P_w$ is obtained by identifying the central vertices of the star graphs $K_{1,m}$ and $K_{1,n}$ with v_1 and v_w respectively.*

Definition 1.2. *Let P_m be a path on m vertices. The graph $\langle P_m \tilde{o} S_n \rangle$ is obtained from P_m and m copies of $K_{1,n}$ by joining the central vertex of i^{th} copy of $K_{1,n}$ with i^{th} vertex of P_m by an edge.*

Definition 1.3. *The graph mP_n is the disjoint union of m copies of the path P_n.*

Definition 1.4. *The wheel graph W_n is a graph with n vertices ($n \geq 4$), formed by connecting a single vertex to all vertices of a $(n-1)$ cycle.*

Definition 1.5. *A graph is obtained from a fan by joining a path of length m, P_m to a middle vertex of a path P_n in fan F_n is called an umbrella graph and denoted by $U(m, n)$.*

Definition 1.6. *The book graph B_m is defined as the cartesian product graph $S_{m+1} \times P_2$, where S_m is a star graph and P_2 is the path graph on two vetices.*

Definition 1.7. *The cartesian product of two paths is known as grid graph $P_m \times P_n$. In particular the graph $L_n = P_m \times P_2$ is known as ladder graph.*

2. Skolem Odd Difference Mean Graphs

In this section we prove that graphs $B(m, n) : P_w$, $\langle P_m \tilde{o} S_n \rangle$, mP_n, $mP_n \cup tP_s$ and $mK_{1,n} \cup tK_{1,s}$ admit skolem odd difference mean labeling. If $G(p, q)$ is a skolem odd differences mean graph then $p \geq q$. Also, we prove that wheel, umbrella, B_n and L_n are not skolem odd difference mean graph.

Theorem 2.1. *The graph $B(m, n) : P_w$ is a skolem odd difference mean graph.*

Proof. Let $V(B(m,n):P_w) = \{u_i, v_j, t_k : 1 \le i \le m, 1 \le j \le n, 1 \le k \le w\}$ and $E(B(m,n):P_w) = \{t_1u_i, t_wv_j, t_kt_{k+1} : 1 \le i \le m, 1 \le j \le n, 1 \le k \le w-1\}$ be the set of vertices and edges of the graph $B(m,n):P_w$. Define $f : V(B(m,n):P_w) \to \{0,1,2,3,...,p+3q-3 = 4(m+n+w)-6\}$ as follows:

$f(t_{2i-1}) = 4(i-1)$ for $1 \le i \le \lceil \frac{w}{2} \rceil$,
$f(t_{2i}) = 4(n+w-i)-2$ for $1 \le i \le \lfloor \frac{w}{2} \rfloor$,
$f(u_i) = 4(m+n+w-i)-2$ for $1 \le i \le m$,
$f(v_i) = 2w+4(i-1)$ for $1 \le i \le n$.

Let $e_i = t_it_{i+1}$ for $1 \le i \le w-1$.

For each vertex label, the induced edge label f^* is defined as follows:
$f^*(e_i) = 2(n+w-i)-1$ for $1 \le i \le w-1$,
$f^*(t_1u_i) = 2(m+n+w-i)-1$ for $1 \le i \le m$,
$f^*(t_wv_i) = 2(n-i)+1$ for $1 \le i \le n$.

It can be verified that f is a skolem odd difference mean labeling of $B(m,n):P_w$. The skolem odd difference mean labeling of $B(3,4):P_5$ is shown in Figure 1.

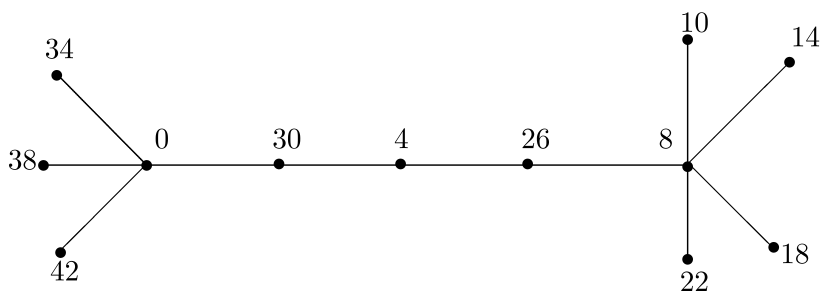

Figure 1 : Skolem odd difference mean labeling of $B(3,4):P_5$

□

Theorem 2.2. *The graph $\langle P_m \tilde{o} S_n \rangle$ is a skolem odd difference mean graph.*

Proof. Let $w_j, u_j, v_i^j (1 \le j \le m, 1 \le i \le n)$ be the vertices of $\langle P_m \tilde{o} S_n \rangle$. $w_ju_j, u_ku_{k+1}, w_jv_i^j : 1 \le j \le m, 1 \le k \le m-1, 1 \le i \le n$ be the edges of $\langle P_m \tilde{o} S_n \rangle$.

Define $f : V(\langle P_m \tilde{o} S_n\rangle) \to \{0, 1, 2, 3, ..., p + 3q - 3 = 4mn + 8m - 6\}$ as follows:

$f(u_{2i-1}) = 4n + 4(n + 2)(i - 1)$ for $1 \leq i \leq \lceil \frac{m}{2} \rceil$,
$f(u_{2i}) = 4(n + 2)(m - i + 1) - 10$ for $1 \leq i \leq \lfloor \frac{m}{2} \rfloor$,
$f(w_{2j-1}) = 4(n + 2)(m - j + 1) - 6$ for $1 \leq j \leq \lceil \frac{m}{2} \rceil$,
$f(w_{2j}) = 4(n + 1) + 4(n + 2)(j - 1)$ for $1 \leq j \leq \lfloor \frac{m}{2} \rfloor$,
$f(v_i^j) = 2(n + 2)(j - 1) + 4(i - 1)$ for $1 \leq i \leq n$, $1 \leq j \leq m$ and j is odd,
$f(v_i^j) = 2(n + 2)(2m + 2 - j) - 2(2i + 5)$ for $1 \leq i \leq n$, $1 \leq j \leq m$ and j is even.

Let $e_i^j = w_j v_i^j$ for $1 \leq i \leq n$, $1 \leq j \leq m$ and $e_j = w_j u_j$ for $1 \leq j \leq m$.

For each vertex labeling, the induced edge label f^* is defined as follows.
$f^*(u_j u_{j+1}) = 2(n + 2)(m - j + 1) - 2n - 5$ for $1 \leq j \leq m - 1$,
$f^*(e_j) = 2(n + 2)(m - j + 1) - 2n - 3$ for $1 \leq j \leq m$ and j is odd,
$f^*(e_j) = 2(n + 2)(m - j + 2) - 2n - 7$ for $1 \leq j \leq m$ and j is even,
$f^*(e_i^j) = 2(n + 2)(m - j + 1) - 2i - 1$ for $1 \leq i \leq n$, $1 \leq j \leq m$ and j is odd,
$f^*(e_i^j) = 2(n+2)(m-j+2) - 2(n+i) - 7$ for $1 \leq i \leq n$, $1 \leq j \leq m$ and j is even.

It can be verified that f is a skolem odd difference mean labeling of $\langle P_m \tilde{o} S_n\rangle$. The skolem odd difference mean labeling of $\langle P_4 \tilde{o} S_3\rangle$ is shown in Figure 2.

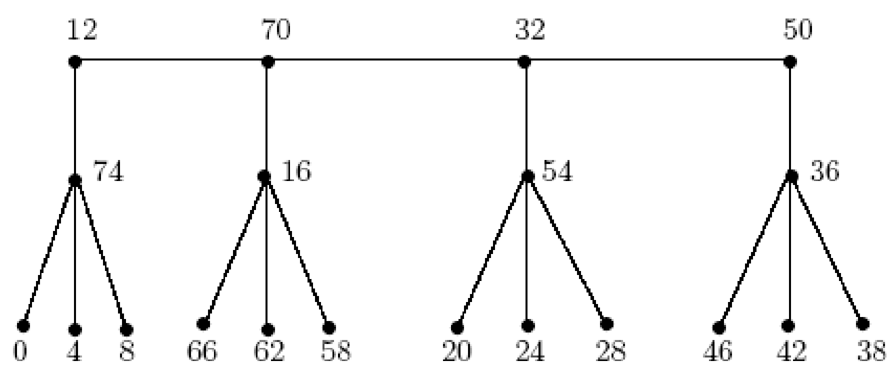

Figure 2: Skolem odd difference mean labeling of $\langle P_4 \tilde{o} S_3\rangle$

Theorem 2.3. *The graph mP_n is a skolem odd difference mean graph.*

Proof. Let $v_1^1, v_2^1, ..., v_n^1$ and $v_1^2, v_2^2, ..., v_n^2$ upto $v_1^m, v_2^m, ..., v_n^m$ be the vertices of the graph mP_n.
Let $V(mP_n) = \{v_i^j : 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(mP_n) = \{v_i^j v_{i+1}^j : 1 \leq i \leq n-1, 1 \leq j \leq m\}$.

Define $f : V(mP_n) \to \{0, 1, 2, ..., 4mn - 3(m+1)\}$ as follows:

When n is odd and m is either odd or even,
$f(v_{2i-1}^j) = (m-1) + 2n(j-1) + 4(i-1)$ for $1 \leq i \leq \lceil \frac{n}{2} \rceil$, $1 \leq j \leq m$,
$f(v_{2i}^j) = 4mn - 3(m+1) - 2(n-2)(j-1) - 4(i-1)$ for $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq j \leq m$.

When n is even and m is either odd or even,
$f(v_{2i-1}^j) = (m-1) + 2(n-1)(j-1) + 4(i-1)$ for $1 \leq i \leq \lceil \frac{n}{2} \rceil$, $1 \leq j \leq m$,
$f(v_{2i}^j) = 4mn - 3(m+1) - 2(n-1)(j-1) - 4(i-1)$ for $1 \leq i \leq \lfloor \frac{n}{2} \rfloor$, $1 \leq j \leq m$.

For each vertex labeling, the induced edge label f^* is defined as follows:
$f^*(v_i^j v_{i+1}^j) = 2(n-1)(m-j+1) - 2i + 1$ for $1 \leq i \leq n-1$, $1 \leq j \leq m$.

It can be verified that f is a skolem odd difference mean labeling of mP_n.
The skolem odd difference mean labeling of $3P_4$ is shown in Figure 3.

□

Theorem 2.4. *The graph $mP_n \cup tP_s$ is a skolem odd difference mean graph if $m, n, t, s \geq 1$.*

Proof. Let $v_1^1, v_2^1, v_3^1, ..., v_n^1$ and $v_1^2, v_2^2, v_3^2, ..., v_n^2$ upto $v_1^m, v_2^m, v_3^m, ..., v_n^m$ and $u_1^1, u_2^1, u_3^1, ..., u_s^1$ and $u_1^2, u_2^2, u_3^2, ..., u_s^2$, upto $u_1^t, u_2^t, u_3^t, ..., u_s^t$ be the vertices of the graph $mP_n \cup tP_s$. Let $E(mP_n \cup tP_s) = \{v_i^j v_{i+1}^j : 1 \leq i \leq n-1, 1 \leq j \leq m$ and $u_i^j u_{i+1}^j : 1 \leq i \leq s-1, 1 \leq j \leq t\}$.

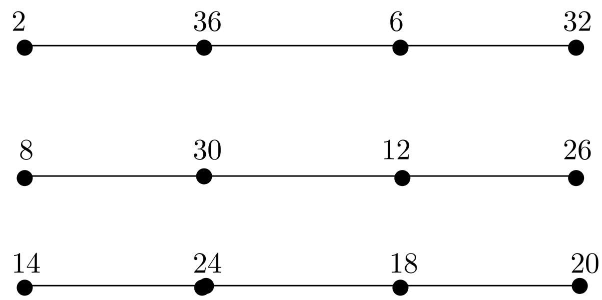

Figure 3 : Skolem odd difference mean labeling of $3P_4$

Define $f : V(mP_n \cup tP_s) \rightarrow \{0, 1, 2, ..., 4(mn + ts) - 3(m + t + 1)\}$ as follows:
$f(v^j_{2i-1}) = 2 + 2(n-1)(j-1) + 4(i-1)$ for $1 \leq j \leq m$, $1 \leq i \leq \frac{n}{2}$ and n is even,
$f(v^j_{2i}) = 4mn - 4m - 2(n-1)(j-1) - 4(i-1)$ for $1 \leq j \leq m$, $1 \leq i \leq \frac{n}{2}$ and n is even,
$f(u^j_{2i-1}) = 1 + 2(s-1)(j-1) + 4(i-1)$ for $1 \leq j \leq t$, $1 \leq i \leq \frac{s}{2}$ and s is even,
$f(u^j_{2i}) = 4m(n-1) + 4t(s-1) - 1 - 2(s-1)(j-1) + 4(i-1)$ for $1 \leq j \leq t$, $1 \leq i \leq \frac{s}{2}$ and s is even,
$f(v^j_{2i-1}) = 2 + 2n(j-1) + 4(i-1)$ for $1 \leq j \leq m$, $1 \leq i \leq \frac{n+1}{2}$ and n is odd,
$f(v^j_{2i}) = 4mn - 4m - 2(n-2)(j-1) - 4(i-1)$ for $1 \leq j \leq m$, $1 \leq i \leq \frac{n-1}{2}$ and n is odd,
$f(u^j_{2i-1}) = 1 + 2s(j-1) + 4(i-1)$ for $1 \leq j \leq t$, $1 \leq i \leq \frac{s+1}{2}$ and s is odd,
$f(u^j_{2i}) = 4m(n-1) + 4t(s-1) - 1 - 2(s-2)(j-1) + 4(i-1)$ for $1 \leq j \leq t$, $1 \leq i \leq \frac{s-1}{2}$ and s is odd.

For each vertex labeling, the induced edge label f^* is defined as follows:
$f^*(E(mP_n \cup tP_s)) = 2m(n-1) + 2t(s-1) - 1 - 2(i-1) : 1 \leq i \leq m(n-1) + t(s-1)$.

It can be verified that f is a skolem odd difference mean labeling of

$mP_n \cup tP_s$. □

Theorem 2.5. *The graph $mK_{1,n} \cup tK_{1,s}$ is a skolem odd difference mean graph if $m, n, t, s \geq 1$.*

Proof. Let $v_1^1, v_2^1, v_3^1, ..., v_n^1$ and $v_1^2, v_2^2, v_3^2, ..., v_n^2$ upto $v_1^t, v_2^t, v_3^t, ..., v_n^t$ and $u_1^1, u_2^1, u_3^1, ..., u_s^1$ and $u_1^2, u_2^2, u_3^2, ..., u_s^2$, upto $u_1^m, u_2^m, u_3^m, ..., u_s^m$ be the vertices of the graph $mK_{1,n} \cup tK_{1,s}$.

Let $E(mK_{1,n} \cup tK_{1,s}) = \{u_1^j u_{1+i}^j : 1 \leq i \leq n, 1 \leq j \leq m$ and $v_1^j v_{1+i}^j : 1 \leq i \leq s, 1 \leq j \leq t\}$.

Define $f : V(mK_{1,n} \cup tK_{1,s}) \to \{0, 1, 2, ..., 4(mn + ts) + m + t - 3\}$ as follows:
$f(u_1^j) = 2(j-1)$ for $1 \leq j \leq m$,
$f(u_{1+i}^j) = 4mn + 4ts - 2 - 4(i-1) - 2(j-1)(2n-1)$ for $1 \leq j \leq m$ and $1 \leq i \leq n$,
$f(v_1^j) = (2j-1)$ for $1 \leq j \leq t$,
$f(v_{1+i}^j) = 4ts - 1 - 4(i-1) - 2(j-1)(2s-1)$ for $1 \leq j \leq t$ and $1 \leq i \leq s$.

For each vertex labeling, the induced edge label f^* is defined as follows:
$f^*(E(mK_{1,n} \cup tK_{1,s})) = (2i-1)$ for $1 \leq i \leq (mn + ts)$.

It can be verified that f is a skolem odd difference mean labeling of $mK_{1,n} \cup tK_{1,s}$. □

Theorem 2.6. *If $G(p, q)$ is a skolem odd difference mean graph then $p \geq q$.*

Proof. Suppose that G is a skolem odd difference mean graph with p vertices and q edges and $p < q$. Let $f : V(G) \to \{0, 1, 2, 3, ..., p + 3q - 3\}$ be skolem odd difference mean labeling of G.
The induced edge labels are $\{1, 3, 5, ..., 2q - 1\}$.
Let uv be an edge of G with $f^*(uv) = 2q - 1$. Take $f(u) = a$ and $f(v) = b$. Without loss of generality assume that $a > b$. Then $0 \leq a, b \leq p + 3q - 3$. Now we consider the following two cases.

Case(i). $a - b$ is odd.
Then $\frac{|f(u)-f(v)|+1}{2} = \frac{a-b+1}{2} = 2q-1$. Hence, $a = 4q+b-3$. As, $a \leq p+3q-3$

we have $q + b \leq p$. Since $p < q$, then $q + b < q$ which implies that $b < 0$ which is a contradiction to $b \geq 0$.

Case(ii). $a - b$ is even.
Then $\frac{|f(u)-f(v)|}{2} = 2q - 1$. Hence, $a = 4q - 2 + b \leq p + 3q - 3$ which implies that $b + q \leq p - 1$. Since $p < q$, then $b < -1$. This is a contradiction to $b \geq 0$.
□

Theorem 2.7. *The wheel graph W_n, $n > 2$ is not a skolem odd difference mean graph.*

Proof. The wheel graph W_n, $n > 2$ has $n + 1$ vertices and $2n$ edges. Suppose that f is a skolem odd difference mean labeling of W_n. Hence the induced edge labels are $\{1, 3, ..., 4n - 1\}$. Let uv be an edge with $f^*(uv) = 4n - 1$. Let $f(u) = a$ and $f(v) = b$. Without loss of generality assume that $a > b$. Then $0 \leq b < a \leq 7n - 2$.
If $\frac{|f(u)-f(v)|}{2} = 4n - 1$ then $\frac{a-b}{2} = 4n - 1$ which implies $a = b + 8n - 2 \geq 8n - 2 > 7n - 2$.
If $\frac{|f(u)-f(v)+1|}{2} = 4n - 1$ then $\frac{a-b+1}{2} = 4n - 1$ which implies $a = b + 8n - 3 \geq 8n - 3 > 8n - 2 > 7n - 2$.
This is a contradiction to $a, b \leq 7n - 2$. Hence the wheel graph W_n, $n > 2$ is not a skolem odd difference mean graph. □

Theorem 2.8. *The umbrella graph $U_{n,m}$, $n > 2$ and $m \geq 1$ is not a skolem odd difference mean graph.*

Proof. The umbrella graph $U_{n,m}$, $n > 2$ and $m \geq 1$, has $m + n$ vertices and $m + 2(n - 1)$ edges. Suppose that f is a skolem odd difference mean labeling of G. Hence the induced edges are $\{1, 3, ..., 2m + 4n - 5\}$.
Let uv be an edge with $f^*(uv) = 2m + 4n - 5$. Let $f(u) = a$ and $f(v) = b$. Without loss of generality assume that $a > b$. Then $0 \leq b < a \leq 4m + 7n - 9$.
If $\frac{|f(u)-f(v)|}{2} = 2m + 4n - 5$ then $\frac{a-b}{2} = 2m + 4n - 5$ this implies that $a = b + 4m + 8n - 10 \geq 4m + 8n - 10 > 4m + 8n - 9 > 4m + 7n - 9$.
If $\frac{|f(u)-f(v)+1|}{2} = 2m + 4n - 5$ then $\frac{a-b+1}{2} = 4n - 1$ this implies that $a = b + 4m + 8n - 11 \geq 4m + 8n - 11 > 4m + 8n - 9 > 4m + 7n - 9$.
This is a contradiction to $a, b \leq 4m + 7n - 9$. Hence the umbrella graph $U_{n,m}$, $n > 2$ and $m \geq 1$ is not a skolem odd difference mean graph. □

Theorem 2.9. *The graph B_n is not a skolem odd difference mean graph if $n \geq 2$.*

Proof. If $n = 1$ then B_1 is a cycle graph that admits skolem odd difference mean graph in [7]. If $n \geq 2$, the number of edges in B_n graph is $q = 3n + 1$. Therefore $p + 3q - 3 = 11n + 2$.
To get $2q - 1 = 6n + 1$ as edge label, the minimum vertex label is $12n + 1$. But $11n + 2 < 12n + 1$ for all $n \geq 2$. Therefore $2q - 1$ cannot occur as an edge label of B_n for $n \geq 2$. Hence B_n is not a skolem odd difference mean graph if $n \geq 2$. □

Theorem 2.10. *The graph L_n is not a skolem odd difference mean graph if $n \geq 3$.*

Proof. If $n = 2$ then L_2 is a cycle graph that admits skolem odd difference mean graph in [7]. The graph $L_n(n \geq 3)$ has $2n$ vertices and $3n - 2$ edges.
For $n \geq 3$, $p + 3q - 3 = 11n - 9$.
That is the maximum possible vertex label of L_n is $11n - 9$.
Therefore, it is not possible to get an edge with label $2q - 1 = 6n - 5$.
Hence L_n $(n \geq 3)$ is not a skolem odd diference mean graph. □

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