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# Totally magic cordial labeling of $mP_n$ and $mK_n$

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**Abstract**

*A graph $G$ is said to have a totally magic cordial labeling with constant $C$ if there exists a mapping $f : V(G) \cup E(G) \rightarrow \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C \pmod 2$ for all $ab \in E(G)$ and $|n_f(0) - n_f(1)| \leq 1$, where $n_f(i)$ $(i = 0,1)$ is the sum of the number of vertices and edges with label $i$. In this paper we establish that $mP_n$ and $mK_n$ are totally magic cordial for various values of $m$ and $n$.*

**Keywords :** *Binary magic total labeling; cordial labeling; totally magic cordial labeling; totally magic cordial deficiency of a graph.*

**AMS Subject Classification 05C78**.

## 1. Introduction

All graphs in this paper are finite, simple and undirected. The graph $G$ has vertex set $V = V(G)$ and edge set $E = E(G)$ and we write $p$ for $|V|$ and $q$ for $|E|$. A general reference for graph theoretic notions is [3]. The concept of cordial labeling was introduced by Cahit [1]. A binary vertex labeling $f : V(G) \rightarrow \{0,1\}$ induces an edge labeling $f^* : E(G) \rightarrow \{0,1\}$ defined by $f^*(uv) = |f(u) - f(v)|$. Such labeling is called cordial if the conditions $|v_f(0) - v_f(1)| \leq 1$ and $|e_{f^*}(0) - e_{f^*}(1)| \leq 1$ are satisfied, where $v_f(i)$ and $e_{f^*}(i)(i = 0,1)$ are the number of vertices and edges with label $i$, respectively. A graph is called cordial if it admits cordial labeling. Also, Cahit [2] introduced the notion of totally magic cordial labeling (TMC) based on cordial labeling.

A graph $G$ is said to have totally magic cordial labeling with constant $C$ if there exists a mapping $f : V(G) \cup E(G) \rightarrow \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C \pmod 2$ for all $ab \in E(G)$ and $|n_f(0) - n_f(1)| \leq 1$, where $n_f(i)$ $(i = 0,1)$ is the sum of the number of vertices and edges with label $i$.

In [4] it is proved that the complete graph $K_n$ is TMC if and only if

$\sqrt{4k+1}$ has an integer value when $n = 4k$,

$\sqrt{k+1}$ or $\sqrt{k}$ has an integer value when $n = 4k+1$,

$\sqrt{4k+5}$ or $\sqrt{4k+1}$ has an integer value when $n = 4k+2$,

$\sqrt{k+1}$ has an integer value when $n = 4k+3$.

Jeyanthi and Angel Benseera [5] established totally magic cordial labeling of one-point union of $n$-copies of cycles, complete graphs and wheels. In [7] we gave necessary condition for an odd graph to be not totally magic cordial.

In [6] we defined binary magic total labeling of a graph $G$ as follows: A binary magic total labeling of a graph $G$ is a function $f : V(G) \cup E(G) \rightarrow \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C \pmod 2$ for all $ab \in E(G)$.

Also, in [6] we defined totally magic cordial deficiency of a graph as the minimum number of vertices taken over all binary magic total labeling of $G$, which it is necessary to add inorder that $G'$ become totally magic cordial is the totally magic cordial deficiency of $G$, denoted by $\mu_T(G)$. That is, $\mu_T(G) = \min\{|n_f(0) - n_f(1)| - 1\}$ such that $f$ is a binary magic total labeling of $G$. Further, we determined totally magic cordial deficiency of complete graphs, wheels and one-point union of complete graphs and wheels.

In this paper we establish the totally magic cordial labeling of $mP_n$, the disjoint union of $m$ copies of path $P_n$ and $mK_n$, the disjoint union of $m$ copies of complete graph $K_n$.

## 2. Totally magic cordial labeling of $mP_n$

In this section, we give some sufficient conditions for $mP_n$ to be TMC by means of the solution of a system which comprises an equation and an inequality.

**Theorem 2.1.** *Let $G$ be the disjoint union of $m$ copies of the path $P_n$ of $n$ vertices and for $i = 1, 2, ..., k.$, let $f_i$ be the binary magic total labeling of $P_n$. Let $\gamma_i = n_{f_i}(0) - n_{f_i}(1)$ for $i = 1, 2, ..., k$ then $G$ is TMC if the system (2.1) has a nonnegative integral solution for $x_i$'s:*

$$\left|\sum_{i=1}^{k} \gamma_i x_i\right| \leq 1 \text{ and } \sum_{i=1}^{k} x_i = m \tag{2.1}$$

**Proof.** Suppose $x_i = \delta_i$, $i = 1, 2, ..., k$ is a nonnegative integral solution of the system (2.1). We label $\delta_i$ copies of $P_n$ with $f_i$ $(i = 1, 2, ..., k)$. As each copy has the property $f_i(a)+f_i(b)+f_i(ab) \equiv C(\text{mod } 2)$ for all $i = 1, 2, ..., k$, the disjoint union of $m$ copies of the path $P_n$ of $n$ vertices is TMC. □

The following table shows the values of $\gamma_i$ for distinct possible binary magic total labelings $f_i$ of the path $P_n$:

| i | $n_{f_i}(0)$ | $n_{f_i}(1)$ | $\gamma_i$ |
|---|---|---|---|
| 1 | 0 | $2n-1$ | $-2n+1$ |
| 2 | 2 | $2n-3$ | $-2n+5$ |
| 3 | 3 | $2n-4$ | $-2n+7$ |
| 4 | 4 | $2n-5$ | $-2n+9$ |
| . | . | . | . |
| . | . | . | . |
| . | . | . | . |
| $n-1$ | $n-1$ | $n$ | $-1$ |
| $n$ | $n$ | $n-1$ | 1 |

**Corollary 2.2.** *The graph $mP_2$ is TMC if $m \not\equiv 2(\text{mod } 4)$.*

**Proof.** The two values of $\gamma_i$ corresponding to the binary magic total labelings of $P_2$ are -3 and 1. Therefore, the system (2.1) in Theorem 2.1 becomes $|-3x_1 + x_2| \leq 1$ such that $x_1+x_2 = m$. When $m = 4t$, then $x_1 = t$ and $x_2 = 3t$ is a solution. When $m = 4t + 1$, then $x_1 = t$ and $x_2 = 3t + 1$ is a solution. When $m = 4t + 2$, then the system has no solution. When $m = 4t + 3$, then $x_1 = t + 1$ and $x_2 = 3t + 2$ is a solution. Hence, $mP_2$ is TMC if $m \not\equiv 2(\text{mod } 4)$. □

**Corollary 2.3.** *The graph $mP_n$ is TMC for all $m \geq 1$ and $n \geq 3$.*

**Proof.** For any $n \geq 3$, using the binary magic total labelings with $\gamma_i$ values as -1 and 1, we get the system $|-x_1 + x_2| \leq 1$, $x_1 + x_2 = m$. Thus for any $m \geq 1$, the above system has solution. Hence, $mP_n$ is TMC for all $m \geq 1$ and $n \geq 3$. □

## 3. Totally magic cordial labeling of $mK_n$

In this section, we establish the TMC labeling of the disjoint union of $m$ copies of complete graph $K_n$ using the solution of a system which comprises an equation and an inequality.

Let $f_i$ be a TMC labeling of the $i^{th}$ copy of $mK_n$. Without loss of generality, we assume that $C = 1$. Then for any edge $e = uv \in E(K_n)$, we have either $f_i(e) = f_i(u) = f_i(v) = 1$ or $f_i(e) = f_i(u) = 0$ and $f_i(v) = 1$ or $f_i(e) = f_i(v) = 0$ and $f_i(u) = 1$ or $f_i(u) = f_i(v) = 0$ and $f_i(e) = 1$. Hence, under the labeling $f_i$, the complete graph can be decomposed as $K_n = K_p \cup K_r \cup K_{p,r}$ where $K_p$ is the subgraph whose vertices and edges are labeled with 1, $K_r$ is the subgraph whose vertices are labeled with 0 and its edges are labeled with 1 and $K_{p,r}$ is the subgraph of $K_n$ with the bipartition $V(K_p) \cup V(K_r)$ in which the edges are labeled with 0. Thus, we have $n_{f_i}(0) = r + pr$ and $n_{f_i}(1) = p + \frac{p(p-1)}{2} + \frac{r(r-1)}{2}$.

**Theorem 3.1.** *Let $G$ be the disjoint union of $m$ copies of the complete graph $K_n$ of $n$ vertices and for $i = 1, 2, ..., k$, $f_i$ be the binary magic total labeling of the $i^{th}$ copy of $K_n$. Let $n_{f_i}(0) = \alpha_i$ for $i = 1, 2, ..., k$, then $G$ is TMC if the system (3.1) has a nonnegative integral solution for $x_i$'s:*

$$\left|\sum_{i=1}^{k} \left[2\alpha_i - \frac{n^2+n}{2}\right] x_i\right| \leq 1 \text{ and } \sum_{i=1}^{k} x_i = m \tag{3.1}$$

**Proof.** Suppose $x_i = \delta_i$, $i = 1, 2, ..., k$ is a nonnegative integral solution of the system (3.1), then we label the $\delta_i$ copies of $K_n$ with $f_i$ $(i = 1, 2, ..., k)$. We have, $n_{f_i}(1) = \frac{n^2+n}{2} - \alpha_i$. Thus, $n_{f_i}(0) - n_{f_i}(1) = 2\alpha_i - \frac{n^2+n}{2}$. As each copy has the property $f_i(a) + f_i(b) + f_i(ab) \equiv C (\text{mod } 2)$, the disjoint union of $m$ copies of the complete graph $K_n$ is TMC. □

**Theorem 3.2.** *If $\sqrt{n+1}$ has an integer value then the disjoint union of $m$ copies of $K_n$, $mK_n$ is TMC for all $m \geq 1$.*

**Proof.** The system (3.1) has solution when $\alpha_i = \frac{n^2+n}{4}$. Thus if there exists a positive integer $t$, $1 \leq t \leq n$ such that $t(n - t + 1) = \frac{n^2+n}{4}$, then $mK_n$ is TMC. By solving the above equation we get, $t = \frac{n+1}{2} \pm \frac{\sqrt{n+1}}{2}$. Hence, if $\sqrt{n+1}$ has an integer value then $mK_n$ is TMC for all $m \geq 1$. □

The following table shows the values of $\alpha_i$ and $\beta_i$ for distinct possible binary magic total labelings $f_i$ of the complete graph $K_n$:

| $i$ | $p$ | $r$ | $\alpha_i$ | $\beta_i$ |
|---|---|---|---|---|
| 1 | 0 | $n$ | $n$ | $\frac{n^2-n}{2}$ |
| 2 | 1 | $n-1$ | $2 \times (n-1)$ | $\frac{n^2-3n+4}{2}$ |
| 3 | 2 | $n-2$ | $3 \times (n-2)$ | $\frac{n^2-5n+12}{2}$ |
| 4 | 3 | $n-3$ | $4 \times (n-3)$ | $\frac{n^2-7n+24}{2}$ |
| . | . | . | . | . |
| . | . | . | . | . |
| . | . | . | . | . |
| $\left\lfloor \frac{n+1}{2} \right\rfloor$ | $\left\lfloor \frac{n-1}{2} \right\rfloor$ | $\left\lceil \frac{n+1}{2} \right\rceil$ | $\left\lfloor \frac{n-1}{2} \right\rfloor \times \left\lceil \frac{n+1}{2} \right\rceil$ | $\frac{\left[\left(\lfloor \frac{n-1}{2} \rfloor\right)^2 + \left(\lceil \frac{n+1}{2} \rceil\right)^2 + \lfloor \frac{n-1}{2} \rfloor + \lceil \frac{n+1}{2} \rceil\right]}{2}$ |

**Corollary 3.3.** *Let $f_1$ and $f_2$ be binary magic total labelings of $mK_n$. Let $n_{f_i}(0) = \alpha_i$, $i = 1, 2$ be such that $\alpha_1 + \alpha_2 = \frac{n^2+n}{2}$, then $mK_n$ is TMC if and only if $m$ is even.*

**Proof.** Let $m = 2t$. We assume that $\alpha_1 + \alpha_2 = \frac{n^2+n}{2}$. Then $\left|\sum_{i=1}^{k}\left[2\alpha_i - \frac{n^2+n}{2}\right]x_i\right| \leq 1$ and $\sum_{i=1}^{k} x_i = m$ implies that $\left|\left[2\alpha_1 - \frac{n^2+n}{2}\right]x_1 + \left[\frac{n^2+n}{2} - 2\alpha_1\right]x_2\right| \leq 1$ and $x_1 + x_2 = 2t$. Clearly, $x_1 = t$ and $x_2 = t$ satisfy the above system. Also, if $m$ is odd there is no solution. Hence, $mK_n$ is TMC if and only if $m$ is even. □

**Corollary 3.4.** *The graph $mK_{j^2}(j \geq 1)$ is TMC if and only if $m$ is even.*

**Proof.** Let $j^2 = n$. We consider the labelings $f_1$ and $f_2$ with $\alpha_1 = r(j^2 - r + 1)$ and $\alpha_2 = \alpha_1 + j$ where $r = \frac{j(j-1)}{2}$. Clearly, $\alpha_1 + \alpha_2 = \frac{n^2+n}{2}$. Hence by Corollary 3.3, $mK_{j^2}$ is TMC if and only if $m$ is even. □

**Illustration 3.5.** *We consider the graph $mK_4$. Clearly, $j = 2$ and $r = 1$. Thus $\alpha_1 = n_{f_1}(0) = 4$ and $\alpha_2 = n_{f_2}(0) = 6$. Therefore, $\alpha_1 + \alpha_2 = \frac{4^2+4}{2} = 10$. Hence, under the labeling $f_1$, all the four vertices of $K_4$ can be labeled with 0 and under the labeling $f_2$, only one vertex can be labeled with 1 and the remaining vertices can be labeled with 0. Therefore, by Corollary 3.3, $mK_4$ is TMC if and only if $m$ is even.*

**Corollary 3.6.** *The graph $mK_{j^2+3}(j \geq 2)$ is TMC if and only if $m$ is even.*

**Proof.** Let $j^2 + 3 = n$. We consider the labelings $f_1$ and $f_2$ with $\alpha_1 = r(j^2 - r + 4)$ and $\alpha_2 = \alpha_1 + 2j$ where $r = \frac{j(j-1)}{2} + 1$. Clearly, $\alpha_1 + \alpha_2 = \frac{n^2+n}{2}$. Thus by Corollary 3.3, $mK_{j^2+3}$ is TMC if and only if $m$ is even. □

**Corollary 3.7.** *The graph $mK_{j^2+8}(j \geq 1)$ is TMC if and only if $m$ is even.*

**Proof.** Let $j^2 + 8 = n$. We consider the labelings $f_1$ and $f_2$ with $\alpha_1 = \frac{3j}{2} + \frac{n^2+n}{4}$ and $\alpha_2 = \frac{n^2+n}{4} - \frac{3j}{2}$. Clearly, $\alpha_1 + \alpha_2 = \frac{n^2+n}{2}$. Hence by Corollary 3.3, $mK_{j^2+8}$ is TMC if and only if $m$ is even. □

**Corollary 3.8.** *Let $f_i$, $i = 1, 2, 3, 4$ be the binary magic total labelings of $mK_n$. Let $n_{f_i}(0) = \alpha_i$, $i = 1, 2, 3, 4$ be such that $\sum_{i=1}^{4} \alpha_i = n^2 + n$, then $mK_n$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $m = 4t$. We assume that $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = n^2 + n$. Then $\left|\sum_{i=1}^{4}\left[2\alpha_i - \frac{n^2+n}{2}\right] x_i\right| \leq 1$ such that $\sum_{i=1}^{4} x_i = 4t$. Clearly, $x_1 = x_2 = x_3 = x_4 = t$ is a solution of the above system. Also when $m \not\equiv 0 (\text{mod } 4)$ the system has no solution. Thus $mK_n$ is TMC if and only if $m \equiv 0(\text{mod } 4)$. □

**Corollary 3.9.** *The graph $mK_{j^2+2} (j \geq 2)$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $j^2 + 2 = n$. We consider the labelings $f_1$ and $f_2$ with $\alpha_1 = r(n - r + 1)$ and $\alpha_2 = \alpha_1 + 2j - 2$ where $r = \frac{j^2-j+2}{2}$. We can easily prove that $\alpha_1 + \alpha_2 = n^2 + n$. Hence, $x_1 = 3t$ and $x_2 = t$ is a solution of the system (3.1). Thus, $mK_{j^2+2}$ is TMC if $m \equiv 0(\text{mod } 4)$. □

**Theorem 3.10.** *The graph $mK_{j^2+4}$ $(j \geq 2)$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $j^2 + 4 = n$. We consider the labelings $f_1$,$f_2$,$f_3$ and $f_4$ with $\alpha_1 = r(n-r+1)$, $\alpha_2 = \alpha_1 + j + 2$, $\alpha_3 = \alpha_1 + 2j + 2$ and $\alpha_4 = \alpha_1 + 3j$. We can easily prove that $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = n^2 + n$. Hence by Corollary 3.8, $mK_{j^2+4}$ is TMC if and only if $m \equiv 0(\text{mod } 4)$. □

**Theorem 3.11.** *If $mK_n$ and $m^{'}K_n$ are TMC and $m$ or $m^{'}$ is even, then $(m + m^{'})K_n$ is TMC.*

**Proof.** Let $f$ and $f^{'}$ be the TMC labelings of $mK_n$ and $m^{'}K_n$ respectively with $C = 1$. We assume that $m$ is even. Then, $n_f(0) = n_f(1)$. For $m^{'}K_n$, we have $n_{f'}(0) = n_{f'}(1)$, $n_{f'}(0) = n_{f'}(1) + 1$ or $n_{f'}(0) = n_{f'}(1) - 1$. Let $f^{''}$ be a binary magic total labeling of $(m + m^{'})K_n$ with $C = 1$. Clearly, $n_{f''}(0) = n_f(0) + n_{f'}(0)$ and $n_{f''}(1) = n_f(1) + n_{f'}(1)$. Therefore, $n_{f''}(0) = n_{f''}(1)$ or $n_{f''}(0) = n_{f''}(1) + 1$ or $n_{f''}(0) = n_{f''}(1) - 1$ are derived from $n_{f'}(0) = n_{f'}(1)$ or $n_{f'}(0) = n_{f'}(1) + 1$ or $n_{f'}(0) = n_{f'}(1) - 1$ respectively. Hence, $(m + m^{'})K_n$ is TMC with $C = 1$. □

**Theorem 3.12.** *[7] If $G$ is an odd graph with $p + q \equiv 2$ (mod 4), then $G$ is not TMC.*

**Corollary 3.13.** *If $m \equiv 1(mod\ 2)$ and $n \equiv 4(mod\ 8)$ then $mK_n$ is not TMC.*

**Proof.** Proof follows from Theorem 3.12. □

**Theorem 3.14.** *Let $f_i$, $i = 1, 2, 3$ be the binary magic total labelings of $mK_n$. Let $n_{f_i}(0) = \alpha_i$, $i = 1, 2, 3$ be such that $\alpha_1 + 2\alpha_2 + \alpha_3 = n^2 + n$, then $mK_n$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $m = 4t$. We assume that $\alpha_1 + 2\alpha_2 + \alpha_3 = n^2 + n$. Then

$$\left|\sum_{i=1}^{3}\left[2\alpha_i - \frac{n^2+n}{2}\right]x_i\right| \leq 1$$

such that $\sum\limits_{i=1}^{3} x_i = 4t$. Clearly, $x_1 = t$, $x_2 = 2t$ and $x_3 = t$ is a solution of the above system. When $m \not\equiv 0(\text{mod } 4)$ the system has no solution. Thus $mK_n$ is TMC if and only if $m \equiv 0(\text{mod } 4)$. □

**Corollary 3.15.** *The graph $mK_{j^2+1}$ $(j \geq 2)$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $j^2 + 1 = n$. Consider the labelings $f_1$, $f_2$ and $f_3$ with $\alpha_1 = r(n - r + 1)$, $\alpha_2 = \alpha_1 + j + 1$ and $\alpha_3 = \alpha_1 + 2j$ where $r = \frac{j^2-j}{2}$. We can easily prove that $\alpha_1 + 2\alpha_2 + \alpha_3 = n^2 + n$. Hence by Theorem 3.14, $mK_n$ is TMC if and only if $m \equiv 0(\text{mod } 4)$. □

**Theorem 3.16.** *Let $f_i$, $i = 1, 2, 3$ be binary magic total labelings of $mK_n$. Let $n_{f_i}(0) = \alpha_i$, $i = 1, 2, 3$ be such that $\alpha_1 + \alpha_2 + 2\alpha_3 = n^2 + n$, then $mK_n$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $m = 4t$. We assume that $\alpha_1 + \alpha_2 + 2\alpha_3 = n^2 + n$. Then

$$\left|\sum_{i=1}^{3}\left[2\alpha_i - \frac{n^2+n}{2}\right]x_i\right| \leq 1$$

such that $\sum\limits_{i=1}^{3} x_i = 4t$. Clearly, $x_1 = t$, $x_2 = t$ and $x_3 = 2t$ is a solution of the above system. Also when $m \not\equiv 0(\text{mod } 4)$ the system has no solution. Thus $mK_n$ is TMC if and only if $m \equiv 0(\text{mod } 4)$. □

**Corollary 3.17.** *The graph $mK_{j^2+5}$ $(j \geq 3)$ is TMC if and only if $m \equiv 0(mod\ 4)$.*

**Proof.** Let $j^2+5=n$. Consider the labelings $f_1$, $f_2$ and $f_3$ with $\alpha_1 = r(n-r+1)$, $\alpha_2 = \alpha_1 + 2j + 4$ and $\alpha_3 = \alpha_1 + 3j + 6$ where $r = \frac{j^2-j+2}{2}$. We can easily prove that $\alpha_1 + 2\alpha_2 + 2\alpha_3 = n^2 + n$. Hence by Theorem 3.16, $mK_n$ is TMC if and only if $m \equiv 0(\text{mod } 4)$. □

We conclude this paper with the following conjecture:

**Conjecture 3.18.** *The graphs $mK_{j^2+k}$ $(j \geq 5)$ for $k = 6, 7, 9, 10, ..., 2j-1$ and $m \geq 1$ admit totally magic cordial labeling.*

## References


[1] I. Cahit, Cordial graphs: A weaker version of graceful and harmonious graphs, *Ars Combin.*, 23, pp. 201–207, (1987).

[2] I. Cahit, Some totally modular cordial graphs, *Discuss. Math. Graph Theory*, 22, pp. 247–258, (2002).

[3] F. Harary, *Graph Theory*, Addison-Wesley Publishing Co., (1969).

[4] P. Jeyanthi, N. Angel Benseera and M. Immaculate Mary, On totally magic cordial labeling, *SUT Journal of Mathematics*, 49 **(1)**, pp. 13–18, (2013).

[5] P. Jeyanthi and N. Angel Benseera, Totally magic cordial labeling of one-point union of $n$ copies of a graph, *Opuscula Mathematica*, 34 (1), pp. 115–122, (2014).

[6] P. Jeyanthi and N. Angel Benseera, Totally magic cordial deficiency of some graphs, *Utilitas Mathematica*, (to appear).

[7] P. Jeyanthi and N. Angel Benseera, Totally magic cordial labeling of some graphs, *Journal of Algorithms and Computation*, 46 (1), pp. 1-8, (2015).

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