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On some seminormed sequence spaces defined by Orlicz function

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Abstract

The sequence space BV_σ was introduced and studied by Mursaleen [9]. In this paper we extend BV_σ to $BV_\sigma(M,p,q,r)$ on a seminormed complex linear space by using orlicz function. We give various properties and some inclusion relations on this space.

1. Introduction

Let ℓ_∞, c and c_0 be respectively the Banach spaces of bounded, convergent and null sequences $x = (x_k)$ with $(x_k) \in \mathbf{R}$ or $\mathbf{C}$ the usual norm $\|x\| = \sup_k |x_k|$, where $k \in \mathbf{N} = 1, 2, 3...$, the positive integers. Let σ be an injection of the set of positive integers $\mathbf{N}$ into itself having no finite orbit and T be the operator defined on ℓ_∞ by $T((x_n)_{n=1}^{\infty}) = (x_{\sigma(n)})_{n=1}^{\infty}$.

A positive linear functional ϕ with $\|\phi\| = 1$ is called a σ-mean or an invariant mean if $\phi(x) = \phi(Tx)$ for all $x \in \ell_\infty$.

A sequence x is said to be σ-convergent, denoted by $x \in V_\sigma$, if $\phi(x)$ takes the same value, called $\sigma - \lim x$, for all σ-means ϕ.(Schaefer [14])

$$V_\sigma = \left\{ x = (x_n) : \sum_{m=1}^{\infty} t_{m,n}(x) = L \text{ uniformly in } n,\ L = \sigma - \lim x \right\}.$$

Where $m \geq 0,\ n \geq 0$

$$t_{m,n}(x) = \frac{x_n + x_{\sigma(n)} + ... + x_{\sigma^m(n)}}{m+1}, \quad \text{and} \quad t_{-1,n} = 0.$$

Where $\sigma^m(n)$ denotes the m - iterative of σ at n. In particular, if σ is the translation a σ-mean is often called a Banach limit and V_σ reduces to f, the set of almost convergent sequence [5]. Subsequently invariant means have been studied by Ahmad and Mursaleen [1], Mursaleen [8], Rami [12] and many others.

The concept of paranormed is closely related to linear metric spaces. It is generalization of that of absolute value. Let X be a linear space. A function $g : X \to R$ is called paranorme, if

$(P1)$ $g(x) \geq 0$, for all $x \in X$,

$(P2)$ $g(-x) = g(x)$, for all $x \in X$,

$(P3)$ $g(x+y) \le g(x) + g(y)$, for all $x, y \in X$,

$(P4)$ if λ_n is a sequence of scalar with $\lambda_n \to \lambda$ $(n \to \infty)$ and (x_n) is a sequence of vector with $g(x_n - x) \to 0$ $(n \to \infty)$ then $g(\lambda_n x_n - \lambda x) \to 0$ $(n \to \infty)$.

A paranormed g for which $g(x) = 0$ implies $x = 0$ is called a total paranorm on X and pair (X, g) is called a totally paranormed space. It is well known that the metric of any linear metric space is given by total paranorm (cf [15, Theorem 10.4.2, p-183]).

A map $M : R \to [0, +\infty]$ is called to be an orlicz function if M is even, convex left continuous on R_+, continuous at zero, convex $M(0) = 0$ and $M(u) \to \infty$ as $u \to \infty$.

If M takes the value zero only at zero we write $M > 0$ and if M takes only finite value we will write $M < \infty$. [2,3,6,7,10,13]

W. Orlicz [11] used the idea of orlicz function to construct the space (L^M) Lindenstrauss and Tzafriri [4] use the idea of Orlicz function and defined the sequence space ℓ_M such as

$$\ell_M = \left\{ x = (x_i) : \sum_{i=1}^{\infty} M\Big(\frac{|x_i|}{\rho}\Big) < \infty, \text{ for some } \rho > 0 \right\}.$$

The space ℓ_M with the norm $\|x\| = \inf \left\{ \rho > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{\rho}\right) \le 1 \right\}$ becomes a Banach space which is called an Orlicz sequence space.The space ℓ_M is closely related to the sequence space ℓ_p, which is an Orlicz sequence spaces with $M(x) = x^p$ for $1 \le p \le \infty$.

The Δ_2-condition is equivalent to $M(Lx) \le KLM(x)$, for all $x \ge 0$ and for $L > 1$.

An Orlicz function M can be represented in the following integral from

$$M(x) = \int_0^x \eta(t)dt,$$

where η is known as the kernal of M is right differentiable for $t \ge 0$, $\eta(0) =$

0, $\eta(t) > 0$, η is non- decreasing and $\eta(t) \to \infty$ as $t \to \infty$. Note that an Orlicz function satisfies the inequality

$$M(\lambda x) \leq \lambda M(x) \text{ for all } \lambda \text{ with } 0 < \lambda < 1.$$

Let E be a sequence space. Then E is called

(i) A sequence space E is said to be symmetric if $(x_n) \in E$ implies $(x_{\pi(n)}) \in E$, where $\pi(n)$ is a permutation of the elements of $\mathbf{N}$,

(ii) Solid (or normal), if $(\alpha_k x_k) \in E$, whenever $(x_k) \in E$ for all sequences of scalar (α_k) with $|\alpha_k| \leq 1$ for all $k \in \mathbf{N}$.

Lemma 1.1. A sequence space E is solid implies E is monotone.

Mursaleen [9] defined the sequence space.

$$BV_\sigma = \left\{ x \in \ell_\infty : \sum_m |\phi_{m,n}(x)| < \infty \text{ uniformly in } n \right\},$$

Where

$$\phi_{m,n}(x) = t_{m,n}(x) - t_{m-1,n}(x)$$

assuming

$$t_{m,n}(x) = 0, \text{ for } m = -1.$$

A straight forward calculation shows that

$$\phi_{m,n} = \begin{cases} \frac{1}{m(m+1)} \sum_{j=1}^{m} j(x_{\sigma^j(n)} - x_{\sigma^{j-1}(n)}) & (m \geq 1) \\ x_n & (m = 0). \end{cases}$$

Note that for any sequence x, y and scalar λ we have

$$\phi_{m,n}(x+y) = \phi_{m,n}(x) + \phi_{m,n}(y) \text{ and } \phi_{m,n}(\lambda x) = \lambda\phi_{m,n}(x).$$

2. Main Results

Let M be an Orlicz function, $p = (p_m)$ be any sequence of strictly positive real numbers, $r \geq 0$ and (X,q) be a seminorm space over the field **C** of complex number with seminorm q. Now we define the following sequence spaces,

$$BV_\sigma(M,p,q,r) = \Bigg\{x = (x_k) : \sum_{m=1}^{\infty} \frac{1}{m^r}\left[M\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\right)\right]^{p_m} < \infty, \text{ uniformly in } n \text{ and for som } \rho > 0\Bigg\}.$$

For $M(x) = x$, we get

$$BV_\sigma(p,q,r) = \Bigg\{x = (x_k) : \sum_{m=1}^{\infty} \frac{1}{m^r}\left[\left(q\Big(|\phi_{m,n}(x)|\Big)\right)\right]^{p_m} < \infty, \text{ uniformly in } n \text{ and for som } \rho > 0\Bigg\}.$$

For $p_m = 1$ for all m, we get

$$BV_\sigma(M,q,r) = \Bigg\{x = (x_k) : \sum_{m=1}^{\infty} \frac{1}{m^r}\left[M\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\right)\right] < \infty, \text{ uniformly in } n \text{ and for som } \rho > 0\Bigg\}.$$

For $r=0$, we get

$$BV_{\sigma}(M,p,q)=\Bigg\{x=(x_k):\sum_{m=1}^{\infty}\Bigg[M\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg]^{p_m}$$
$$<\infty,\ \text{uniformly in } n \text{ and for som } \rho>0\Bigg\}.$$

For $M(x)=x$ and $r=0$ we get

$$BV_{\sigma}(p,q)=\Bigg\{x=(x_k):\sum_{m=1}^{\infty}\Bigg[\Bigg(q\Big(|\phi_{m,n}(x)|\Big)\Bigg)\Bigg]^{p_m}$$
$$<\infty,\ \text{uniformly in } n \text{ and for som } \rho>0\Bigg\}.$$

For $p_m=1$ for all m and $r=0$ we get

$$BV_{\sigma}(M,q)=\Bigg\{x=(x_k):\sum_{m=1}^{\infty}\Bigg[M\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg]$$
$$<\infty,\ \text{uniformly in } n \text{ and for som } \rho>0\Bigg\}.$$

For $M(x)=x,\ p_m=1$ for all m , $r=0$ and $q(x)=|x|$ we get

$$BV_{\sigma}=\Bigg\{x=(x_k):\sum_{m=1}^{\infty}|\phi_{m,n}(x)|$$
$$<\infty,\ \text{uniformly in } n \text{ and for som } \rho>0\Bigg\}.$$

Theorem 2.1. The sequence space $BV_\sigma(M,p,q,r)$ is a linear space over the field $\mathbf{C}$ of complex numbers.

Proof. Let $x,y \in BV_\sigma(M,p,q,r)$ and $\alpha,\beta \in \mathbf{C}$. Then there exist positive numbers
ρ_1 and ρ_2 such that

$$\sum_{m=1}^{\infty} \frac{1}{m^r}\left[M\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho_1}\Big)\right)\right]^{p_m} < \infty$$

and

$$\sum_{m=1}^{\infty} \frac{1}{m^r}\left[M\left(q\Big(\frac{|\phi_{m,n}(y)|}{\rho_2}\Big)\right)\right]^{p_m} < \infty \text{ uniformly in } n.$$

Define $\rho_3 = \max\Big(2|\alpha|\rho_1,\ 2|\beta|\rho_2\Big)$. Since M is non-decreasing and convex we have

$$\sum_{m=1}^{\infty} \frac{1}{m^r}\left[M\left(q\left(\frac{|\alpha\phi_{m,n}(x)+\beta\phi_{m,n}(y)|}{\rho_3}\right)\right)\right]^{p_m}$$

$$\leq \sum_{m=1}^{\infty} \frac{1}{m^r}\left[M\left(q\left(\frac{|\alpha\phi_{m,n}(x)|}{\rho_3}+\frac{|\beta\phi_{m,n}(y)|}{\rho_3}\right)\right)\right]^{p_m}$$

$$\leq \sum_{m=1}^{\infty} \frac{1}{m^r}\frac{1}{2}\left[M\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho_3}\Big)\right)+M\left(q\Big(\frac{|\beta\phi_{m,n}(y)|}{\rho_3}\Big)\right)\right] < \infty \text{ uniformly in } n.$$

This proves that $BV_\sigma(M,p,q,r)$ is a linear space over the field $\mathbf{C}$ of complex numbers.

Theorem 2.2. For any Orlicz function M and a bounded sequence $p = (p_m)$ of strictly positive real numbers, $BV_\sigma(M,p,q,r)$ is a paramormed (need not be total paranormed) space with

$$g(x) = \inf_{n\geq 1}\left\{\rho^{p_n/k} : \left(\sum_{m=1}^{\infty} m^{-r}\left[M\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\right)\right]^{p_m}\right)^{1/K} \leq 1, \text{ uniformly in } n\right\}.$$

Where $K = \max\Big(1, \sup p_m\Big)$

Proof. It is clear that $g(x) = g(-x)$. Since $M(0) = 0$, we get

$$\inf\left\{\rho^{p_n/K}\right\} = 0 \quad \text{for } x = 0$$

By using Theorem 1 , for $\alpha = \beta = 1$, we get

$$g(x+y) \leq g(x) + g(y)$$

For the continuity of scalar multiplication let $l \neq 0$ be any complex number. Then by definition we have

$$g(lx) = \inf_{n\geq 1}\left\{\rho^{p_n/k} : \left(\sum_{m=1}^{\infty} m^{-r}\left[M\left(q\Big(\frac{|\phi_{m,n}(lx)|}{\rho}\Big)\right)\right]^{p_m}\right)^{1/K} \leq 1, \text{ uniformly in } n\right\}$$

$$g(lx) = \inf_{n\geq 1}\left\{\Big(s|l|\Big)^{p_n/k} : \left(\sum_{m=1}^{\infty} m^{-r}\left[M\left(q\Big(\frac{|\phi_{m,n}(lx)|}{(s|l|)}\Big)\right)\right]^{p_m}\right)^{1/K} \leq 1,\right.$$

$$\text{uniformly in } n\Bigg\}$$

Where $s = \frac{\rho}{|l|}$. Since $|l|^{p_n} \leq \max\Big(1, |l|^H\Big)$, we have

$$g(lx) \leq \max\Big(1, |l|^H\Big) \inf_{n\geq 1}\Bigg\{s^{p_n/k} : \Bigg(\sum_{m=1}^{\infty} m^{-r}\Bigg[M\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{s}\Big)\Bigg)\Bigg]^{p_m}\Bigg)^{1/K} \leq 1,$$

$$\text{uniformly in } n\Bigg\}$$

$$= \max\Big(1, |l|^H\Big) g(x)$$

and therefore $g(lx)$ converges to zero when $g(x)$ converges to zero in $BV_\sigma(M, p, q, r)$.

Now let x be fixed element in $BV_\sigma(M, p, q, r)$.Then there exists $\rho > 0$ such that

$$g(x) = \inf_{n\geq 1}\Bigg\{\rho^{p_n/k} : \Bigg(\sum_{m=1}^{\infty} m^{-r}\Bigg[M\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg]^{p_m}\Bigg)^{1/K} \leq 1,$$

$$\text{uniformly in } n\Bigg\}$$

Now

$$g(lx) = \inf_{n\geq 1}\Bigg\{\rho^{p_n/k} : \Bigg(\sum_{m=1}^{\infty} m^{-r}\Bigg[M\Bigg(q\Big(\frac{|\phi_{m,n}(lx)|}{\rho}\Big)\Bigg)\Bigg]^{p_m}\Bigg)^{1/K} \leq 1,$$

$$\text{uniformly in } n\Bigg\} \to 0$$

as $l \to 0$.

This completes the proof.

Theorem 2.3. Suppose that $0 < p_m \leq t_m < \infty$ for each $m \in \mathbf{N}$ and $r \geq 0$. Then

(a) $BV_\sigma(M,p,q) \subseteq BV_\sigma(M,t,q)$,

(b) $BV_\sigma(M,q) \subseteq BV_\sigma(M,q,r)$.

Proof.(i) suppose that $x \in BV_\sigma(M,p,q)$. This implies that $\left[M\left(q\left(\frac{|\phi_{i,n}(x)|}{\rho}\right)\right)\right]^{p_m} \leq 1$ for sufficiently large values of i, say that $i \geq m_0$ for some fixed $m_0 \in \mathbf{N}$. Since M is non- decreasing, we have

$$\sum_{m=m_0}^{\infty}\left[M\Big(q\Big(\frac{|\phi_{i,n}(x)|}{\rho}\Big)\Big)\right]^{t_m} \leq \sum_{m=m_0}^{\infty}\left[M\left(q\Big(\frac{|\phi_{i,n}(x)|}{\rho}\Big)\right)\right]^{p_m} < \infty.$$

Hence $x \in BV_\sigma(M,t,q)$.

The proof (ii) is trivial.

The following result is consequence of the above result.

Corollary 1. If $0 < p_m \leq 1$ for each m, then $BV_\sigma(M,p,q) \subseteq BV_\sigma(M,q)$. If $p_m \geq 1$ for all m, then $BV_\sigma(M,q) \subseteq BV_\sigma(M,p,q)$.

Theorem 2.4. The sequence space $BV_\sigma(M,p,q,r)$ is solid.

Proof. Let $x \in BV_\sigma(M,p,q,r)$. This implies that

$$\sum_{m=1}^{\infty} m^{-r}\left[M\left(q\Big(\frac{|\phi_{k,n}(x)|}{\rho}\Big)\right)\right]^{p_m} < \infty.$$

Let (α_m) be sequence of scalars such that $|\alpha_m| \leq 1$ for all $m \in \mathbf{N}$. Then the result follows from the following inequality.

$$\sum_{m=1}^{\infty} m^{-r}\left[M\left(q\Big(\frac{|\alpha_m \phi_{k,n}(x)|}{\rho}\Big)\right)\right]^{p_m} \leq \sum_{m=1}^{\infty} m^{-r}\left[M\left(q\Big(\frac{|\phi_{k,n}(x)|}{\rho}\Big)\right)\right]^{p_m} < \infty.$$

Hence $\alpha x \in BV_\sigma(M,p,q,r)$ for all sequences of scalar (α_m) with $|\alpha_m| \leq 1$ for all $m \in \mathbf{N}$, whenever $x \in BV_\sigma(M,p,q,r)$.

From Theorem 4 and Lemma (1.1) we have:

Corollary 2. The sequence spaces $BV_\sigma(M,p,q,r)$ is monotone.

Theorem 2.5. Let M_1 and M_2 be Orlicz functions satisfying Δ_2-condition
and $r, r_1, r_2 \geq 0$. Then we have

(i) If $r > 1$ then $BV_\sigma(M_1,p,q,r) \subseteq BV_\sigma(MoM_1,p,q,r)$,

(ii) $BV_\sigma(M_1,p,q,r) \cap BV_\sigma(M_2,p,q,r) \subseteq BV_\sigma(M_1+M_2,p,q,r)$,

(iii) If $r_1 \leq r_2$ then $BV_\sigma(M,p,q,r_1) \subseteq BV_\sigma(M,p,q,r_2)$.

Proof. (i).Since M is continuous at 0 from right, for $\epsilon > 0$ there exist $0 < \delta < 1$ such that $0 \leq c \leq \delta$ implies $M(c) < \epsilon$. If we define

$$I_1 = \left\{m \in \mathbf{N} : M_1\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\right) \leq \delta \ \text{ for som } \rho > 0\right\},$$

$$I_2 = \left\{m \in \mathbf{N} : M_1\left(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\right) > \delta \ \text{ for som } \rho > 0\right\}.$$

then, when $M_1\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg) > \delta$, we get

$$M\Bigg[M_1\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg] \leq \Bigg(\frac{2M(1)}{\delta}\Bigg)\Bigg[M_1\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg].$$

Hence for $x \in BV_\sigma(M_1, p, q, r)$ and $r > 1$

$$\sum_{m=1}^{\infty} m^{-r}\Bigg[MoM_1\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg]^{p_m}$$

$$= \sum_{m\in I_1}^{\infty} m^{-r}\Bigg[MoM_1\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg]^{p_m} + \sum_{m\in I_2}^{\infty} m^{-r}\Bigg[MoM_1\Big(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Big)\Bigg]^{p_m}$$

$$\leq \sum_{m\in I_1}^{m\in I_2} m^{-r}\Bigg[\epsilon\Bigg]^{p_m} + \sum_{m\in I_2}^{\infty}\Bigg(\frac{2M(1)}{\delta}\Bigg)\Bigg[M_1\Bigg(q\Big(\frac{|\phi_{m,n}(x)|}{\rho}\Big)\Bigg)\Bigg]^{p_m}$$

$$\leq \max\Big(\epsilon^h, \epsilon^H\Big)\sum_{m=1}^{\infty} m^{-r} + \max\Bigg\{\Bigg[\frac{2M(1)}{\delta}\Bigg]^h, \Bigg[\frac{2M(1)}{\delta}\Bigg]^H\Bigg\}$$

where $0 < h = \inf p_m \leq p_m \leq H = \sup_m p_m < \infty$.

(ii) The proof follows from the following inequality:

$$m^{-r}\Bigg[(M_1 + M_2)\Bigg(q\Bigg(\frac{|\phi_{m,n}(x)|}{\rho}\Bigg)\Bigg)\Bigg]^{p_m} \leq Cm^{-r}\Bigg[M_1\Bigg(q\Bigg(\frac{|\phi_{m,n}(x)|}{\rho}\Bigg)\Bigg)\Bigg]^{p_m} + Cm^{-r}\Bigg[M_2\Bigg(q\Bigg(\frac{|\phi_{m,n}(x)|}{\rho}\Bigg)\Bigg)\Bigg]^{p_m}.$$

(iii) The proof is straight forward.

Corollary 3. Let M be an Orlicz function satisfying Δ_2- condition. Then we have

(i) If $r > 1$ then $BV_\sigma(p, q, r) \subseteq BV_\sigma(M, p, q, r)$, (*ii*) $BV_\sigma(M,p,q) \subseteq BV_\sigma(M,p,q,r)$,

(*iii*) $BV_\sigma(p,q) \subseteq BV_\sigma(p,q,r)$,

(*iv*) $BV_\sigma(M,q) \subseteq BV_\sigma(M,q,r)$.

The proof is straight forward.

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References

[1] Z. U. Ahmad and M. Mursaleen, An application of banach limits, Proc. Amer. Math. Soc. 103, pp. 244-246, (1983).

[2] S. T. Chem, Geometry of Orlicz Spaces, Dissertationes Math. (The Institute of Mathematics, Polish Academy of Sciences) (1996).

[3] M. A. Krasnoselskii and Rutickii, Ya. B, Convex Functions and Orlicz Spaces, (Gooningen: P.Nordhoff Ltd.) (1961)(translation)

[4] J. Lindenstrauss and L. Tzafriri, On Orlicz sequence spaces, Israel J. Math., 10, pp. 379-390, (1971).

[5] G. G. Lorenz, A contribution to the theory of divergent sequences, Acta Math. 80, pp. 167-190, (1948).

[6] W. A. Luxemburg, Banach Function Spaces, Thesis (Delft), (1995).

[7] L. Maligranda, Orlicz Space and Interpolation, Seminar in Math.5 Campinas (1989).

[8] M. Mursaleen, Matrix Transformation Between some new sequence spaces, Houston J. Math., 9, pp. 505-509, (1983).

[9] M. Mursaleen, On some new invariant matrix methods of summability, Quart. J. Math., Oxford (2) 34, pp. 77-86, (1983).

[10] J. Musielak, Orlicz spaces and modular spaces, Lecture Notes in Math. 1034 (Springer-Verlag)(1983).

[11] W. Orlicz, Über Raume (L^M), Bulletin International del' Académie Polonaise de Sciences et des Letters, Série A, pp. 93-107, (1936).

[12] R. A. Raimi, Invariant means and invariant matrix method of summability, Duke Math. J., 30, pp. 81-94, (1963).

[13] M. M. Rao and Z.D.Ren, Theory of Orlicz spaces (New york, Basel, Hong Kong: Marcal Dekker Inc.) (1991)

[14] P. Schafer, Infinite matrices and invariant means Proc .Ammer. Math. Soc. 36, pp. 104-110, (1972).

[15] A. Wilansky, Summability through FunctionAlalysis,North-Holland Mathematical Studies, 85 (1984)

[16] K.Yosidak, Functional Analysis, Springer- Verlag, Berlian- Heidelberg Newyork., (1971)

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