Proyecciones Journal of Mathematics
Vol. 33, N^o 2, pp. 175-187, June 2014.
Universidad Católica del Norte
Antofagasta - Chile
DOI: 10.4067/S0716-09172014000200004

Upper Edge Detour Monophonic Number of a Graph

P. Titus
University College of Engineering Nagercoil, India
and
K. Ganesamoorthy
University V. O. C. College of Engineering Tuticorin, India
Received : November 2013. Accepted : March 2014

Abstract

For a connected graph G of order at least two, a path P is called a monophonic path if it is a chordless path. A longest $x-y$ monophonic path is called an $x-y$ detour monophonic path. A set S of vertices of G is an edge detour monophonic set of G if every edge of G lies on a detour monophonic path joining some pair of vertices in S. The edge detour monophonic number of G is the minimum cardinality of its edge detour monophonic sets and is denoted by $edm(G)$. An edge detour monophonic set S of G is called a minimal edge detour monophonic set if no proper subset of S is an edge detour monophonic set of G. The upper edge detour monophonic number of G, denoted by $edm^+(G)$, is defined as the maximum cardinality of a minimal edge detour monophonic set of G. We determine bounds for it and characterize graphs which realize these bounds. For any three positive integers b, c and n with $2 \leq b \leq n \leq c$, there is a connected graph G with $edm(G) = b$, $edm^+(G) = c$ and a minimal edge detour monophonic set of cardinality n.

Key Words: *edge detour monophonic set, edge detour monophonic number, minimal edge detour monophonic set, upper edge detour monophonic set, upper edge detour monophonic number.*

AMS Subject Classification: *05C12.*

1. Introduction

By a graph $G = (V, E)$ we mean a finite undirected connected graph without loops or multiple edges. The order and size of G are denoted by p and q, respectively. For basic graph theoretic terminology we refer to Harary [1]. For vertices x and y in a connected graph G, the *distance* $d(x, y)$ is the length of a shortest $x-y$ path in G. An $x-y$ path of length $d(x, y)$ is called an $x-y$ *geodesic*. The *neighborhood* of a vertex v is the set $N(v)$ consisting of all vertices u which are adjacent with v. A vertex v is an *extreme vertex* if the subgraph induced by its neighbors is complete. A vertex v is a *semi-extreme vertex* of G if the subgraph $G[S]$ induced by its neighborhood S has a vertex with degree equal to $|S| - 1$. In particular, every extreme vertex is a semi-extreme vertex and a semi-extreme vertex need not be an extreme vertex.

A *chord* of a path P is an edge joining two non-adjacent vertices of P. A path P is called *monophonic* if it is a chordless path. A longest $x - y$ monophonic path is called an $x - y$ *detour monophonic path*. A set S of vertices of a graph G is a *detour monophonic set* if each vertex v of G lies on an $x - y$ detour monophonic path for some $x, y \in S$. The cardinality of a detour monophonic set of G with minimum cardinality is the *detour monophonic number* of G and is denoted by $dm(G)$. The detour monophonic number of a graph was introduced in [4] and further studied in [5].

An *edge monophonic set* of G is a set S of vertices such that every edge of G lies on a monophonic path joining some pair of vertices in S. The *edge monophonic number* of G is the minimum cardinality of its edge monophonic sets and is denoted by $m_1(G)$. An edge monophonic set of cardinality $m_1(G)$ is an m_1-set of G. An *edge detour monophonic set* of G is a set S of vertices such that every edge of G lies on a detour monophonic path joining some pair of vertices in S. The *edge detour monophonic number* of G is the minimum cardinality of its edge detour monophonic sets and is denoted by $edm(G)$. An edge detour monophonic set of cardinality $edm(G)$ is an edm-set of G. The edge detour monophonic number of a graph was introduced and studied in [3].

The following theorems will be used in the sequel.

Theorem 1.1. [2] Each semi-extreme vertex of a graph G belongs to every edge monophonic set of G.

Theorem 1.2. [3] Each semi-extreme vertex of a graph G belongs to every edge detour monophonic set of G.

Theorem 1.3. [3] Let G be a connected graph with cut-vertices and S an edge detour monophonic set of G. If v is a cut-vertex of G, then every component of $G - v$ contains an element of S.

Theorem 1.4. [3] For any connected graph G, no cut-vertex of G belongs to any minimum edge detour monophonic set of G.

Theorem 1.5. [3] If T is a tree with k end-vertices, then $m_1(T) = edm(T) = k$.

Throughout this paper G denotes a connected graph with at least two vertices.

2. Upper edge detour monophonic number

Definition 2.1. Let G be a connected graph with at least two vertices. An edge detour monophonic set S of G is called a *minimal edge detour monophonic set* if no proper subset of S is an edge detour monophonic set of G. The *upper edge detour monophonic number* of G, denoted by $edm^+(G)$, is defined as the maximum cardinality of a minimal edge detour monophonic set of G.

Example 2.2. For the graph G given in Figure 2.1, the minimal edge detour monophonic sets are $S_1 = \{x, z\}$ and $S_2 = \{y, u, v\}$. Hence the upper edge detour monophonic number of G is 3.

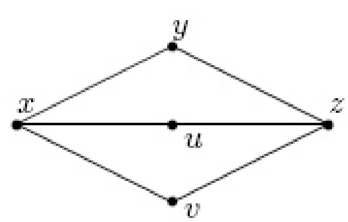

Figure 2.1: G

Note 2.3 Every minimum edge detour monophonic set is a minimal edge detour monophonic set, but the converse is not true. For the graph G given in Figure 2.1, S_2 is a minimal edge detour monophonic set but it is not a minimum edge detour monophonic set of G.

Since every minimal edge detour monophonic set of G is an edge detour monophonic set of G, we have the following theorems.

Theorem 2.3. Each semi-extreme vertex of a connected graph G belongs to every minimal edge detour monophonic set of G.

Proof. This follows from Theorem 1.2. □

Corollary 2.4. For the complete graph K_p, $edm^+(K_p) = p$.

Proof. Since every vertex of K_p is a semi-extreme vertex, the result follows from Theorem 2.3. □

Theorem 2.5. Let G be a connected graph with cut-vertices and let S be a minimal edge detour monophonic set of G. If v is a cut-vertex of G, then every component of $G - v$ contains an element of S.

Proof. This follows from Theorem 1.3. □

Corollary 2.6. Let G be a connected graph with cut-vertices and let S be a minimal edge detour monophonic set of G. Then every branch of G contains an element of S.

Proof. This follows from Theorem 2.5. □

Theorem 2.7. No cut-vertex of a connected graph G belongs to any minimal edge detour monophonic set of G.

Proof. This follows from Theorem 1.4. □

Corollary 2.8. For any tree T with k end-vertices, $edm(T) = edm^+(T) = k$.

Proof. Since every vertex of T is either a semi-extreme vertex or a cut-vertex, the result follows from Theorems 2.3 and 2.7. □

We denote the vertex connectivity of a connected graph G by $\kappa(G)$ or κ.

Theorem 2.9. If G is a non-complete connected graph such that it has a minimum cut set consisting of κ vertices, then $edm^+(G) \leq p - \kappa$.

Proof. Since G is a non-complete connected graph, it is clear that $1 \leq \kappa \leq p-2$. Let U $=\{u_1, u_2, u_3, ..., u_\kappa\}$ be a minimum cut set of G. Let $G_1, G_2, ..., G_r (r \geq 2)$ be the components of $G - U$ and let $S = V(G) - U$. Then every vertex $u_i (1 \leq i \leq \kappa)$ is adjacent to at least one vertex of G_j for each j $(1 \leq j \leq r)$. It is clear that S is an edge detour monophonic set of G and so $edm^+(G) \leq |S| = p - \kappa$. □

Remark 2.10. The bound in Theorem 2.9 is sharp for the graph G given in Figure 2.1.

Theorem 2.11. For any connected graph G, $2 \leq edm(G) \leq edm^+(G) \leq p$.

Proof. It is clear from the definition of minimum edge detour monophonic set that $edm(G) \geq 2$. Since every minimal edge detour monophonic set is an edge detour monophonic set of G, $edm(G) \leq edm^+(G)$. Also, since $V(G)$ induces an edge detour monophonic set of G, it is clear that $edm^+(G) \leq p$. Thus $2 \leq edm(G) \leq edm^+(G) \leq p$. □

Remark 2.12. The bounds in Theorem 2.11 are all sharp for K_2. Furthermore, for any tree T with k end-vertices $edm(T) = edm^+(T) = k$ (notice that a non-trivial path is a tree with two end-vertices) and for the complete graph K_p, $edm^+(K_p) = p$.

Theorem 2.13. For a connected graph G, $edm(G) = p$ if and only if $edm^+(G) = p$.

Proof. Let $edm^+(G) = p$. Then $S = V(G)$ is the unique minimal edge detour monophonic set of G. Since no proper subset of S is an edge detour monophonic set, it is clear that S is the unique minimum edge detour monophonic set of G and so $edm(G) = p$. The converse follows from Theorem 2.11. □

Theorem 2.14. If G is a connected graph with $edm(G) = p - 1$, then $edm^+(G) = p - 1$.

Proof. Since $edm(G) = p-1$, it follows from Theorem 2.11 that $edm^+(G) = p$ or $p-1$. If $edm^+(G) = p$, then by Theorem 2.13, $edm(G) = p$, which is a contradiction. Hence $edm^+(G) = p-1$. □

Theorem 2.15. For the complete bipartite graph $G = K_{m,n}$,
(i) $edm^+(G) = 2$ if $m = n = 1$.
(ii) $edm^+(G) = n$ if $m = 1, n \geq 2$.
(iii) $edm^+(G) = max\{m, n\}$ if $m, n \geq 2$.

Proof. (i) and (ii) follows from Corollary 2.8.
(iii) Let $m, n \geq 2$. Assume without loss of generality that $m \leq n$. Let $X = \{x_1, x_2, \ldots, x_m\}$ and $Y = \{y_1, y_2, \ldots, y_n\}$ be the bipartition of G. Let $S = Y$. We prove that S is a minimal edge detour monophonic set of G. Any edge $x_iy_j (1 \leq i \leq m, 1 \leq j \leq n)$ lies on the detour monophonic path y_j, x_i, y_k for any $k \neq j$ so that S is an edge detour monophonic set of G. Let $S' \subset S$. Then there exists a vertex $y_j \in S$ such that $y_j \notin S'$. Clearly, the edge x_iy_j for every i, does not lie on any detour monophonic path joining a pair of vertices in S'. Thus S' is not an edge detour monophonic set of G. Hence S is a minimal edge detour monophonic set of G and so $edm^+(G) \geq n$.

Let S_1 be any minimal edge detour monophonic set of G with $|S_1| > n$. Since any edge $x_jy_i (1 \leq i \leq n)$ for every j, lies on the detour monophonic path x_j, y_i, x_k for $j \neq k$, it follows that X is an edge detour monophonic set of G. Hence S_1 cannot contain X. Similarly, since Y is a minimal edge detour monophonic set of G, S_1 cannot contain Y. Hence $S_1 \subseteq X' \cup Y'$, where $X' \subset X$ and $Y' \subset Y$. Hence there exists a vertex $x_i \in X (1 \leq i \leq m)$ and a vertex $y_j \in Y (1 \leq j \leq n)$ such that $x_i, y_j \notin S_1$. It is easily seen that, the edge x_iy_j does not lie on any $x - y$ detour monophonic path, for any $x, y \in S_1$. Thus S_1 is not an edge detour monophonic set of G, which is a contradiction. Therefore, any minimal edge detour monophonic set of G contains at most n elements so that $edm^+(G) \leq n$. Hence $edm^+(G) = n$. □

Theorem 2.16. For any three positive integers a, b, c with $2 \leq a \leq b \leq c$, there is a connected graph G with $m_1(G) = a$, $edm(G) = b$ and $edm^+(G) = c$.

Proof. **Case 1.** $2 \leq a = b = c$. Let G be any tree with a end-vertices. Then by Theorem 1.5 and Corollary 2.8, G has the desired properties.

Case 2. $2 \le a = b < c$. Let $P_3 : x, y, z$ be the path of order 3. Let G be the graph obtained by adding $c-1$ new vertices $v_1, v_2, \dots, v_{a-1}, w_1, w_2, \dots, w_{c-a}$ to P_3 and joining each $w_i (1 \le i \le c-a)$ to both x, z; and also joining each $v_i (1 \le i \le a-1)$ to x. The graph G is shown in Figure 2.2. Let $S = \{v_1, v_2, \dots, v_{a-1}\}$ be the set of all end-vertices of G.

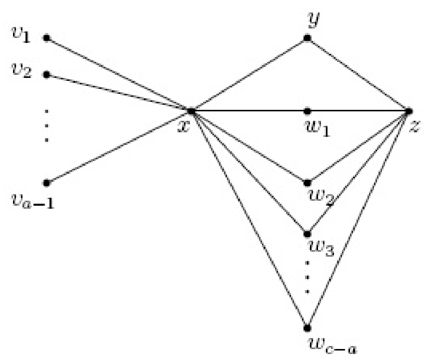

Figure 2.2: G

Then by Theorems 1.1, 1.2 and 2.3, S is contained in every edge monophonic set, every edge detour monophonic set and every minimal edge detour monophonic set of G. It is clear that S is not an edge monophonic set of G. It is easily verified that $S' = S \cup \{z\}$ is a minimum edge monophonic set of G and also a minimum edge detour monophonic set of G. Thus $m_1(G) = edm(G) = a$.

Next we show that $edm^+(G) = b$. Clearly $T = S \cup \{y, w_1, w_2, \dots, w_{c-a}\}$ is an edge detour monophonic set of G. We claim that T is a minimal edge detour monophonic set of G. Let W be any proper subset of T. Then there exists a vertex, say v, such that $v \in T$ and $v \notin W$. By Theorem 2.3, $v \in \{y, w_1, w_2, \dots, w_{c-a}\}$. It is easily verified that the edge vz is not an internal edge of any $x-y$ detour monophonic path for some $x, y \in W$, it follows that W is not an edge detour monophonic set of G. Hence T is a minimal edge detour monophonic set of G and so $edm^+(G) \ge c$.

Now, we prove that $edm^+(G) = c$. Suppose that $edm^+(G) > c$. Let N be a minimal edge detour monophonic set of G with $|N| > c$. Then there exists at least one vertex, say $u \in N$ such that $u \notin T$. Then by Theorem 2.7, $u \neq x$ and so $u = z$. Clearly $S \cup \{z\}$ is an edge detour monophonic set of G and it is a proper subset of N, which is a contradiction to N a minimal edge detour monophonic set of G. Hence $edm^+(G) = c$.

Case 3. $2 \leq a < b = c$. Let $C_i : u_i, v_i, w_i, x_i, y_i, u_i$ $(1 \leq i \leq b-a)$ be $b-a$ copies of a cycle of order 5. Let H be the graph obtained from $C_i(1 \leq i \leq b-a)$ by joining the vertices w_{i-1} of C_{i-1} and u_i of $C_i(2 \leq i \leq b-a)$. Let G be the graph obtained from H by adding a new vertices $z, z_1, z_2, \ldots, z_{a-1}$ and (i) joining z to u_1, (ii) joining each $z_j(1 \leq j \leq a-1)$ to w_{b-a}. The graph G is shown in Figure 2.3. Let $S = \{z, z_1, z_2, \ldots, z_{a-1}\}$ be the set of all extreme vertices of G. Then by Theorem 1.1, every edge monophonic set of G contains S. Clearly, S is an edge monophonic set of G and so $m_1(G) = a$.

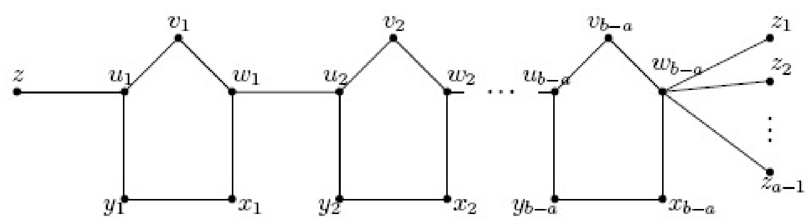

Figure 2.3: G

By Theorem 1.2 and Theorem 2.3, every edge detour monophonic set and every minimal edge detour monophonic set of G contains S. Clearly, S is not a minimal and minimum edge detour monophonic set of G. We observe that every minimal and minimum edge detour monophonic set of G contains exactly one vertex from each set $\{v_i, x_i, y_i\}$ for every $i(1 \leq i \leq b-a)$. Thus $edm(G) \geq b$ and $edm^+(G) \geq b$. On the other hand, $S' = S \cup \{v_1, v_2, \cdots v_{b-a}\}$ is a minimum edge detour monophonic set of G, it follows that $edm(G) \leq b$. Thus $edm(G) = b$. By Theorem 2.7, no cut-vertex of G belongs to any minimal edge detour monophonic set of G. It follows that there does not exist a minimal edge detour monophonic set N of G with $|N| > b$. Hence $edm^+(G) = b$.

Case 4. $2 \leq a < b < c$. Let $V(K_2) = \{x, y\}$ and $V(K_{c-b+1}) = \{l_1, l_2, \ldots l_{c-b+1}\}$. Let $H = \overline{K}_{c-b+1} + \overline{K}_2$. Let $C_i : u_i, v_i, w_i, x_i, y_i, u_i(1 \leq i \leq b-a)$ be $b-a$ copies of a cycle of order 5. Let H' be the graph obtained from

$C_i(1 \le i \le b-a)$ by joining the vertices w_{i-1} of C_{i-1} and u_i of $C_i(2 \le i \le b-a)$. Let G be the graph obtained by joining the vertices w_{b-a} from H' and x from H, and then adding $a-1$ new vertices $z, z_1, z_2, \ldots, z_{a-2}$; and (i) joining z to u_1, (ii) joining $z_j(1 \le j \le a-2)$ to x. The graph G is shown in Figure 2.4. Let $S = \{z, z_1, z_2, \ldots, z_{a-2}\}$ be the set of all end-vertices of G. Then by Theorem 1.1, every edge monophonic set of G contains S. Clearly, S is not an edge monophonic set of G. Let $S' = S \cup \{y\}$. It is easily verified that S' is an edge monophonic set of G and so $m_1(G) = a$.

By Theorem 1.2 and Theorem 2.3, every edge detour monophonic set of G and every minimal edge detour monophonic set of G contains S. Clearly, S is not an edge detour monophonic set of G. We observe that every minimum edge detour monophonic set of G contains y and exactly one vertex from $\{v_i, x_i, y_i\}$ for every $i(1 \le i \le b-a)$. Thus $edm(G) \ge b$. On the other hand, $S' = S \cup \{v_1, v_2, \cdots v_{b-a}, y\}$ is a minimum edge detour monophonic set of G and so $edm(G) = b$.

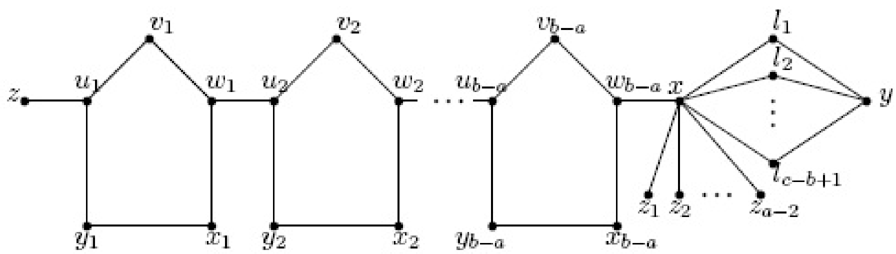

Figure 2.4: G

Now, $T = S \cup \{v_1, v_2, \ldots, v_{b-a}, l_1, l_2, \ldots, l_{c-b+1}\}$ is an edge detour monophonic set of G. We show that T is a minimal edge detour monophonic set of G. Let W be any proper subset of T. Then there exists at least one vertex, say $v \in T$, such that $v \notin W$. If $v = v_i(1 \le i \le b-a)$, the edge vu_i is not an internal edge of any $x-y$ detour monophonic path for some $x, y \in W$, it follows that W is not an edge detour monophonic set of G. If $v = l_i(1 \le i \le c-b+1)$, the edge vy is not an internal edge of any $x-y$ detour monophonic path for some $x, y \in W$, it follows that W is not an edge detour monophonic set of G. Hence T is a minimal edge detour monophonic set of G and so $edm^+(G) \ge c$.

Next we show that there is no minimal edge detour monophonic set X of G with $|X| \geq c+1$. Suppose that there exists a minimal edge detour monophonic set X of G such that $|X| \geq c+1$. Then there exists at least one vertex, say, $v \in X$ such that $v \notin T$. We observe that every minimal edge detour monophonic set contains exactly one element from $\{v_i, x_i, y_i\}$ for every $i(1 \leq i \leq b-a)$. Hence by Theorem 2.7, $v = y$. Clearly, $(X - \{l_1, l_2, \ldots, l_{c-b+1}\}) \cup \{y\}$ is a minimal edge detour monophonic set of G, which is a contradiction. Therefore $edm^+(G) = c$. □

Theorem 2.17. For any three positive integers b, c and n with $2 \leq b \leq n \leq c$, there is a connected graph G with $edm(G) = b$, $edm^+(G) = c$ and a minimal edge detour monophonic set of cardinality n.

Proof. We consider four cases.

Case 1. $b = n = c$. Let G be any tree with b end-vertices. Then by Corollary 2.8, G has the desired properties.

Case 2. $b = n < c$. For the graph G given in Figure 2.2 of Theorem 2.16, it is proved that $edm(G) = b, edm^+(G) = c$ and $S = \{z, v_1, v_2, \ldots, v_{b-1}\}$ is a minimal edge detour monophonic set of cardinality n.

Case 3. $b < n = c$. For the graph G given in Figure 2.2 of Theorem 2.16, it is proved that $edm(G) = b, edm^+(G) = c$ and $S = \{v_1, v_2, \ldots, v_{b-1}, y, w_1, w_2, \ldots, w_{c-b}\}$ is a minimal edge detour monophonic set of cardinality n.

Case 4. $b < n < c$. Let $l = n - b + 1$ and $m = c - n + 1$.

Let $F_1 = mK_1 + \overline{K_2}$, where $U_1 = V(\overline{K_2}) = \{x, u_1\}$ and $X = V(mK_1) = \{x_1, x_2, \ldots, x_m\}$. Similarly, $F_2 = lK_1 + \overline{K_2}$, where $U_2 = V(\overline{K_2}) = \{u_2, y\}$ and $Y = V(lK_1) = \{y_1, y_2, \ldots, y_l\}$. Let $K_{1,b-2}$ be the star at the vertex u and let $S = \{w_1, w_2, \ldots, w_{b-2}\}$ be the set of end-vertices of $K_{1,b-2}$. Let G be the graph obtained from $K_{1,b-2}$, F_1 and F_2 by identifying the vertices u from $K_{1,b-2}$, u_1 from F_1 and u_2 from F_2. The graph G is shown in Figure 2.5. It follows from Theorem 2.3, every minimal edge detour monophonic set contains S.

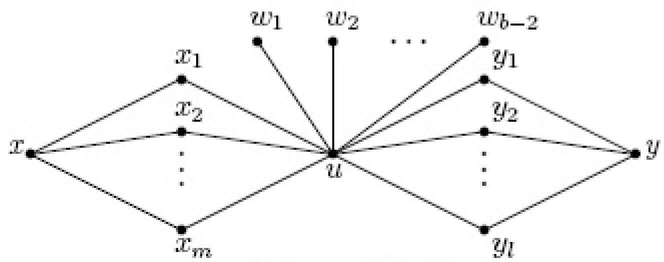

Figure 2.5: G

First we show that $edm(G) = b$. It is clear that S is not an edge detour monophonic set of G. Also, for any $v \in V(G) - S$, $S \cup \{v\}$ is not an edge detour monophonic set of G. Let $S' = S \cup \{x, y\}$. It is easily verified that S' is a minimum edge detour monophonic set of G and so $edm(G) = b$.

Next, we show that $edm^+(G) = c$. Let $T = S \cup X \cup Y$. It is clear that T is an edge detour monophonic set of G. We claim that T is a minimal edge detour monophonic set of G. Let W be any proper subset of T. Then there exists a vertex, say, $v \in T$ such that $v \notin W$. Assume first that $v = x_i$ for some $i (1 \leq i \leq m)$ or $v = y_j$ for some $j (1 \leq j \leq l)$. Then the edge uv is not an internal edge of any detour monophonic path joining a pair of vertices in W. If $v = w_i$ for some $i (1 \leq i \leq b-2)$, then the edge uw_i is not an internal edge of any $x - y$ detour monophonic path for some $x, y \in W$. Hence T is a minimal edge detour monophonic set of G and so $edm^+(G) \geq |T| = b - 2 + l + m = c$.

Now, we prove that $edm^+(G) = c$. Suppose that $edm^+(G) > c$. Let T' be a minimal edge detour monophonic set of G with $|T'| > c$. Then there exists at least one vertex, say $v \in T'$ such that $v \notin T$. Also, by Theorem 2.7, $v \in \{x, y\}$. If $v = x$, then $T' - X$ is an edge detour monophonic set of G and it is a proper subset of T', which is a contradiction to T' a minimal edge detour monophonic set of G. Similarly, if $v = y$, then $T' - Y$ is an edge detour monophonic set of G and it is a proper subset T', which is a contradiction. Hence $edm^+(G) = c$.

Next we show that there is a minimal edge detour monophonic set of cardinality n. Let $P = \{w_1, w_2, ..., w_{b-2}, x, y_1, y_2, ..., y_l\}$. It is clear that P is an edge detour monophonic set of G. We claim that P is a minimal edge detour monophonic set of G. Assume, to the contrary, that P is not a minimal edge detour monophonic set of G. Then there is a proper subset P' of P such that P' is an edge detour monophonic set of G. Let $v \in P$ and $v \notin P'$. By Theorem 1.2, clearly $v = x$ or $v = y_i$ for some $i = 1, 2, \ldots, l$. If $v = x$, then the edges vx_j and $x_j u (1 \leq j \leq m)$ are not internal edges of any $s - t$ detour monophonic path for some $s, t \in P'$. If $v = y_i$ for some $i = 1, 2, \ldots l$, then the edge vu is not an internal edge of any $s - t$ detour monophonic path for some $s, t \in P'$. Hence P is a minimal edge detour monophonic set of G with cardinality $|P| = n$. □

References

[1] F. Harary, Graph Theory, Addison-Wesley, (1969).

[2] A. P. Santhakumaran, P. Titus and P. Balakrishnan, Some Realisation Results on Edge Monophonic Number of a Graph, communicated.

[3] A. P. Santhakumaran, P. Titus, K. Ganesamoorthy and P. Balakrishnan, Edge Detour Monophonic Number of a Graph, Proyecciones Journal of Mathematics, Vol. 32, No. 2, pp. 183-198, (2013).

[4] P. Titus, K. Ganesamoorthy and P. Balakrishnan, The Detour Monophonic Number of a Graph, J. Combin. Math. Combin. Comput. 83, pp. 179-188, (2013).

[5] P. Titus and K. Ganesamoorthy, On the Detour Monophonic Number of a Graph, Ars Combinatoria, to appear.

P. Titus
Department of Mathematics
University College of Engineering Nagercoil
Anna University, Tirunelveli Region
Nagercoil - 629 004,
India
e-mail: titusvino@yahoo.com

and

K. Ganesamoorthy
Department of Mathematics
University V.O.C. College of Engineering
Anna University, Tirunelveli Region
Tuticorin - 628 008,
India
e-mail : kvgm_2005@yahoo.co.in