Proyecciones Journal of Mathematics
Vol. 34, N^o 1, pp. 15-24, March 2015.
Universidad Católica del Norte
Antofagasta - Chile
DOI: 10.4067/S0716-09172015000100002

On some maps concerning g$\beta\theta$-open sets

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Received : January 2014. Accepted : February 2015

Abstract

In this paper, we consider a new notion of $\beta\theta$-open maps via the concept of $g\beta\theta$-closed sets which we call approximately $\beta\theta$-open maps. We study some of its fundamental properties. It turns out that we can use this notion to obtain a new characterization of $\beta\theta$-$T_{\frac{1}{2}}$ spaces.

1991 Math. Subject Classification : *54C10, 54D10.*

Key Words and Phrases : *Topological spaces, $g\beta\theta$-closed sets, $\beta\theta$-open sets, $\beta\theta$-open maps, β-irresolute maps, $\beta\theta$-$T_{\frac{1}{2}}$ spaces.*

1. Introduction and Preliminaries

In 2003, T. Noiri [10] introduced a new class of sets called $\beta\theta$-open in topological space. In M. Caldas [4, 5], a $\beta\theta$-open function is defined as follows: A function $f : (X,\tau) \to (Y,\sigma)$ is $\beta\theta$-open if $f(A)$ is $\beta\theta$-open in Y for

open set A of X, and weakly $\beta\theta$-open functions which is independent with weakly open functions given by D.A. Rose [11]. In this direction, we introduce the notion of ap-$\beta\theta$-open maps by using g$\beta\theta$-closed sets and study some of their basic properties. Finally we characterize the class of $\beta\theta$-$T_{\frac{1}{2}}$ spaces in terms of ap-$\beta\theta$-open maps.

Throughout the present paper, spaces (X,τ) and (Y,σ) (or simply, X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated. Abd. El Monsef et al. [1] and Andrijević [2] introduced the notion of β-open set, which Andrijević called semipreopen, completely independent of each other. In this paper, we adopt the word β-open for the sake of clarity. A subset A of a topological space (X,τ) is called β-open if $A \subseteq$Cl(Int(Cl(A))), where Cl(A) and Int(A) denote the closure and the interior of A, respectively. The complement of a β-open set is called β-closed. The family of all β-open sets of a space X is denoted by $\beta O(X,\tau)$ or $\beta O(X)$. We set $\beta O(X,x) = \{U : x \in U \in \beta O(X)\}$. The intersection of all β-closed sets containing A is called the β-closure of A and is denoted by $\beta Cl(A)$. The β-interior of a subset $A \subset X$, denoted by $\beta Int(A)$, is the union of all β-open sets contained in A.

Now we recall the well-known notions given by T.Noiri [10] which will be used in the sequel.

Definition 1. *[10]. Let A a subset of X. The β-θ-*closure *of A, denoted by $\beta Cl_\theta(A)$, is the set of all $x \in X$ such that $\beta Cl(O) \cap A \neq \emptyset$ for every $O \in \beta O(X,x)$. A subset A is called β-θ-*closed *if $A = \beta Cl_\theta(A)$. The set $\{x \in X \mid \beta Cl_\theta(O) \subset A$ for some $O \in \beta O(X,x)\}$ is called the β- θ-*interior *of A and is denoted by $\beta \mathrm{Int}_\theta(A)$. A subset A is called β-θ-*open *if $A = \beta\mathrm{Int}_\theta(A)$.*

Theorem 1.1. *[10]. For any subset A of X:*
(1) $\beta Cl_\theta(\beta Cl_\theta(A)) = \beta Cl_\theta(A)$.
(2) $\beta Cl_\theta(A)$ is $\beta\theta$-closed.
(3) Intersection of arbitrary collection of $\beta\theta$-closed set in X is $\beta\theta$-closed.

(4) $\beta Cl_\theta(A)$ is the intersection of all $\beta\theta$-closed sets each containing A.
(5) If $A \in \beta O(X)$ then, $\beta Cl(A) = \beta Cl_\theta(A)$.

A map $f : (X, \tau) \to (Y, \sigma)$ is called β-irresolute [10] if $f^{-1}(O)$ is β-open in (X, τ) for every $O \in \beta O(Y, \tau)$.

A map $f : (X, \tau) \to (Y, \sigma)$ is called pre-$\beta\theta$-closed (resp. pre-$\beta\theta$-open) [6, 7] if for every $\beta\theta$-closed (resp. $\beta\theta$-open) set B of $(X, \tau), f(B)$ is $\beta\theta$-closed (resp. $\beta\theta$-open) in (Y, σ).

For other advances on topological spaces obtained by our research group we recommend [3, 4, 8, 10].

2. Ap-$\beta\theta$-open maps.

We introduce the following notion:

Definition 2. *A subset A of a topological space (X, τ) is called* generalized $\beta\theta$-closed *(briefly $g\beta\theta$-closed) if $\beta Cl_\theta(A) \subset O$ whenever $A \subset O$ and $O \in \beta\theta O(X, \tau)$.*

Lemma 2.1. *(i) Every $\beta\theta$-closed set is $g\beta\theta$-closed.*
(ii) [10] Every $\beta\theta$-closed set is β-closed.
But not conversely.

Proof. (i) Let $A \subset X$ be $\beta\theta$-closed. Then $A = \beta\mathrm{Cl}_\theta(A)$. Let $A \subset O$ and $O \in \beta\theta O(X, \tau)$. It follows that $\beta\mathrm{Cl}_\theta(A) \subset O$. This means that A is g$\beta\theta$-closed.
(ii) See Remark 3.2 in [10].

Example 2.2. *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. If $A = \{a, b\}$. Then $\beta Cl(A) = X$ and so A is not β-closed. Hence A is not $\beta\theta$-closed. Since X is the only $\beta\theta$-open containing A. A is $g\beta\theta$-closed.*

Example 2.3. *Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. If $A = \{c\}$, then A is a β-closed set which is not $\beta\theta$-closed.*

Recall that, the complement of a generalized $\beta\theta$-closed set is called generalized $\beta\theta$-open (briefly g$\beta\theta$-open).

Theorem 2.4. *A set $A \subset (X, \tau)$ is $g\beta\theta$-open if and only if $F \subset \beta Int_\theta(A)$ whenever F is $\beta\theta$-closed in X and $F \subset A$.*

Proof. Necessity. Let A be g$\beta\theta$-open and $F \subset A$, where F is $\beta\theta$-closed. It is obvious that A^c is contained in F^c. This implies that $\beta Cl_\theta(A^c) \subset F^c$. Hence $\beta Cl_\theta(A^c) = (\beta Int_\theta(A))^c \subset F^c$, i.e. $F \subset \beta Int_\theta(A)$.

Sufficiency. If F is a $\beta\theta$-closed set with $F \subset \beta Int_\theta(A)$ whenever $F \subset A$, then it follows that $A^c \subset F^c$ and $(\beta Int_\theta(A))^c \subset F^c$ i.e., $\beta Cl_\theta(A^c) \subset F^c$. Therefore A^c is g$\beta\theta$-closed and therefore A is g$\beta\theta$-open. Hence the proof. $\square$

Definition 3. *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be approximately $\beta\theta$-open (briefly ap-$\beta\theta$-open) if $\beta Cl_\theta(B) \subseteq f(A)$ whenever B is a g$\beta\theta$-closed subset of (Y, σ), A is a $\beta\theta$-open subset of (X, τ) and $B \subseteq f(A)$.*

Theorem 2.5. *$f : (X, \tau) \to (Y, \sigma)$ is ap-$\beta\theta$-open if $f(O) \in \beta\theta C(Y, \sigma)$ for every $\beta\theta$-open subset O of (X, τ).*

Proof. Let $B \subseteq f(A)$, where A is a $\beta\theta$-open subset of (X, τ) and B is a g$\beta\theta$-closed subset of (Y, σ). Therefore $\beta Cl_\theta(B) \subseteq \beta Cl_\theta(f(A)) = f(A)$. Thus f is ap-$\beta\theta$-open. $\square$

Clearly pre-$\beta\theta$-open maps are ap-$\beta\theta$-open, but not conversely.

Example 2.6. *Let $X = \{a, b\}$ be the Sierpinski space with the topology, $\tau = \{\emptyset, \{a\}, X\}$. Let $f : X \to X$ be defined by $f(a) = b$ and $f(b) = a$. Since the image of every $\beta\theta$-open set is $\beta\theta$-closed, then f is ap-$\beta\theta$-open. However $\{a\}$ is $\beta\theta$-open in (X, τ) but $f(\{a\})$ is not $\beta\theta$-open in (Y, σ). Therefore f is not pre-$\beta\theta$-open.*

Remark 2.7. *It is clear that the converse of Theorem 2.5 does not hold: Let (X, τ) be the topological space defined in Example 2.6. Then the identity map on (X, τ) is ap-$\beta\theta$-open, but $f(a) \notin \beta\theta C(X, \tau)$ for $\{a\}$ a $\beta\theta$-open set of (X, τ).*

In the following theorem, we show that under certain conditions the converse of Theorem 2.5 is true.

Theorem 2.8. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map. If the $\beta\theta$-open and $\beta\theta$-closed sets of (Y, σ) coincide, then f is ap-$\beta\theta$-open if and only if $f(A) \in \beta\theta C(Y, \sigma)$ for every $\beta\theta$-open subset A of (X, τ).*

Proof. Suppose that f is ap-$\beta\theta$-open. Let A be an arbitrary subset of (Y, σ) such that $A \subseteq Q$ where $Q \in \beta\theta O(Y, \sigma)$. Then by hypothesis

$\beta Cl_\theta(A) \subseteq \beta Cl_\theta(Q) = Q$. Therefore all subset of (Y, σ) are g$\beta\theta$-closed and hence all are g$\beta\theta$-open. So for any $O \in \beta\theta O(X, \tau)$,$f(O)$ is g$\beta\theta$-closed in (Y, σ). Since f is ap-$\beta\theta$-open $\beta Cl_\theta(f(O)) \subseteq f(O)$. Therefore $\beta Cl_\theta(f(O)) = f(O)$, i.e., $f(O)$ is $\beta\theta$-closed in (Y, σ). The converse is obvious by Theorem 2.5. □

As an immediate consequence of Theorem 2.8, we have the following:

Corollary 2.9. *Let $f : (X, \tau) \to (Y, \sigma)$ be a map. If the $\beta\theta$-closed and $\beta\theta$-open sets of (Y, σ) coincide, then f is ap-$\beta\theta$-open if and only if, f is pre-$\beta\theta$-open.*

Definition 4. *A map $f : (X, \tau) \to (Y, \sigma)$ is called contra pre-$\beta\theta$-open if $f(O)$ is $\beta\theta$-closed in (Y, σ) for each set $O \in \beta\theta O(X, \tau)$.*

Remark 2.10. *In fact contra pre-$\beta\theta$-open maps and pre-$\beta\theta$-open are independent notions.*
Example 2.6 above shows that contra-pre-$\beta\theta$-openness does not imply pre-$\beta\theta$-openness while the converse is shown in the following example.

Example 2.11. *The identity map on the same topological space (X, τ) where $\tau = \{\emptyset, \{a\}, X\}$ is an example of a pre-$\beta\theta$-open map which is not contra pre-$\beta\theta$-open.*

Remark 2.12. *By Theorem 2.5 and Remark 2.7, we have that every contra pre-$\beta\theta$-open map is ap-$\beta\theta$-open, the converse does not hold.*

The following diagram holds :

$$
\begin{array}{ccccc}
 & & \textit{contra pre } \beta\theta\textit{-open map} & & \\
 & \not\nearrow \ \not\swarrow & & \not\nwarrow \ \searrow & \\
\beta\theta\textit{-open map} & & \not\uparrow \ \not\downarrow & & \textit{ap-}\beta\theta\textit{-open map} \\
 & \not\searrow \ \not\nwarrow & & \not\swarrow \ \nearrow & \\
 & & \textit{pre } \beta\theta\textit{-open map} & &
\end{array}
$$

The next theorem establishes conditions under which the inverse map of every g$\beta\theta$-open set from codomain is a g$\beta\theta$-open set.

Theorem 2.13. *If a map $f : (X, \tau) \to (Y, \sigma)$ is surjective β-irresolute and ap-$\beta\theta$-open, then $f^{-1}(A)$ is g$\beta\theta$-open whenever A is g$\beta\theta$-open subset of (Y, σ).*

Proof. Let A be a g$\beta\theta$-open subset of (Y, σ). Suppose that $F \subseteq f^{-1}(A)$, where $F \in \beta\theta C(X, \tau)$. Taking complements we obtain $f^{-1}(A^c) \subseteq F^c$ or $A^c \subseteq f(F^c)$. Since f is an ap-$\beta\theta$-open and $(\beta Int_\theta(A))^c = \beta Cl_\theta(A^c) \subseteq f(F^c)$. It follows that $(f^{-1}(\beta Int_\theta(A)))^c \subseteq F^c$ and hence $F \subseteq f^{-1}(\beta Int_\theta(A))$. Since f is β-irresolute $f^{-1}(\beta Int_\theta(A))$ is $\beta\theta$-open. Thus, we have $F \subseteq f^{-1}(\beta Int_\theta(A)) = \beta Int_\theta(f^{-1}(\beta Int_\theta(A))) \subseteq \beta Int_\theta(f^{-1}(A))$. This implies that $f^{-1}(A)$ is g$\beta\theta$-open in (X, τ). □

A map $f : (X, \tau) \to (Y, \sigma)$ is called g$\beta\theta$-irresolute if $f^{-1}(O)$ is g$\beta\theta$-closed in (X, τ) for every g$\beta\theta$-closed in (Y, σ).

Theorem 2.14. *Let $f : (X, \tau) \to (Y, \sigma)$, $g : (Y, \sigma) \to (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \to (Z, \gamma)$. Then*
(i) $g \circ f$ is ap-$\beta\theta$-open, if f is pre-$\beta\theta$-open and g is ap-$\beta\theta$-open.
(ii) $g \circ f$ is ap-$\beta\theta$-open, if f is ap-$\beta\theta$-open and g is bijective pre-$\beta\theta$-closed and g$\beta\theta$-irresolute.

Proof. (i) Suppose that A is an arbitrary $\beta\theta$-open subset in (X, τ) and B a g$\beta\theta$-closed subset of (Z, γ) for which $B \subseteq g \circ f(A)$. Then $f(A)$ is $\beta\theta$-open in (Y, σ) since f is pre-$\beta\theta$-open. Since g is ap-$\beta\theta$-open, $\beta Cl_\theta(B) \subseteq g(f(A))$. This implies that $g \circ f$ is ap-$\beta\theta$-open.
(ii) Let A be an arbitrary $\beta\theta$-open subset of (X, τ) and B a g$\beta\theta$-closed subset of (Z, γ) for which $B \subseteq g \circ f(A)$. Hence $g^{-1}(B) \subseteq f(A)$. Then $\beta Cl_\theta(g^{-1}(B)) \subseteq f(A)$ because $g^{-1}(B)$ is g$\beta\theta$-closed and f is ap-$\beta\theta$-open. Hence we have ,
$\beta Cl_\theta(B) \subseteq \beta Cl_\theta(gg^{-1}(B)) \subseteq g(\beta Cl_\theta(g^{-1}(B))) \subseteq g(f(A)) = (g \circ f)(A)$.
This implies that $g \circ f$ is ap-$\beta\theta$-open. □

Now we state the following two theorems whose proof are straightforward and hence omitted.

Theorem 2.15. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \gamma)$ be two maps such that*
$g \circ f : (X, \tau) \to (Z, \gamma)$. Then,
(i) $g \circ f$ is contra pre $\beta\theta$-open, if f is pre $\beta\theta$-open and g is contra pre $\beta\theta$-open.
(ii) $g \circ f$ is contra pre $\beta\theta$-open, if f is contra pre $\beta\theta$-open and g is pre $\beta\theta$-closed.

Theorem 2.16. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \gamma)$ be two maps such that $g \circ f : (X, \tau) \to (Z, \gamma)$ is contra pre $\beta\theta$-open.*
(i) If f is a β-irresolute surjection, then g is contra pre $\beta\theta$-open.
(ii) If g is a β-irresolute injection, then f is contra pre $\beta\theta$-open .

Definition 5. *Let (X, τ) and (Y, σ) be topological spaces. A map $f : (X, \tau) \to (Y, \sigma)$ is said to have an $\beta\theta$-closed graph if its $Gr(f) = \{(x, y) : y = f(x), x \in X\}$ is $\beta\theta$-closed in the product space $(X \times Y, \tau_p)$, where τ_p denotes the product topology.*

We say that the product space $X = X_1 \times ... \times X_n$ has Property $P_{\beta\theta}$ if A_i is a $\beta\theta$-open set in a topological spaces X_i, for $i = 1, 2, .., n$, then $A_1 \times ... \times A_n$ is also $\beta\theta$-open in the product space $X = X_1 \times ... \times X_n$.

It is well-known that the graph $Gr(f)$ of f is a closed set of $X \times Y$, whenever f is continuous and Y is Hausdorff. The following theorem is a modification of this fact, i.e., we give a condition under which a contra pre $\beta\theta$-open map has $\beta\theta$-closed graph.

Theorem 2.17. *If $f : (X, \tau) \to (Y, \sigma)$ is a contra pre $\beta\theta$-open map with $\beta\theta$-closed fibers, then the graph $Gr(f)$ of f is $\beta\theta$-closed in the product space $X \times Y$. which have the property $P_{\beta\theta}$.*

Proof. Let $(x, y) \in X \times Y \backslash Gr(f)$. Then $x \in (f^{-1}\{y\})^c$. Since fibers are $\beta\theta$-closed, there is an $\beta\theta$-open set O for which $x \in O \subseteq (f^{-1}\{y\})^c$. Set $A = (f(O))^c$. Then A is an $\beta\theta$-open set in Y containing y, since f is contra pre $\beta\theta$-open. Therefore, we obtain that $(x, y) \in O \times A \subseteq X \times Y \backslash Gr(f)$, where $O \times A \in \beta\theta O(X \times Y, \tau_p)$. This implies that $X \times Y \backslash Gr(f)$ is $\beta\theta$-open in $X \times Y$. Hence $Gr(f)$ is a $\beta\theta$-closed set of $X \times Y$. □

3. A characterization of $\beta\theta$-$T_{\frac{1}{2}}$ spaces

Lemma 3.1. *Let A be a $g\beta\theta$-closed subset of (X, τ). Then $\beta Cl_\theta(A) \setminus A$ does not contain a nonempty $\beta\theta$-closed set.*

Proof. Let F be a $\beta\theta$-closed set such that $F \subset \beta Cl_\theta(A) \setminus A$. Clearly $A \subset F^c$, where A is g$\beta\theta$-closed and F^c is $\beta\theta$-open . Thus $\beta Cl_\theta(A) \subset F^c$, or equivalently $F \subset (\beta Cl_\theta(A))^c$. Since by assumption $F \subset \beta Cl_\theta(A)$, then $F \subset (\beta Cl_\theta(A))^c \cap \beta Cl_\theta(A) = \emptyset$. This shows that F coincides with the void-set. □

Definition 6. *A topological space (X, τ) is $\beta\theta$-$T_{\frac{1}{2}}$ if every g$\beta\theta$-closed set is $\beta\theta$-closed.*

Theorem 3.2. *For a topological space (X, τ) the following conditions are equivalent:*
(i) X is $\beta\theta$-$T_{\frac{1}{2}}$,
(ii) For each $x \in X$, $\{x\}$ is $\beta\theta$-closed or $\beta\theta$-open.

Proof. $(i) \Rightarrow (ii)$: Suppose that for some $x \in X$, $\{x\}$ is not $\beta\theta$-closed. Then $\{x\}^c$ is not $\beta\theta$-open. Since X is the only $\beta\theta$-open containing $\{x\}^c$, the set $\{x\}^c$ is g$\beta\theta$-closed and so it is $\beta\theta$-closed in the $\beta\theta$-$T_{\frac{1}{2}}$ space (X, τ). Therefore $\{x\}^c$ is $\beta\theta$-closed or equivalently $\{x\}$ is $\beta\theta$-open.
$(ii) \Rightarrow (i)$: Let $A \subset X$ be g$\beta\theta$-closed. Let $x \in \beta Cl_\theta(A)$. We will show that $x \in A$. By the hypothesis, the singleton $\{x\}$ is either $\beta\theta$-closed or $\beta\theta$-open. We consider these two cases.

Case 1. $\{x\}$ is $\beta\theta$-closed: Then if $x \notin A$, there exists a $\beta\theta$-closed set in $\beta Cl_\theta(A) - A$. By Lemma 3.1 $x \in A$.

Case 2. $\{x\}$ is $\beta\theta$-open: Since $x \in \beta Cl_\theta(A)$, then $\{x\} \bigcap A \neq \emptyset$. Thus $x \in A$.

Hence in both cases, we have $x \in A$, i.e., $\beta Cl_\theta(A) \subset A$ or equivalently A is $\beta\theta$-closed since the inclusion $A \subset \beta Cl_\theta(A)$ is trivial. $\square$

In the following theorem, we give a new characterization of $\beta\theta$-$T_{\frac{1}{2}}$ spaces by using the notion of ap-$\beta\theta$-open maps.

Theorem 3.3. *Let (Y, σ) be a topological space. Then the following statements are equivalent.*
(i) (Y, σ) is a $\beta\theta$-$T_{\frac{1}{2}}$ space,
(ii) For every space (X, τ) and every map $f : (X, \tau) \to (Y, \sigma)$, f is ap-$\beta\theta$-open.

Proof. $(i) \to (ii)$: Let B be a g$\beta\theta$-closed subset of (Y, σ) and suppose that $B \subseteq f(A)$ where $A \in \beta\theta O(X, \tau)$. Since (Y, σ) is a $\beta\theta$-$T_{\frac{1}{2}}$ space, B is $\beta\theta$-closed (i.e., $B = \beta Cl_\theta(B)$). Therefore $\beta Cl_\theta(B) \subseteq f(A)$. Then f is ap-$\beta\theta$-open.
$(ii) \to (i)$: Let B be a g$\beta\theta$-closed subset of (Y, σ) and let X be the set Y with the topology $\tau = \{\emptyset, B, X\}$. Finally let $f : (X, \tau) \to (Y, \sigma)$ be the identity map. By assumption f is ap-$\beta\theta$-open. Since B is g$\beta\theta$-closed in (Y, σ) and $\beta\theta$-open in (X, τ) and $B \subseteq f(B)$, it follows that $\beta Cl_\theta(B) \subseteq$

$f(B) = B$. Hence B is $\beta\theta$-closed in (Y, σ). Therefore (Y, σ) is a $\beta\theta$-$T_{\frac{1}{2}}$ space. □

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