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Skolem Difference Mean Graphs

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Abstract

A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, \cdots, p+q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, \cdots, q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. In this paper, we prove $C_n@P_m (n \geq 3, m \geq 1)$, $T\langle K_{1,n_1} : K_{1,n_2} : \cdots : K_{1,n_m}\rangle$, $T\langle K_{1,n_1} \circ K_{1,n_2} \circ \circ \circ K_{1,n_m}\rangle$, $St(n_1, n_2, \cdots, n_m)$ and $Bt(\underbrace{n, n, \cdots, n}_{m \text{ times}})$ are skolem difference mean graphs.

Keywords : *Mean labeling, skolem difference mean labeling, skolem difference mean graph, extra skolem difference mean labeling.*

AMS Subject Classification : *05C78*

1. Introduction

Throughout this paper by a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary[1]. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions. There are several types of labeling. An excellent survey of graph labeling is available in [2]. The concept of mean labeling was introduced by Somasundaram and Ponraj [8]. A graph $G = (V, E)$ with p vertices and q edges is called a mean graph if there is an injective function f that maps $V(G)$ to $\{0, 1, 2, \cdots, q\}$ such that each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if $f(u) + f(v)$ is even and $\frac{f(u)+f(v)+1}{2}$ if $f(u)+f(v)$ is odd. Then the resulting edge labels are distinct. The notion of skolem difference mean labeling was due to Murugan and Subramanian [3]. A graph $G = (V, E)$ with p vertices and q edges is said to have skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $1, 2, 3, \cdots, p+q$ in such a way that for each edge $e = uv$, let $f^*(e) = \left\lceil \frac{|f(u)-f(v)|}{2} \right\rceil$ and the resulting labels of the edges are distinct and are from $1, 2, 3, \cdots, q$. A graph that admits a skolem difference mean labeling is called a skolem difference mean graph. Further studies on skolem difference mean labeling are available in [4]-[7].

In this paper, we extend the study on skolem difference mean labeling and prove that $C_n@P_m (n \geq 3, m \geq 1)$, $T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle$, $T\langle K_{1,n_1} \circ K_{1,n_2} \circ \circ \circ K_{1,n_m}\rangle$, $St(n_1, n_2, \cdots, n_m)$ and $Bt(\underbrace{n, n, \cdots, n}_{m \text{ times}})$ are skolem difference mean graphs.
We use the following definitions in the subsequent section.

Definition 1.1. *The graph $C_n@P_m$ is obtained by identifying one pendant vertex of the path P_m with a vertex of the cycle C_n.*

Definition 1.2. *The shrub $St(n_1, n_2, \cdots, n_m)$ is a graph obtained by connecting a vertex v_0 to the central vertex of each of m number of stars.*

Definition 1.3. *The banana tree $Bt(\underbrace{n_1, n_2, \cdots, n_m}_{m \text{ times}})$ is a graph obtained by connecting a vertex v_0 to one leaf of each of m number of stars.*

Definition 1.4. *Let $G = (V, E)$ be a skolem difference mean graph with p vertices and q edges. If one of the skolem difference mean labeling of G*

satisfies the condition that all the labels of the vertices are odd, then we call this skolem difference mean labeling an extra skolem difference mean labeling and call the graph G an extra skolem difference mean graph [7].

An extra skolem difference mean labeling of P_6 is given in Figure 1

Figure 1

2. Main Results

Theorem 2.1. *The graph $C_n@P_m(n \geq 3, m \geq 1)$ is a skolem difference mean graph.*

Proof. **Case (i):** n is odd.
Let $n = 2k+1$. Let $u_1, u_2, \cdots, u_k, v_k, v_{k-1}, \cdots, v_1, v_0$ be the vertices of C_{2k+1} and let $w_1, w_2, \cdots, w_m$ be the vertices of P_m. Then $C_n@P_m$ is obtained by identifying w_1 of P_m with v_0 of C_{2k+1}, which has $n+m-1$ edges. $E(C_{2k+1}@P_m) = \{v_iv_{i+1} | 1 \leq i \leq k-1\} \cup \{u_iu_{i+1} | 1 \leq i \leq k-1\} \cup \{w_jw_{j+1} | 1 \leq j \leq m-1\} \cup \{v_0v_1, v_0u_1, u_kv_k\}$. Define $f : V(C_{2k+1}@P_m) \to \{1, 2, 3, \cdots, p+q = 2n+2m-2\}$ as follows:
$f(w_{2i-1}) = 2i-1$ for $1 \leq i \leq \lceil \frac{m}{2} \rceil$,
$f(w_{2i}) = 2m+3-2i$ for $1 \leq i \leq \lfloor \frac{m}{2} \rfloor$,
$f(u_{2i-1}) = 2m+2n+2-4i$ for $1 \leq i \leq \left\lceil \frac{k}{2} \right\rceil$,
$f(u_{2i}) = 4i$ for $1 \leq i \leq \left\lfloor \frac{k}{2} \right\rfloor$,
$f(v_{2i-1}) = 2m+2n+1-4i$ for $1 \leq i \leq \left\lceil \frac{k}{2} \right\rceil$,
$f(v_{2i}) = 4i+2$ for $1 \leq i \leq \left\lfloor \frac{k}{2} \right\rfloor$.
For each vertex label f, the induced edge label f^* is calculated as follows:
$f^*(w_iw_{i+1}) = m+1-i$ for $1 \leq i \leq m-1$,
$f^*(u_kv_k) = 1$,
$f^*(u_iu_{i+1}) = n+m-1-2i$ for $1 \leq i \leq k-1$,

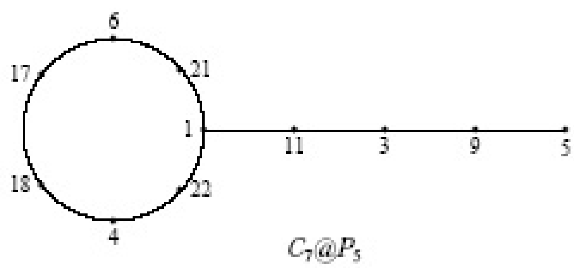
$f^*(v_iv_{i+1}) = n + m - 2 - 2i$ for $1 \leq i \leq k-1$,
$f^*(u_1v_0) = n + m - 1$, $f^*(v_1v_0) = n + m - 2$.

Case (ii): n is even.
Let $n = 2k$, $k > 1$. Let $v_0, v_1, v_2, \cdots, v_{k-1}, u_0, u_{k-1}, u_{k-2}, \cdots, u_2, u_1$ be the vertices of C_{2k} and let $w_1, w_2, \cdots, w_m$ be the vertices of P_m. Then $C_n@P_m$ is obtained by identifying w_1 of P_m with v_0 of C_{2k}, which has $n + m - 1$ edges. $E(C_{2k}@P_m) = \{v_iv_{i+1} | 1 \leq i \leq k-2\} \cup \{u_iu_{i+1} | 1 \leq i \leq k-2\} \cup \{w_jw_{j+1} | 1 \leq j \leq m-1\} \cup \{v_0v_1, v_0u_1, u_0u_{k-1}, u_0v_{k-1}\}$.
Subcase (i): $k > 1$ is odd.
Define $f : V(C_{2k}@P_m) \rightarrow \{1, 2, 3, \cdots, p + q = 2n + 2m - 2\}$ as follows:
$f(w_{2i-1}) = 2i - 1$ for $1 \leq i \leq \lceil \frac{m}{2} \rceil$,
$f(w_{2i}) = 2m + 1 - 2i$ for $1 \leq i \leq \lfloor \frac{m}{2} \rfloor$,
$f(u_{2i-1}) = 2m + 2n + 2 - 4i$ for $1 \leq i \leq \frac{k-1}{2}$,
$f(u_{2i}) = 4i$ for $1 \leq i \leq \frac{k-1}{2}$,
$f(v_{2i-1}) = 2m + 2n + 1 - 4i$ for $1 \leq i \leq \frac{k-1}{2}$,
$f(v_{2i}) = 4i + 2$ for $1 \leq i \leq \frac{k-1}{2}$, and $f(u_0) = 2(n + m - k)$.
For each vertex label f, the induced edge label f^* is calculated as follows:
$f^*(w_iw_{i+1}) = m - i$ for $1 \leq i \leq m-1$,
$f^*(u_iu_{i+1}) = n + m - 1 - 2i$ for $1 \leq i \leq k-2$,
$f^*(v_iv_{i+1}) = n + m - 2 - 2i$ for $1 \leq i \leq k-2$,
$f^*(v_0u_1) = n + m - 1$,
$f^*(v_0v_1) = n + m - 2$,
$f^*(u_{k-1}u_0) = m + 1$,
$f^*(v_{k-1}u_0) = m$.

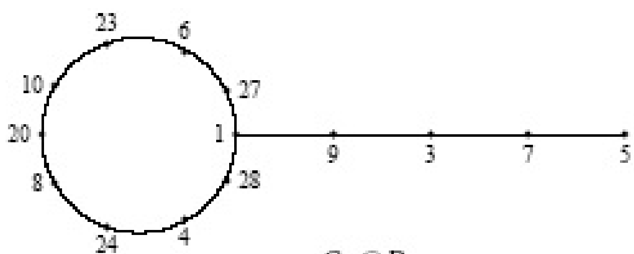
Subcase (ii): $k > 1$ is even.
Define $f : V(C_{2k}@P_m) \rightarrow \{1, 2, 3, \cdots, p + q = 2n + 2m - 2\}$ as follows:
$f(w_{2i-1}) = 2i - 1$ for $1 \leq i \leq \lceil \frac{m}{2} \rceil$,
$f(w_{2i}) = 2m + 1 - 2i$ for $1 \leq i \leq \lfloor \frac{m}{2} \rfloor$,
$f(u_{2i-1}) = 2m + 2n + 2 - 4i$ for $1 \leq i \leq \frac{k}{2}$,
$f(u_{2i}) = 4i$ for $1 \leq i \leq \frac{k-2}{2}$,
$f(v_{2i-1}) = 2m + 2n - 4i$ for $1 \leq i \leq \frac{k}{2}$,
$f(v_{2i}) = 4i + 1$ for $1 \leq i \leq \frac{k-2}{2}$,
and $f(u_0) = 2k$.
For each vertex label f, the induced edge label f^* is calculated as follows:
$f^*(w_iw_{i+1}) = m - i$ for $1 \leq i \leq m-1$,
$f^*(u_iu_{i+1}) = n + m - 1 - 2i$ for $1 \leq i \leq k-2$,
$f^*(v_iv_{i+1}) = n + m - 2 - 2i$ for $1 \leq i \leq k-2$,

$f^*(v_0u_1) = n + m - 1,$
$f^*(v_0v_1) = n + m - 2,$
$f^*(u_{k-1}u_0) = m + 1,$
$f^*(v_{k-1}u_0) = m.$
It can be verified that f is a skolem difference mean labeling. Hence $C_n@P_m$ is a skolem difference mean graph. □

Skolem difference mean labelings of $C_7@P_5$ and $C_{10}@P_5$ are shown in Figure 2.



$C_7@P_5$



$C_{10}@P_5$

Figure 2

In the following theorem, we prove that the graph $T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle$, obtained from the stars $K_{1,n_1}, K_{1,n_2}, ..., K_{1,n_m}$ by joining the central vertices of K_{1,n_j} and $K_{1,n_{j+1}}$ to a new vertex w_j for $1 \leq j \leq m-1$ is an extra skolem difference mean graph.

Theorem 2.2. *The graph $T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle$ is an extra skolem difference mean graph.*

Proof. Let $u_i^j (1 \leq i \leq n_j)$ be the pendant vertices and $v_j (1 \leq j \leq m)$ be the central vertex of the star $K_{1,n_j} (1 \leq j \leq m)$. Then

$T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle$ is a graph obtained by joining v_j and v_{j+1} to a new vertex $w_j (1 \le j \le m-1)$ by an edge. Define
$f : V(T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle) \to$
$\left\{1, 2, 3, \cdots, p+q = 2\sum\limits_{k=1}^{m} n_k + 4m - 3\right\}$ as follows:

$f(v_j) = 2\sum\limits_{k=1}^{m} n_k + 4m - 3 - 2(j-1)$ for $1 \le j \le m$,
$f(w_1) = 2n_1 + 1$,
$f(w_j) = 2n_1 + 1 + 2\sum\limits_{k=2}^{j} (n_k + 1)$ for $2 \le j \le m-1$,
$f(u_i^1) = 2i - 1$ for $1 \le i \le n_1$
$f(u_i^j) = 2\sum\limits_{k=1}^{j-1} (n_k + 1) + 2i - 1$ for $1 \le i \le n_j$ and $2 \le j \le m$.
For each vertex label f , the induced edge label f^* is calculated as follows:

Let $e_i^j (1 \le i \le n_j$ and $1 \le j \le m)$ be the edges joining the vertices v_j with u_i^j. $f^*(e_i^1) = \sum\limits_{k=1}^{m} n_k + 2m - i - 1$ for $1 \le i \le n_1$,
$f^*(e_i^j) = n_j + n_{j+1} + \cdots + n_m + 2m - 2j - i + 1$ for $1 \le i \le n_j$ and $2 \le j \le m$,
$f^*(v_1 w_1) = n_2 + n_3 + \cdots + n_m + 2(m-1)$,
$f^*(v_j w_j) = n_{j+1} + n_{j+2} + \cdots + n_m + 2(m-j)$ for $2 \le j \le m-1$,
$f^*(w_1 v_2) = n_2 + n_3 + \cdots + n_m + 2m - 3$,
$f^*(w_j v_{j+1}) = n_{j+1} + n_{j+2} + \cdots + n_m + 2(m-j) - 1$ for $2 \le j \le m-1$.
It can be verified that $T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle$ is an extra skolem difference mean labeling. Hence $T\langle K_{1,n_1} : K_{1,n_2} : K_{1,n_3} : \cdots : K_{1,n_m}\rangle$ is an extra skolem difference mean graph. □

Corollary 2.3. *The graph* $T\left\langle \underbrace{K_{1,n} : K_{1,n} : K_{1,n} : \cdots : K_{1,n}}_{m \text{ times}} \right\rangle$ *is an extra skolem difference mean graph.*

An extra skolem difference mean labeling of $T\langle K_{1,7} : K_{1,7} : K_{1,7}\rangle$ is shown in Figure 3.

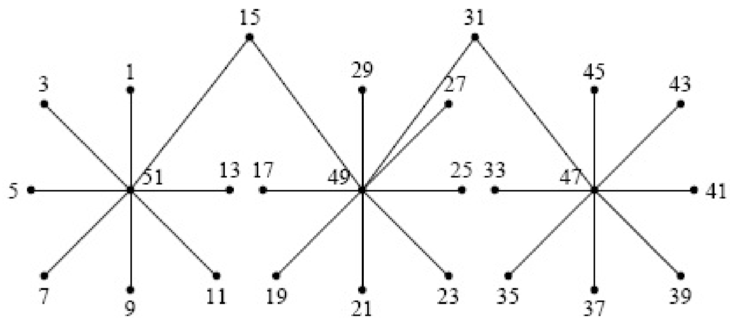


Figure 3

The graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ \circ \circ K_{1,n_m}\rangle$ is obtained from the stars $K_{1,n_1}, K_{1,n_2}, \cdots, K_{1,n_m}$ by joining a leaf of K_{1,n_j} and a leaf of $K_{1,n_{j+1}}$ to a new vertex $w_j (1 \leq j \leq m-1)$ by an edge.

Theorem 2.4. *The graph $T\langle K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \circ \circ K_{1,n_m}\rangle$ is an extra skolem difference mean graph.*

Proof. Let $u_i^j (1 \leq i \leq n_j)$ be the pendant vertices and $v_j (1 \leq j \leq m)$ be the central vertex of the star $K_{1,n_j} (1 \leq j \leq m)$. Then $T\langle K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \circ \circ K_{1,n_m}\rangle$ is a graph obtained by joining $u_{n_j}^j$ and u_1^{j+1} to a new vertex $w_j (1 \leq j \leq m-1)$ by an edge. Define $f : V(T\langle K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \circ \circ K_{1,n_m}\rangle) \to \left\{1, 2, 3, \cdots, p+q = 2\sum_{k=1}^{m} n_k + 4m - 3\right\}$ as follows:

$f(v_j) = 2\sum_{k=1}^{m} n_k + 4m - 4j + 1$ for $1 \leq j \leq m$,

$f(w_j) = 2\sum_{k=1}^{m} n_k + 4m - 4j - 1$ for $1 \leq j \leq m-1$,

$f(u_i^1) = 2i - 1$ for $1 \leq i \leq n_1$

$f(u_i^j) = 2\sum_{k=1}^{j-1} n_k + 2i - 1$ for $1 \leq i \leq n_j$ and $2 \leq j \leq m$.

Let $e_i^j = v_j u_i^j$ for $1 \leq i \leq n_j$ and $1 \leq j \leq m$. For each vertex label f, the induced edge label f^* is calculated as follows: $f^*(e_i^1) = \sum_{k=1}^{m} n_k + 2m - i - 1$ for $1 \leq i \leq n_1$,

$f^*(e_i^j) = n_j + n_{j+1} + \cdots + n_m + 2m - 2j - i + 1$ for $2 \leq j \leq m-1$ and

$1 \le i \le n_j$,
$f^*(u^1_{n_1}w_1) = \sum\limits_{k=1}^{m} n_k + 2m - 2 - n_1$,
$f^*(u^j_{n_j}w_j) = n_{j+1} + n_{j+2} + \cdots + n_m + 2(m-j)$ for $2 \le j \le m-1$,
$f^*(w_1u^2_1) = n_2 + n_3 + \cdots + n_m + 2m - 3$,
$f^*(w_ju^{j+1}_1) = n_{j+1} + n_{j+2} + \cdots + n_m + 2m - 2j - 1$ for $2 \le j \le m-1$.

It can be verified that $T\langle K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \circ \circ K_{1,n_m}\rangle$ is an extra skolem difference mean labeling. Hence $T\langle K_{1,n_1} \circ K_{1,n_2} \circ K_{1,n_3} \circ \circ \circ K_{1,n_m}\rangle$ is an extra skolem difference mean graph. □

Corollary 2.5. *The graph* $T\left\langle \underbrace{K_{1,n} \circ K_{1,n} \circ K_{1,n} \circ \circ \circ K_{1,n}}_{m \text{ times}} \right\rangle$ *is a skolem difference mean graph.*

A skolem difference mean labeling of $T\langle K_{1,6} \circ K_{1,6} \circ K_{1,6} \circ K_{1,6}\rangle$ is shown in Figure 4.

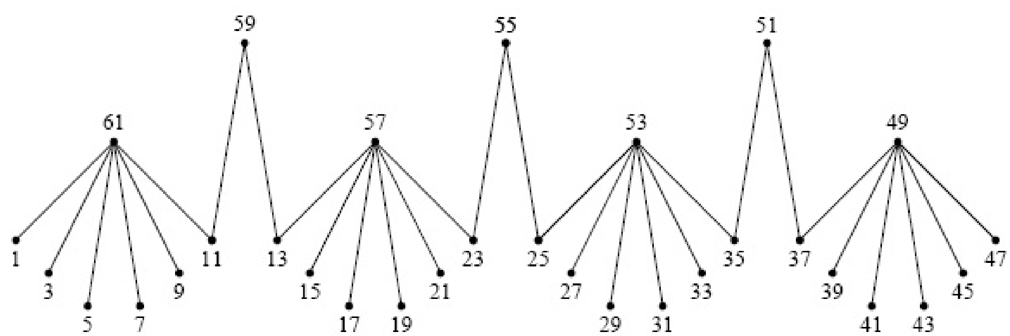


Figure 4

Theorem 2.6. *If G is a graph having p vertices and q edges with $q > p$ then G is not a skolem difference mean graph.*

Proof. Let G be a (p, q) graph with $q > p$. The minimum possible vertex label of G is 1 and the maximum possible vertex label of G is $p+q$. Therefore, the maximum possible edge label of G is $\left\lceil \frac{p+q-1}{2} \right\rceil < \left\lceil \frac{2q-1}{2} \right\rceil = q$. That is, G has no edge having the label q and hence G is not a skolem difference mean graph. $\square$

Corollary 2.7. *The complete graph K_n is a skolem difference mean graph if and only if $n \leq 3$.*

Proof. K_2 is P_2 and K_3 is C_3. The graph K_n has n vertices and $\frac{n(n-1)}{2}$ edges. For $n \geq 4$, $\frac{n(n-1)}{2} > n$. By theorem 2.6, K_n, $n \geq 4$ is not a skolem difference mean graph. $\square$

Theorem 2.8. *The shrub $St(n_1, n_2, \cdots, n_m)$ is a skolem difference mean graph.*

Proof. Let v_0, v_j, u_i^j $(1 \leq j \leq m, 1 \leq i \leq n_j)$ be the vertices of $St(n_1, n_2, \cdots, n_m)$. Then
$E(St(n_1, n_2, \cdots, n_m)) = \{v_0 v_j | 1 \leq j \leq m\} \cup \left\{v_j u_i^j | 1 \leq i \leq n_j\right\}$.

Define $f : V(St(n_1, n_2, \cdots, n_m)) \to \left\{1, 2, 3, \cdots, p+q = 2\sum\limits_{j=1}^{m} n_j + 2m + 1\right\}$ as follows:

$f(v_0) = 2m + 2\sum\limits_{j=1}^{m} n_j + 1$,
$f(v_j) = 2j - 1, 1 \leq j \leq m$,
$f(u_i^j) = 2(n_j + n_{j+1} + \cdots + n_m) + 2(j - i)$, $1 \leq i \leq n_j - 1$ and $1 \leq j \leq m$,
$f(u_{n_j}^j) = 2(n_{j+1} + n_{j+2} + \cdots + n_m) + 2j + 1$ for $1 \leq j \leq m-1$,
$f(u_{n_m}^m) = 2m + 1$.

Let $e_i^j = v_j u_i^j$ for $1 \leq i \leq n_j$ and $1 \leq j \leq m$. For each vertex label f, the induced edge label f^* is defined as follows: $f^*(v_0 v_j) = m + n_1 + n_2 + \cdots + n_m - j + 1$, $1 \leq j \leq m$
$f^*(e_i^j) = n_j + n_{j+1} + \cdots + n_m - i + 1$, $1 \leq i \leq n_j$ and $1 \leq j \leq m$.

It can be verified that $St(n_1, n_2, \cdots, n_m)$ is a skolem difference mean graph. $\square$

The skolem difference mean labeling of $St(2,5,5,3)$ is shown in Figure 5.

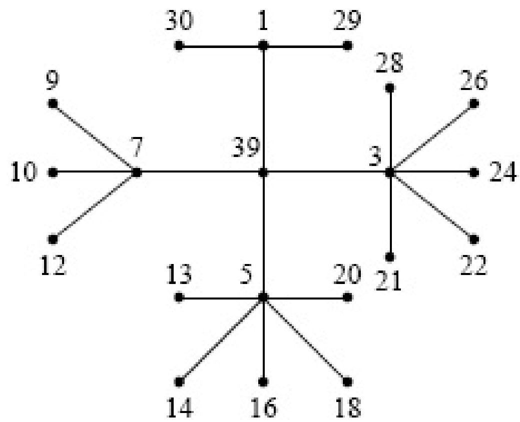


Figure 5

Theorem 2.9. *The banana tree* $Bt(\underbrace{n,n,\cdots,n}_{m\ times})$ *is a skolem difference mean graph.*

Proof. Let v_0, v_j, u_i^j $(1 \leq j \leq m, 1 \leq i \leq n)$ be the vertices of $Bt(\underbrace{n,n,\cdots,n}_{m \text{ times}})$. Then $E(Bt(\underbrace{n,n,\cdots,n}_{m \text{ times}})) = \Big\{v_0u_1^j, v_ju_i^j | 1 \leq i \leq n \text{ and } 1 \leq j \leq m\Big\}$.
Define $f : V(Bt(\underbrace{n,n,\cdots,n}_{m \text{ times}})) \to \{1,2,3,\cdots,p+q = 2m(n+1)+1\}$ as follows:

$f(v_0) = 2m(n+1)+1$,
$f(v_j) = 2m(n-1)+4j-1$, for $1 \leq j \leq m$,
$f(u_1^j) = 2j-1$, $1 \leq j \leq m$,
$f(u_i^j) = (2i-2)m+2j-1$, for $2 \leq i \leq n-1$ and $1 \leq j \leq m$,
$f(u_n^j) = 2m(n+1)-2(m-j)$ for $1 \leq j \leq m$. Let $e_i^j = v_ju_i^j$ for $1 \leq i \leq n$ and $1 \leq j \leq m$. For each vertex label f , the induced edge label f^* is calculated as follows: $f^*(v_0u_1^j) = m(n+1)-j+1$, for $1 \leq j \leq m$
$f^*(e_i^j) = m(n-i)+j$, for $1 \leq i \leq n-1$ and $1 \leq j \leq m$,
$f^*(v_ju_n^j) = m-j+1$ for $1 \leq j \leq m$.

It can be verified that $Bt(\underbrace{n, n, \cdots, n}_{m \text{ times}})$ is a skolem difference mean graph. □

A skolem difference mean labeling of Bt(3,3,3,3,3,3,3,3) is shown in Figure 6.

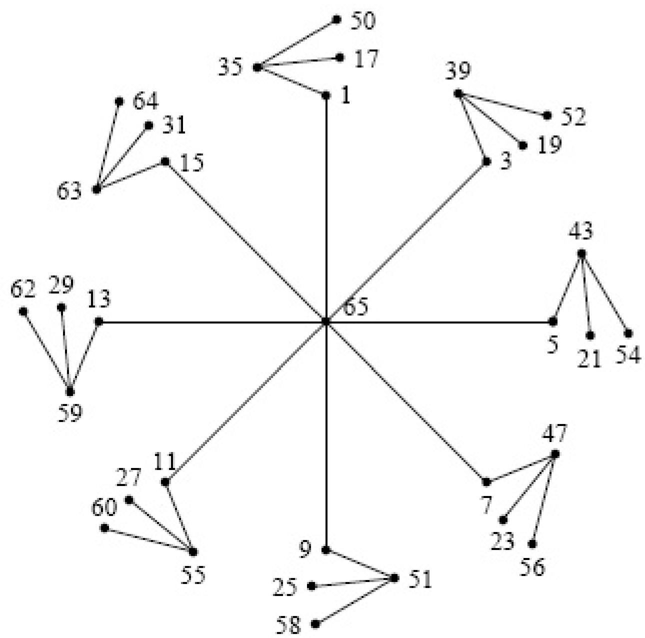


Figure 6

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