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Equi independent equitable dominating sets in graphs

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Abstract

We introduce the concept of an equi independent equitable dominating set and define equi independent equitable domination number. We also investigate the graph families whose equi independent equitable domination number and equitable domination number are same.

Keywords : *Equi independent equitable domination number, equitable domination number, domination number.*

AMS subject classification number : *05C69; 05C38.*

1. Introduction

In any group of people equality of status, power, wealth etc. is desirable and it is also a remarkable feature of constitution of a democratic nation. Keeping this idea in to center Sampathkumar has introduced the concept of equitable domination. On the other hand some social status like marital relationship must be enjoyed individually. Also the information and password security for financial matters must be independent in order to retain uniqueness. Such issues have motivated the concept of independent domination which was formalized by Berge [1] and Ore [6] while the independent domination and the notation $i(G)$ were introduced by Cockayne and Hedetniemi in [2, 3]. A survey on independent domination can be found in Goddard and Henning [4].

Is it possible to think about a set of people who can enjoy both the characteristics namely, equality and independence simultaneously? In attempt to answer this question, the idea of equitable independent equitable domination was conceived by Swaminathan and Dharmalingam [7]. We formalize this concept and term as an equi independent equitable domination. We provide some prerequisites for the initiation of new concept.

In this paper, the term graph we mean simple, finite, connected and undirected graph $G = (V(G), E(G))$. For terminology and notation not defined here we follow West [11] and Haynes et al. [5]. For every vertex $v \in V(G)$, the *open neighbourhood set* $N(v)$ is the set of all vertices adjacent to v in G. That is, $N(v) = \{u \in V(G)/uv \in E(G)\}$. The *closed neighbourhood set* $N[v]$ of v is defined as $N[v] = N(v) \cup \{v\}$. A set $D \subseteq V(G)$ is called a *dominating set* if every vertex in $V(G) - D$ is adjacent to at least one vertex in D. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. A subset D of $V(G)$ is called an *equitable dominating set* if for every $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of such a dominating set is denoted by γ^e and is called the *equitable domination number* of G. A vertex $u \in V(G)$ is said to be degree equitable with a vertex $v \in V(G)$ if $|deg(u) - deg(v)| \leq 1$. An equitable dominating set D is said to be a *minimal equitable dominating set* if no proper subset of D is an equitable dominating set.

Swaminathan and Dharmalingam [7] have derived following necessary and sufficient condition for minimal equitable dominating set.

Theorem 1.1. *An equitable dominating set D is minimal if and only if for every vertex $u \in D$ one of the following holds.*

(i) Either $N(u) \cap D = \emptyset$ *or* $|\deg(v) - \deg(u)| \geq 2$ *for all* $N(u) \cap D$.
(ii) There exists a vertex $v \in V(G) - D$ *such that* $N(v) \cap D = \{u\}$ *and* $|\deg(v) - \deg(u)| \leq 1$.

A vertex $v \in V(G)$ is called *equitable isolates* if $|d(v) - d(u)| \geq 2$ for every $u \in N(v)$. Note that equitable isolates must belongs to any equitable dominating set of graph G. Obviously isolated vertices are equitable isolated vertices. Hence $I_s \subseteq I_e \subseteq D$ for every equitable dominating set D, where I_s, I_e are the set of all isolated vertices and set of all equitable isolates of G respectively.

Theorem 1.2. *[7] A graph G has a unique minimal equitable dominating set if and only if the set of all equitable isolates forms an equitable dominating set.*

The *equitable neighbourhood* of v denoted by $N^e(v)$ is defined as $N^e(v) = \{u \in V(G)/u \in N(v), |deg(v) - deg(u)| \leq 1\}$. The cardinality of $N^e(v)$ is denoted by $\deg^e_G(v)$. The maximum and minimum equitable degree of graph G are denoted by $\Delta^e(G)$ and $\delta^e(G)$ respectively which are defined as $\Delta^e(G) = \max\limits_{v \in V(G)} |N^e(v)|$, $\delta^e(G) = \min\limits_{v \in V(G)} |N^e(v)|$.

Remark 1.3. $\Delta^e(G) \leq \Delta(G)$

Remark 1.4. *For a connected graph* G, $\delta^e(G)$ *may be zero. For star graph* $K_{1,n}$, $\Delta^e(G) = \delta^e(G) = 0$.

Swaminathan and Dharamlingam [7] have also introduced the concept of equitable independent set. According to them a subset S of $V(G)$ is called an *equitable independent set* if for any $u \in S$, $v \notin N^e(u)$ for all $v \in S - \{u\}$. The maximum cardinality of an equitable independent set is denoted by $\beta^e(G)$.

Remark 1.5. *Every independent set is an equitable independent set.*

Theorem 1.6. *[7] Let S be a maximal equitable independent set. Then S is a minimal equitable dominating set.*

Definition 1.7. *The helm* H_n *is the graph obtained from a wheel* W_n *by attaching a pendant edge to each of the rim vertex.*

Definition 1.8. *The closed helm* CH_n *is the graph obtained from helm* H_n *by joining each pendant vertex to form a cycle.*

An equitable dominating set D is said to be *equi independent equitable dominating set* if it is also equitable independent set. The minimum cardinality of an equi independent equitable dominating is called *equi independent equitable domination number* which denoted by i^e.

Illustration 1.9. *In Figure 1, $D = \{v_1, v_2, v_3, v_5, v_7, v_9\}$ is equitable independent set as well as equitable dominating set for graph G.*

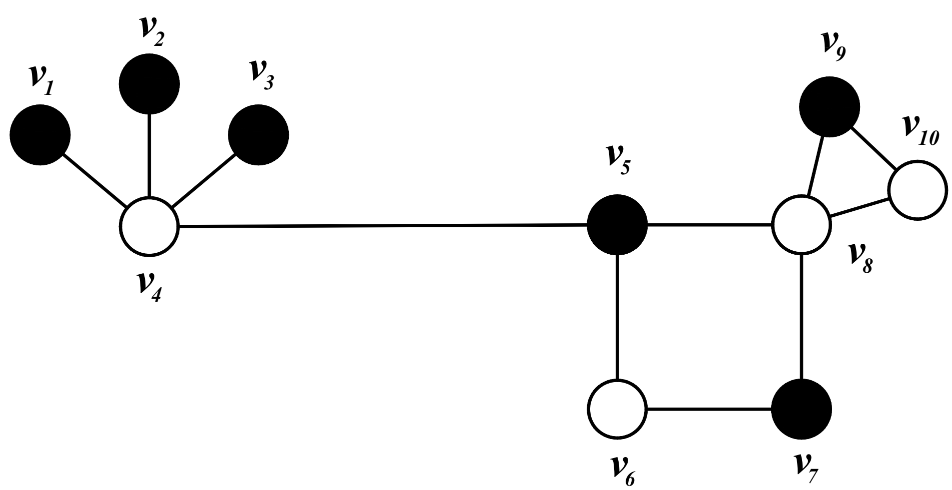

Figure 1: *Equi independent equitable dominating set of graph G*

The equi independent equitable domination number have been investigated for various graph families in [8, 9, 10].

2. Main Results

Theorem 2.1. *Let G be a graph in which all the vertices are equitable isolates then $i^e(G) = \gamma^e(G) = n$.*

Proof. *Let G be a graph in which all the vertices are equitable isolates then each vertex can equitably dominate it self only. While every pair of vertices $u, v \in V(G)$ are equitably non adjacent. Which implies that $V(G)$ is γ^e-set of G and maximal independent set. Hence $i^e(G) = \gamma^e(G) = n$.* □

We define $n(e_i)$ as number of equitable isolates in graph G. The following theorem gives the lower bound for an equi independent equitable dominating set.

Theorem 2.2. $i^e(G) \geq \gamma^e(G) \geq n(e_i) + i^e(G - I_e)$.

Proof. *Let G be a graph. If graph G have equitable isolates then they must be in any equi independent equitable dominating set. Therefore any equi independent equitable dominating set must contain equitable isolates and i^e-set of $G - I_e$. While if graph G is without equitable isolates then $n(e_i) = 0$. Hence $i^e(G) \geq \gamma^e(G) \geq n(e_i) + i^e(G - I_e)$.* □

Remark 2.3. *The graph shown in figure 1 achieves the lower bound of the above theorem.*

Theorem 2.4. $i^e(P_n) = \gamma^e(P_n) = \gamma(P_n)$.

Proof. *Let D be any γ-set of path P_n. Then it is an equi independent equitable dominating set of P_n as $d(v_i) = 1\,\text{or}\,2$ for every i. Also it is equitable independent set as any γ-set of P_n is independent set. Hence $i^e(P_n) = \gamma^e(P_n) = \gamma(P_n)$.* □

Theorem 2.5. $i^e(C_n) = \gamma^e(C_n) = \gamma(C_n)$.

Proof. *Let D be a γ-set of C_n. Since C_n is 2-regular graph, any γ-set of C_n is an equitable dominating set of C_n. Also any γ-set of C_n is an independent set. Hence $i^e(P_n) = \gamma^e(P_n) = \gamma(P_n)$.* □

Theorem 2.6. $i^e(K_n) = \gamma^e(K_n) = \gamma(K_n)$.

Proof. *Any singleton subset of $V(K_n)$ form equitable independent as well as equitable dominating set. Hence $i^e(K_n) = \gamma^e(K_n) = \gamma(K_n)$.* □

Theorem 2.7. $i^e(W_n) = \gamma^e(W_n) = \gamma(W_n) = 1$ *for* $n = 3, 4$.

Proof. *Let $v_1, v_2, \ldots, v_i$ be the rim vertices of W_i and v be a apex vertex of W_i where $i = 3, 4$. Observe that $D = \{v\}$ is an equitable independent set as being singleton set and it is an equitable dominating set as it is apex vertex with $|d(v) - d(v_i)| = 1$. Hence $i^e(W_n) = \gamma^e(W_n) = \gamma(W_n) = 1$.* □

Theorem 2.8. $i^e(W_n) = \gamma^e(W_n) = \lceil \frac{n}{3} \rceil + 1$ *for* $n > 4$.

Proof. *Let $v_1, v_2, \ldots, v_n$ be the rim vertices, v be a apex vertex of W_n. Observe that apex vertex v is equitable isolates. Therefore $\gamma^e(W_n) \geq \gamma^e(C_n) + 1$. Let $S = \gamma^e-$set of C_n and $D = S \cup \{v\}$ with $|D| = \gamma^e(C_n) + 1$. Observe that D is an equitable dominating set and it is an equitable independent set as it contain equitable isolates vertex v and γ^e-set of C_n, which is maximal equitable independent set of C_n. Hence $i^e(W_n) = \gamma^e(W_n) = \lceil \frac{n}{3} \rceil + 1$.* □

Theorem 2.9. $i^e(K_{m,n}) = \gamma^e(K_{m,n})$.

Proof. *Let $K_{m,n}$ be the complete bipartite graph with m vertices in one partition say V_1 and n vertices in other partition say V_2. Therefore $V(K_{m,n}) = V_1 \cup V_2$. Observe that* $\deg(u) = \begin{cases} n & \text{if } u \in V_1 \\ m & \text{if } u \in V_2 \end{cases}$

Let D be any γ^e-set of $K_{m,n}$.

Case 1: $|m - n| \leq 1$.
In this case $\gamma^e(K_{m,n}) = 2$ with $D = \{u, v / u \in V_1, v \in V_2\}$. Therefore D is an independent set which implies that D is an equitable independent set. Therefore D is an equi independent equitable dominating set. Hence $i^e(K_{m,n}) = \gamma^e(K_m, n)$.

Case 2: $|m - n| > 2$
In this case $\gamma^e(k_{m,n}) = m + n$. Therefore γ^e-set of $K_{m,n} = i^e$-set of $K_{m,n}$ $= V(K_{m,n})$, as all the vertices are equitable isolates. Hence $i^e(K_{m,n}) = \gamma^e(K_m, n)$. □

Theorem 2.10. $i^e(H_n) = \begin{cases} \gamma^e(H_n) = n + 1 & \text{for } n = 3, 4, 5 \\ \gamma^e(C_n) + n + 1 & \text{for } n \geq 6 \end{cases}$

Proof. *Let $v_1, v_2, \ldots, v_n$ be rim vertices, $u_1, u_2, \ldots, u_n$ be the pendant vertices and v be the apex vertex of helm H_n.*

Case 1: $n = 3, 4, 5$
In this case vertices $u_1, u_2, \ldots, u_i$ are equitable isolates for $i = 3, 4$. Therefore they must belongs to any equitable dominating set. Let $D = \{u_1, u_2, \ldots, u_i, v\}$ for $i = 3, 4$. Observe that D is equitable dominating set as apex vertex v equitably dominate rim vertices $v_1, v_2, \ldots, v_i$ for $i = 3, 4$. Also D is independent set which implies that D is equitable independent set. Therefore D is an equi independent equitable dominating set. Hence $i^e(H_n) = \gamma^e(H_n) = n + 1$.

Case 2: $n \geq 6$
In this case $d(u_i) = 1, d(v_i) = 4$ and $d(v) = n$. This implies that pendant vertices $u_1, u_2, \ldots, u_n$ and apex vertex v are equitable isolates. Therefore they must belongs to any equitable dominating set. Also subgraph induced by $V(H_n) - \{u_1, u_2, \ldots, u_n, v\}$ is cycle C_n. Which implies that $\gamma^e(H_n) \geq \gamma^e(C_n) + n + 1$. Let S be a γ^e-set of C_n and $D = \{u_1, u_2, \ldots, u_n\} \cup S$. Observe that D is an equitable dominating set as all pendant vertices and apex vertex are in D, while rim vertices are equitably dominate by set S. Also D is an equitable independent set. Hence D is an equi independent equitable dominating set and $i^e(H_n) = \gamma^e(H_n) + n + 1$. $\square$

Theorem 2.11. $i^e(CH_n) = \begin{cases} \gamma^e(CH_n) = 2 & \text{for } n = 3, 4 \\ \gamma^e(CH_n) = 3 & \text{for } n = 5 \end{cases}$

Proof. *Let v be apex vertex with $d(v) = n$, $v_1, v_2, \ldots, v_n$ be the vertices of degree 4 and $u_1, u_2, \ldots, u_n$ be vertices of degree 3 of CH_n.*

Case 1: $n = 3, 4$
In this case $|V(CH_3)| = 2n + 1$ and none of the vertex having degree $2n$. Therefore $\gamma^e(CH_n) = \gamma(CH_n) > 1$. Consider $D = \{v_1, u_3\}$ with $|D| = 2$. Then $N^e[D] = V(CH_n)$. Therefore D is an equitable dominating set of CH_n. Also vertices v_1 and u_3 are not adjacent to each other. This implies that set D is an independent set as well as an equitable independent set of CH_n. Thus D is an equi independent equitable dominating set of CH_n and $i^e(CH_n) = \gamma^e(CH_n) = 2$.

Case 2: $n = 5$
In this case $|V(CH_5)| = 11$. and none of the vertex having degree 10.

Therefore $\gamma^e(CH_5) = \gamma(CH_5) > 1$. *Consider* $D = \{v, u_1, u_3\}$ *with* $|D| = 3$. *Then* $N^e[D] = V(CH_5)$. *Therefore* D *is an equitable dominating set of* CH_5. *Also vertices* v, u_1, u_3 *are not adjacent to each other. This implies that set* D *is an independent set as well as an equitable independent set of* CH_5. *Thus* D *is an equi independent equitable dominating set of* CH_5 *and* $i^e(CH_5) = \gamma^e(CH_5) = 3$.

□

Theorem 2.12.

$$i^e(CH_n) = \begin{cases} \gamma^e(CH_n) = \begin{cases} 2\left\lfloor \frac{n}{4} \right\rfloor + 3 & \text{for n} \equiv 2, 3 (\bmod 4),\ n \not\equiv 3 \\ \frac{n}{2} + 1 & \text{for}\, n \equiv 0 (\mathrm{mod} 4),\ n \not\equiv 4 \end{cases} \\ \gamma^e(CH_n) + 1 = (2\left\lfloor \frac{n}{4} \right\rfloor + 2) + 1 \ \text{for}\, n \equiv 1 (\mathrm{mod} 4),\ n \not\equiv 5 \end{cases}$$

Proof. *To prove this result we continue with the terminology and notations used in Theorem 2.11. Observe that apex vertex* v *is an equitable isolates. while other vertices are equitably adjacent to each other.*

Case 1: $n \equiv 0 \,(\mathrm{mod}\, 4)$

Here $N^e(v_{4i+1}) = \{v_{4i}, v_{4i+2}, u_{4i+1}\}$ *and* $N^e(u_{4i+3}) = \{u_{4i+2}, u_{4i}, v_{4i+3}\}$. *Let* $D = \{v, v_{4i+1}, u_{4i+3}\}$ *where* $0 \leq i \leq \frac{n}{4} - 1$ *with* $|D| = \frac{n}{2} + 1$. *Here* v *is an equitable isolate and vertices* v_{4i+1}, u_{4i+3} *are not equitably adjacent to each other. Therefore* D *is an equitable independent set. Other hand* $N^e(v) \cup N^e(v_{4i+1}) \cup N^e(u_{4i+3}) = V(CH_n)$, *where* $0 \leq i \leq \frac{n}{4} - 1$. *Therefore* D *is an equitable dominating set. Hence* D *is an equi independent equitable dominating set and* $i^e(CH_n) = \gamma^e(CH_n) = \frac{n}{2} + 1$.

Case 2: $n \equiv 1 \,(\mathrm{mod}\, 4)$

Here $N^e(v_{4i+1}) = \{v_{4i}, v_{4i+2}, u_{4j+1}\}$ *and* $N^e(u_{4j+3}) = \{u_{4j+2}, u_{4j}, v_{4i+3}\}$. *Let* $D = \{v, v_{4i+1}, u_{4j+3}, v_{n-1}, u_n\}$ *where* $0 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor, 0 \leq j < \left\lfloor \frac{n}{4} \right\rfloor$ *with* $|D| = 2\left\lfloor \frac{n}{4} \right\rfloor + 3$. *Here vertex* v *is an equitable isolate and vertices* $v_{4i+1}, u_{4j+3}, v_{n-1}, u_n$ *are not equitably adjacent to each other. Therefore* D *is an equitable independent set. Also* $N^e(v) \cup N^e(v_{4i+1}) \cup N^e(u_{4j+3}) = V(CH_n)$, *where* $0 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor, 0 \leq j < \left\lfloor \frac{n}{4} \right\rfloor$. *Therefore* D *is an equitable dominating set. Hence* D *is an equi independent equitable dominating set and* $i^e(CH_n) = \gamma^e(CH_n) + 1 = (2\left\lfloor \frac{n}{4} \right\rfloor + 2) + 1$.

Case 3: $n \equiv 2 \,(\mathrm{mod}\, 4)$

Here $N^e(v_{4i+1}) = \{v_{4i}, v_{4i+2}, u_{4j+1}\}$ *and* $N^e(u_{4j+3}) = \{u_{4j+2}, u_{4j}, v_{4i+3}\}$. *Let* $D = \{v, v_{4i+1}, u_{4j+3}, u_n\}$ *where* $0 \leq i \leq \left\lfloor \frac{n}{4} \right\rfloor, 0 \leq j < \left\lfloor \frac{n}{4} \right\rfloor$ *with* $|D| = 2\left\lfloor \frac{n}{4} \right\rfloor + 3$. *Here* v *is an equitable isolate and vertices* v_{4i+1}, u_{4j+3}, u_n *are not equitably adjacent to each other. Therefore* D *is an equitable independent set. Also* $N^e(v) \cup N^e(v_{4i+1}) \cup N^e(u_{4j+3}) \cup N^e(u_n) = V(CH_n)$, *where* $0 \leq i \leq$

$\lfloor \frac{n}{4} \rfloor, 0 \le j < \lfloor \frac{n}{4} \rfloor$. *Therefore D is an equitable dominating set. Hence D is an equi independent equitable dominating set and* $i^e(CH_n) = \gamma^e(CH_n) = 2\lfloor \frac{n}{4} \rfloor + 3$.

Case 4: $n \equiv 3\,(\mathrm{mod}\,4)$

Here $N^e(v_{4i+1}) = \{v_{4i}, v_{4i+2}, u_{4i+1}\}$ *and* $N^e(u_{4i+3}) = \{u_{4i+2}, u_{4i}, v_{4i+3}\}$. *Let* $D = \{v, v_{4i+1}, u_{4i+3}\}$ *where* $0 \le i \le \lfloor \frac{n}{4} \rfloor$ *with* $|D| = 2\lfloor \frac{n}{4} \rfloor + 3$. *Here v is an equitable isolate and vertices v_{4i+1}, u_{4i+3} are not equitably adjacent to each other. Therefore D is an equitable independent set. Also* $N^e(v) \cup N^e(v_{4i+1}) \cup N^e(u_{4i+3}) = V(CH_n)$, *where* $0 \le i \le \lfloor \frac{n}{4} \rfloor$. *Therefore D is an equitable dominating set. Thus D is an equi independent equitable dominating set and* $i^e(CH_n) = \gamma^e(CH_n) = 2\lfloor \frac{n}{4} \rfloor + 3$.

Hence

$$i^e(CH_n) = \begin{cases} \gamma^e(CH_n) = \begin{cases} 2\lfloor \frac{n}{4} \rfloor + 3 & \text{for n} \equiv 2, 3(\bmod 4),\ n \not\equiv 3 \\ \frac{n}{2} + 1 & \text{for} n \equiv 0(\mathrm{mod}4),\ n \not\equiv 4 \end{cases} \\ \gamma^e(CH_n) + 1 = (2\lfloor \frac{n}{4} \rfloor + 2) + 1 \text{ for} n \equiv 1(\mathrm{mod}4),\ n \not\equiv 5 \end{cases} \qquad \square$$

Illustration 2.13. *The closed helm graph CH_9 and its i^e-set of CH_9 is shown by grey colored vertices in Figure 2.*

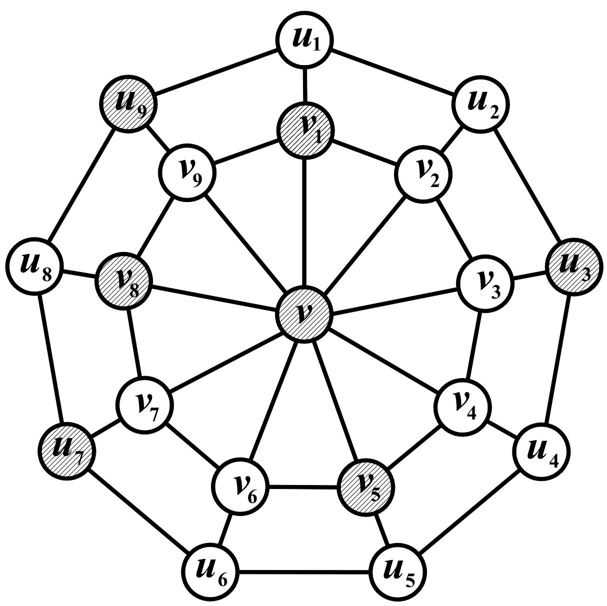

Figure 2: *The closed helm graph CH_9 and its i^e-set of CH_9*

Theorem 2.14. $i^e(Wb_n) = i^e(CH_n) + n$.

Proof. *Let $v_1, v_2, \ldots, v_n$ be the vertices of inner cycle, $u_1, u_2, \ldots, u_n$ be the vertices of outer cycle, $w_1, w_2, \ldots, w_n$ be the pendant vertices and v be a apex vertex of web Wb_n. Observe that pendant vertices are $w_1, w_2, \ldots, w_n$ are equitable isolates. There fore they must belongs to any equitable dominating set of web Wb_n. Which implies that $i^e(Wb_n) \geq i^e(CH_n) + n$.*
Let S be the i^e-set of CH_n and $D = S \cup \{w_1, w_2, \ldots, w_n\}$ with $|D| = i^e(CH_n) + n$. We claim that D is an equi independent equitable dominating set of web Wb_n. Since pendant vertices $w_1, w_2, \ldots, w_n$ are equitable non adjacent to vertices $u_1, u_2, \ldots, u_n$ of outer cycle and set S is an equitable independent set of CH_n. Therefore D is equitable independent set of web Wb_n. While D is equitable dominating set of web Wb_n as pendant vertices $w_1, w_2, \ldots, w_n$ are in D and remaining vertices equitably dominated by set S. Therefore D is an equi independent equitable dominating set of web Wb_n with $|D| = i^e(CH_n) + n$. Hence $i^e(Wb_n) = i^e(CH_n) + n$. □

Theorem 2.15. *For any graph G,* $\left\lceil \frac{n}{1+\Delta^e(G)} \right\rceil \leq \gamma^e(G) \leq n - \Delta^e(G)$.

Proof. *Let D be a γ^e-set of G. Each vertex of G can dominate at most itself and $\Delta^e(G)$ other vertices. Hence $\gamma^e(G) \geq \left\lceil \frac{n}{1+\Delta^e(G)} \right\rceil$.*
Let v be a vertex of maximum equitable degree $\Delta^e(G)$. Then v dominate $N^e[v]$ and the vertices in $V(G) - N^e[v]$ dominate them self. Hence $V(G) - N^e[v]$ is dominating set of cardinality $n - \Delta^e(G)$. Which implies that $\gamma^e(G) \leq n - \Delta^e(G)$. □

Remark 2.16. *Cycle C_n achieve the lower bound and Wheel graph W_4 achieve the upper bound of above Theorem.*

Theorem 2.17. *For any graph G,* $i^e(G) \geq \left\lceil \frac{n}{1+\Delta^e(G)} \right\rceil$.

Theorem 2.18. $i^e(G) \leq n - \Delta^e(G)$.

Proof. *Let G be a graph. Any maximal equitable independent set of G containing a vertex of maximum equitable degree $\Delta^e(G)$ contains at most $n - \Delta^e(G)$ vertices. Hence $i^e(G) \leq n - \Delta^e(G)$.* □

3. Concluding Remarks

A new domination model has been introduced. We establish several general results in the context of newly defined concept. We also establish bounds for the equi independent equitable domination number as well as equitable domination number and also investigate graphs achieving these bounds.

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