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One modulo three mean labeling of transformed trees

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Abstract

A graph G is said to be one modulo three mean graph if there is an injective function ϕ from the vertex set of G to the set $\{a|0 \leq a \leq 3q-2$ and either $a \equiv 0(mod\ 3)$ or $a \equiv 1(mod\ 3)\}$ where q is the number of edges G and ϕ induces a bijection ϕ^* from the edge set of G to $\{a|1 \leq a \leq 3q-2$ and either $a \equiv 1(mod\ 3)\}$ given by $\phi^*(uv) = \left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil$ and the function ϕ is called one modulo three mean labeling of G. In this paper, we prove that the graphs $T \odot \overline{K_n}, T\hat{o}K_{1,n}, T\hat{o}P_n$ and $T\hat{o}2P_n$ are one modulo three mean graphs.

Keywords : *Mean labeling, one modulo three graceful labeling, one modulo three mean labeling, one modulo three mean graphs, transformed tree.*

AMS Subject Classification : *05C78.*

1. Introduction

All graphs considered here are simple, finite, connected and undirected. For a detailed survey of graph labeling we refer to [1]. We follow the basic notations and terminology of graph theory as in Harary [2]. The notion of mean labeling was due to Somasundaram and Ponraj [7]. A graph $G = (V, E)$ with p vertices and q edges is called a mean graph if $f : V(G) \to \{0, 1, 2, 3, \dots q\}$ be an injection. For each edge $e = uv$, let $f^*(e) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Then the resulting edge labels are distinct. The concept of one modulo three graceful labeling was introduced by Swaminathan and Sekar in [8]. A graph $G = (V, E)$ with p vertices and q edges is called an one modulo three graceful if there is a function ϕ from the vertex set of G to $\{0, 1, 3, 4, \dots, 3q-2\}$ in such a way that (i) ϕ is one-one (ii) ϕ induces a bijection ϕ^* from the edge set or fG to $\{1, 4, 7, \dots, 3q-2\}$ where $\phi^*(uv) = |\phi(u) - \phi(v)|$. Motivated by the work of the authors in [7, 8] Jeyanthi and Maheswari defined one modulo three mean labeling in [4] and proved that P_{2n}, comb, bistar $B_{n,n}$, T_p-tree with even number of vertices, C_{4n+1}, ladder L_{n+1}, $K_{1,2n} \times K_2$ are one modulo three mean graphs. Furthermore, they proved that $B_{m,n}, K_{1,n}, K_n, n > 3$ are not one modulo three mean graphs. In [5, 6] it is proved that $DA(Q_n), DA(Q_2) \odot nK_1, DA(Q_m) \odot nK_1, DA(T_2) \odot nK_1, DA(T_m) \odot nK_1, \overline{S}(DA(T_n)), \overline{S}(DA(Q_n)), D(C_n, v'),$ $D(C_n, e'), S'(P_{2n}), NA(Q_m), K_{1,2n} \times P_2, EJ_n, mP_n, m \geq 1, C_m * eC_n (m, n \equiv 1 (mod\ 4))$ and $P_{4m}(+)\overline{K_n}$ graphs are one modulo three mean graphs. In this paper we extend the study on one modulo three mean labeling and prove that graphs $T \odot \overline{K_n}, T\hat{o}K_{1,n}, T\hat{o}P_n$ and $T\hat{o}2P_n$ are one modulo three mean graphs. We use the following definitions in the subsequent section.

Definition 1.1. *The corona $G_1 \odot G_2$ of the graphs G_1 and G_2 is defined as a graph obtained by taking one copy of G_1 (with p vertices) and p copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex of the i^{th} copy of G_2.*

Definition 1.2. *Let G_1 be a graph with p vertices and G_2 be any graph. A graph $G_1 \hat{o} G_2$ is obtained from G_1 and p copies of G_2 by identifying one vertex of i^{th} copy of G_2 with i^{th} vertex of G_1.*

Definition 1.3. [3] *Let T be a tree and u_0 and v_0 be the two adjacent vertices in T. Let u and v be the two pendant vertices of T such that the length of the path u_0-u is equal to the length of the path v_0-v. If the edge u_0v_0 is deleted from T and u and v are joined by an edge uv, then such a* *transformation of T is called an elementary parallel transformation (or an ept) and the edge u_0v_0 is called transformable edge. If by the sequence of ept's, T can be reduced to a path, then T is called a T_p-tree (transformed tree) and such sequence regarded as a composition of mappings (ept's) denoted by P is called a parallel transformation of T. The path, the image of T under P is denoted as $P(T)$. A T_P-tree and the sequence of two ept's reducing it to a path are illustrated in the following figure.*

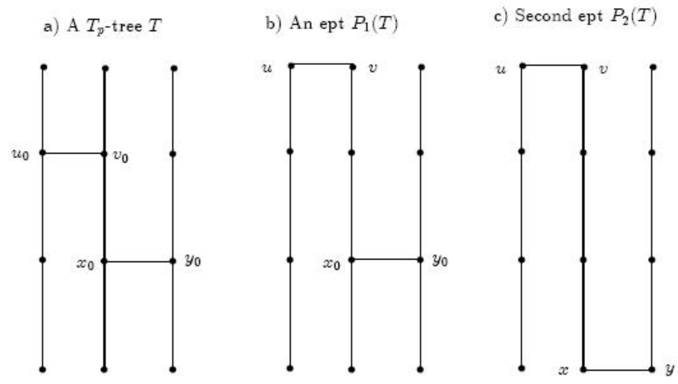

Figure 1

2. Main Results

Theorem 2.1. *Let T be a T_p-tree with even number of vertices. Then the graph $T \odot \overline{K_n}$ is a one modulo three mean graph for all $n \geq 1$.*

Proof. Let T be a T_P-tree with m vertices where m is even. By the definition of T_p-tree, there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \cup E_p$ where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts P used to arrive the path $P(T)$. Clearly, E_d and E_p have the same number of edges.

Now, we denote the vertices of $P(T)$ successively as $u_1, u_2, \ldots, u_m$ starting from one pendant vertex of $P(T)$ right up to the other. Hence the vertex set $V(T) = \{u_1, u_2, u_3, \ldots, u_m\}$ and the edge set $E(T) = \{e_i = u_iu_{i+1} : 1 \leq i \leq m-1\}$. Let $u_{i1}, u_{i2}, \ldots, u_{in}$ be the pendant vertices joined with $u_i (1 \leq i \leq m)$ by an edge. Then, $V(T \odot K_n) = \{u_i, u_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(T \odot K_n) = \{e_i = u_iu_{i+1} : 1 \leq i \leq m-1\} \cup \{e^i_j = u_iu_{ij} : 1 \leq$

$i \leq m, 1 \leq j \leq n\}$. The graph $T \odot \overline{K_n}$ has $mn + m$ vertices $mn + m - 1$ edges.

Define a vertex labeling $\phi : V(T \odot \overline{K_n}) \to \{0, 1, 3, \ldots, 3mn + 3m - 5\}$ as follows:

For $1 \leq i \leq m, 1 \leq j \leq n$ $\phi(u_i) = \begin{cases} 3(n+1)(i-1) & \text{if } i \text{ is odd} \\ 3(n+1)i - 5 & \text{if } i \text{ is even,} \end{cases}$

$\phi(u_{ij}) = \begin{cases} 3(n+1)(i-1) + 6j - 5 & \text{if } i \text{ is odd} \\ 3(n+1)(i-2) + 6j & \text{if } i \text{ is even.} \end{cases}$

For the vertex labeling ϕ, the induced edge labeling ϕ^* is as follows:

$\phi^*(e_j^i) = 3(n+1)(i-1) + 3j - 2$ for $1 \leq i \leq m, 1 \leq j \leq n$ and $\phi^*(e_i) = 3(n+1)i - 2$ for $1 \leq i \leq m - 1$.

Let $u_i u_j$ be an edge of T for some indices i and $j, 1 \leq i < j \leq m$. Let P_1 be the ept that deletes this edge and adds an edge $u_{i+t}u_{j-t}$ where t is the distance of u_i from u_{i+t} and also the distance of u_j from u_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts. Since $u_{i+t}u_{j-t}$ is an edge in the path $P(T)$, it follows that $i + t + 1 = j - t$ which implies $j = i + 2t + 1$. Therefore, i and j are of opposite parity. The induced label of the edge $u_i u_j$ is given by

$\phi^*(u_i u_j) = \phi^*(u_i u_{i+2t+1}) = \left\lceil \frac{\phi(u_i) + \phi(u_{i+2t+1})}{2} \right\rceil$
$= 3(n+1)(i+t) - 2, 1 \leq i \leq m.$
$\phi^*(u_{i+t}u_{j-t}) = \phi^*(u_{i+t}u_{i+t+1}) = \left\lceil \frac{\phi(u_{i+t}) + \phi(u_{i+t+1})}{2} \right\rceil$
$= 3(n+1)(i+t) - 2, 1 \leq i \leq m.$

Therefore, we have $\phi^*(u_i u_j) = \phi^*(u_{i+t}u_{j-t})$.

It can be verified that the induced edge labels of $T \odot \overline{K_n}$ are $1, 4, 7, \ldots,$ $3mn + 3m - 5$. Hence, $T \odot \overline{K_n}$ is a one modulo three mean graph for all $n \geq 1$. $\square$

An example for one modulo three mean labeling of $T \odot \overline{K_4}$ where T is a T_p-tree with 10 vertices is given in Figure 2.

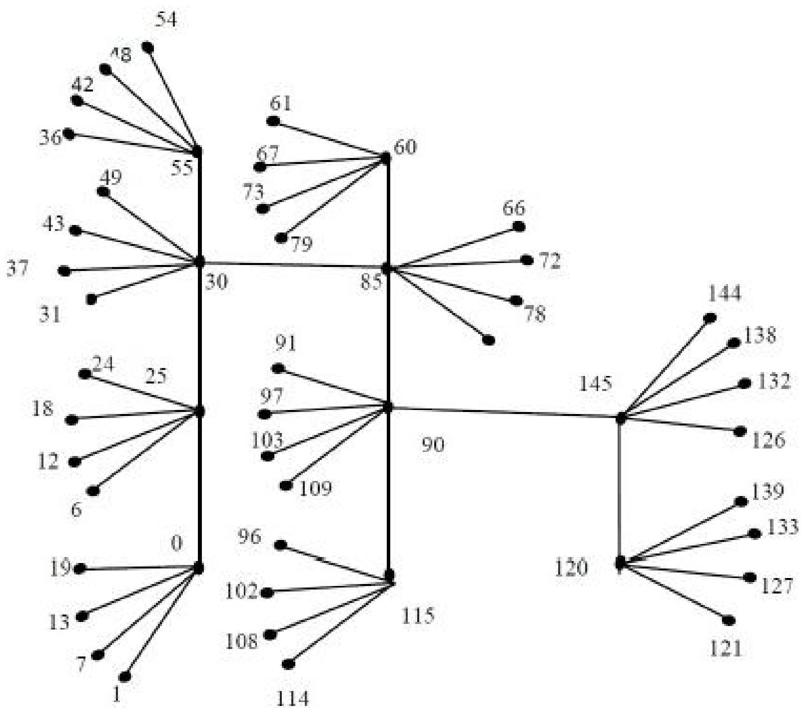

Figure 2

Theorem 2.2. *Let T be a T_p-tree with even number of vertices. Then the graph $T\hat{o}K_{1,n}$ is a one modulo three mean graph.*

Proof. Let T be a T_p-tree with m vertices where m is even. By the definition of transformed tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \cup E_p$ where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly, E_d and E_p have the same number of edges.

Now, we denote the vertices of $P(T)$ successively as $v_1, v_2, \ldots, v_m$ starting from one pendant vertex of $P(T)$ right up to the other. Hence, the vertex set $V(T) = \{v_1, v_2 \ldots, v_m\}$ and the edge set $E(T) = \{e_i = v_iv_{i+1} : 1 \le i \le m-1\}$. Let $u_0^j, u_1^j, u_2^j, \ldots, u_n^j (1 \le j \le m)$ be the vertices of i^{th} copy of $K_{1,n}$ with $u_n^j = v_j$. Then $V(T\hat{o}K_{1,n}) = \{u_i^j : 0 \le i \le n, 1 \le j \le m\}$ and $E(T\hat{o}K_{1,n}) = \{e_i = v_iv_{i+1} : 1 \le i \le m-1, e_i' = v_iu_0^i : 1 \le i \le m\} \cup \{e_j^i =$

$u_0^i u_{ij} : 1 \le i \le m, 1 \le j \le n\}$. The graph $T\hat{o}K_{1,n}$ has $mn+m$ vertices and $mn+m-1$ edges.

Define a vertex labeling $\phi : V(T\hat{o}K_{1,n}) \to \{0, 1, 3, \ldots, 3mn+3m-5\}$ as follows:

For $1 \le i \le m$ $\phi(v_i) = \begin{cases} 3(n+1)(i-1) & \text{if } i \text{ is odd} \\ 3(n+1)i-5 & \text{if } i \text{ is even,} \end{cases}$

$\phi(u_0^j) = \begin{cases} 3(n+1)(j-1)+1 & \text{if } j \text{ is odd, } 1 \le j \le m \\ 3(n+1)j-6 & \text{if } j \text{ is even, } 1 \le j \le m, \end{cases}$

$\phi(u_i^j) = \begin{cases} 3(n+1)(j-1)+6i & \text{if } j \text{ is odd, } 1 \le j \le m, 1 \le i \le n-1 \\ 3(n+1)(j-2)+6i+1 & \text{if } j \text{ is even, } 1 \le j \le m, 1 \le i \le n-1. \end{cases}$

For the vertex labeling ϕ, the induced edge labeling ϕ^* is as follows:

For $1 \le i \le m, 1 \le j \le n$ $\phi^*(e_j^i) = \begin{cases} 3(n+1)(i-1)+3j+1 & \text{if } i \text{ is odd} \\ 3(n+1)(i-1)+3j-2 & \text{if } i \text{ is even,} \end{cases}$

$\phi^*(e_i') = \begin{cases} 3(n+1)(i-1)+1 & \text{if } i \text{ is odd} \\ 3(n+1)i-5 & \text{if } i \text{ is even} \end{cases}$ and

$\phi^*(e_i) = 3(n+1)i-2$ if $1 \le i \le m-1$.

Let v_iv_j be a transformed edge in T for some indices i and $j, 1 \le i < j \le m$. Let P_1 be the ept that deletes the edge v_iv_j and adds an edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts.

Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i+2t+1$. Therefore, i and j are of opposite parity. The induced label of the edge v_iv_j is given by

$\phi^*(v_iv_j) = \phi^*(v_iv_{i+2t+1}) = \left\lceil \frac{\phi(v_i)+\phi(v_{i+2t+1})}{2} \right\rceil$
$= 3(n+1)(i+t)-2, 1 \le i \le m$ and
$\phi^*(v_{i+t}v_{j-t}) = \phi^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{\phi(v_{i+t}+\phi(v_{i+t+1})}{2} \right\rceil$
$= 3(n+1)(i+t)-2, 1 \le i \le m.$

Therefore $\phi^*(v_iv_j) = \phi^*(v_{i+t}v_{j-t})$.

It can be verified that the induced edge labels of $T\hat{o}K_{1,n}$ are $1, 4, 7, \ldots, 3mn+3m-5$. Hence, $T\hat{o}K_{1,n}$ is a one modulo three mean graph. $\square$

An example for one modulo three mean labeling of $T\hat{o}K_{1,4}$ where T is a T_p-tree with 12 vertices is given in Figure 3.

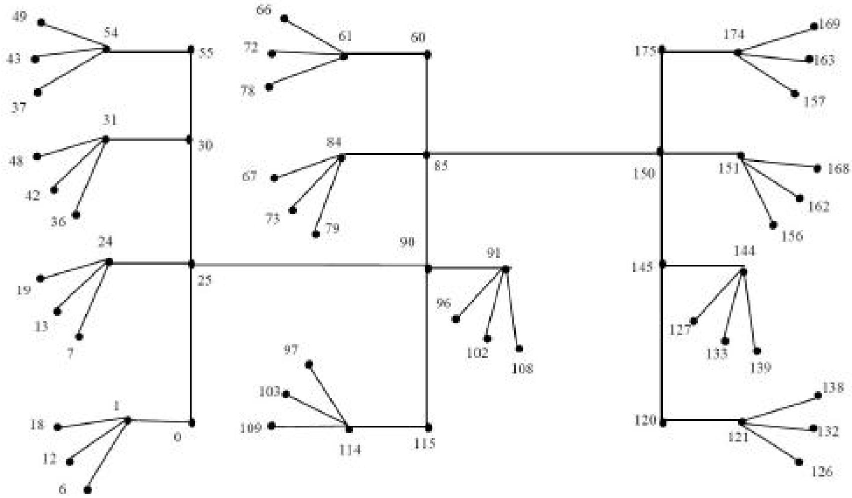

Figure 3

Theorem 2.3. *If T be a T_p-tree with even number of vertices, then the graph $T\hat{o}P_n$ is a one modulo three mean graph.*

Proof. Let T be a T_p-tree with m vertices. By the definition of a transformed tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \cup E_p$ where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly, E_d and E_p have the same number of edges.

Now, denote the vertices of $P(T)$ successively by $v_1, v_2, \ldots, v_m$ starting from one pendant vertex of $P(T)$ right up to the other one. Then the vertex set $V(T) = \{v_1, v_2, \ldots, v_m\}$ and the edge set $E(T) = \{e_i = v_i v_{i+1} : 1 \leq i \leq m-1\}$. Let $u_1^j, u_2^j, \ldots, u_n^j (1 \leq j \leq n)$ be the vertices of j^{th} copy of P_n. Then $V(T\hat{o}P_n) = \{u_i^j : 1 \leq i \leq n, 1 \leq j \leq m$ with $u_n^j = v_j\}$ and $E(T\hat{o}P_n) = \{e_i = v_i v_{i+1} : 1 \leq i \leq m-1\} \cup \{e_j^i = u_i^j u_{i+1}^j : 1 \leq j \leq m, 1 \leq i \leq n-1\}$. The graph $T\hat{o}P_n$ has mn vertices and $mn - 1$ edges.

Define a vertex labeling $\phi : V(T\hat{o}P_n) \to \{0, 1, 3, \ldots, 3mn-5\}$ as follows: For $1 \leq i \leq m, 1 \leq j \leq n$.

When j is odd, $\phi(u_i^j) = \begin{cases} 3(i-1)+3n(j-1) & \text{if } i \text{ is odd} \\ 3(i-2)+3n(j-1)+1 & \text{if } i \text{ is even.} \end{cases}$

When j is even, $\phi(u_i^j) = \begin{cases} 3(nj-i)-2 & \text{if } i \text{ is odd} \\ 3(nj-i) & \text{if } i \text{ is even.} \end{cases}$

For the vertex labeling ϕ, the induced edge labeling ϕ^* is as follows:

For $1 \le i \le n-1, 1 \le j \le m \phi^*(e_i^j) = \begin{cases} 3n(j-1)+3i-2 & \text{if } j \text{ is odd} \\ 3nj-3i-2 & \text{if } j \text{ is even.} \end{cases}$

For $1 \le j \le m-1 \phi^*(e_j) = \begin{cases} 3n(j-1)+3n-2 & \text{if } j \text{ is odd} \\ 3nj-2 & \text{if } j \text{ is even.} \end{cases}$

Let v_iv_j be a transformed edge in T for some indices $i, j, 1 \le i < j \le m$. Let P_1 be the ept that deletes the edge v_iv_j and adds an edge $v_{i+t}v_{j-t}$ where t is the distance of v_i from v_{i+t} and the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts.

Since $v_{i+t}v_{j-t}$ is an edge in the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i+2t+1$. Therefore, i and j are of opposite parity, that is, i is odd and j is even or vice-versa.

The induced label of the edge v_iv_j is given by $\phi^*(v_iv_j) = \phi^*(v_iv_{i+2t+1}) = \left\lceil \frac{\phi(v_i)+\phi(v_{i+2t+1})}{2} \right\rceil$
$= 3n(i+t)-2, 1 \le i \le m$ and
$\phi^*(v_{i+t}v_{j-t}) = \phi^*(v_{i+t}v_{i+t+1}) = \left\lceil \frac{\phi(v_{i+t})+\phi(v_{i+t+1})}{2} \right\rceil$
$= 3n(i+t)-2, 1 \le i \le m$. Therefore, $\phi^*(v_iv_j) = \phi^*(v_{i+t}v_{j-t})$. Let $e_i^j = u_i^j u_{i+1}^j (1 \le i \le n-1, 1 \le j \le m), e_j = v_jv_{j+1} (1 \le j \le m-1)$ be the edges of $T\hat{o}P_n$.

It can be verified that the induced edge labels of $T\hat{o}P_n$ are $1, 4, 7, \ldots,$ $3mn-5$. Hence, $T\hat{o}P_n$ is a one modulo three mean graph. $\square$

An example for one modulo three mean labeling of $T\hat{o}P_5$ where T is a T_p-tree with 10 vertices is given in Figure 4.

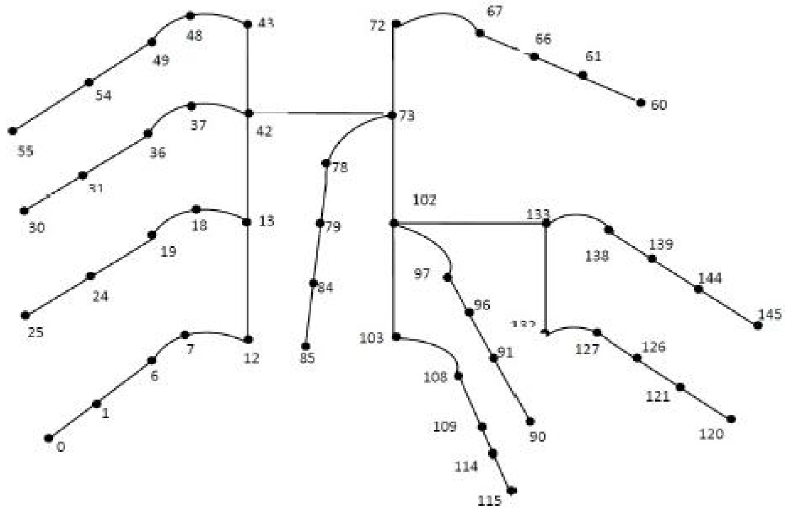

Figure 4

Theorem 2.4. *If T be a T_p-tree with even number of vertices, then the graph $T\hat{o}2P_n$ is a one modulo three mean graph.*

Proof. Let T be a T_p-tree with m vertices where m is even. By the definition of a transformed tree there exists a parallel transformation P of T such that for the path $P(T)$ we have (i) $V(P(T)) = V(T)$ (ii) $E(P(T)) = (E(T) - E_d) \cup E_p$ where E_d is the set of edges deleted from T and E_p is the set of edges newly added through the sequence $P = (P_1, P_2, \ldots, P_k)$ of the epts P used to arrive at the path $P(T)$. Clearly, E_d and E_p have the same number of edges.

Now, denote the vertices of $P(T)$ successively by $v_1, v_2, \ldots, v_m$ starting from one pendant vertex of $P(T)$ right up to the other. Then the vertex set $V(T) = \{v_1, v_2, \ldots, v_m\}$ and the edge set $E(T) = \{e_i = v_i v_{i+1} : 1 \leq i \leq m-1\}$. Let $u^j_{1,1}, u^j_{1,2}, \ldots, u^j_{1,n}$ and $u^j_{2,1}, u^j_{2,2}, \ldots, u^j_{2,n}$ $(1 \leq j \leq m)$ be the vertices of the two vertex disjoint paths joined by the j^{th} vertex of T such that $v_j = u^j_{1,n} = u^j_{2,n}$. Then $V(T\hat{o}2P_n) = \{v_j, u^j_{1,t}, u^j_{2,t} :: 1 \leq i \leq n, 1 \leq j \leq m$ with $u^j_{1,n} = u^j_{2,n} = v_j\}$ and $E(T\hat{o}2P_n) = \{e^j_{1,i} = u^j_{1,i}u^j_{1,i+1}, e^j_{2,i} =$

$u^j_{2,i}, u^j_{2,i+1} : 1 \le i \le n-1, 1 \le j \le m\} \cup \{e_j = v_j v_{j+1} : 1 \le j \le m-1\}$. The graph $T\hat{o}2P_n$ has $2mn - m$ vertices and $m(2n-1) - 1$ edges.

Define a vertex labeling $\phi : V(T\hat{o}P_n) \to \{0, 1, 3, \ldots, 6mn - 3m - 5\}$ as follows: For $1 \le i \le m, 1 \le j \le n$.

When j is odd, $\phi(u^j_{1,i}) = \begin{cases} 3(i-1) + 3(j-1)(n+2) & \text{if } i \text{ is odd} \\ 3(i-2) + 3(j-1)(n+2) + 1 & \text{if } i \text{ is even} \end{cases}$

$\phi(u^j_{2,i}) = \begin{cases} 3(2n-1)j - 3i & \text{if } i \text{ is odd} \\ 3(2n-1)j - 3i - 2 & \text{if } i \text{ is even.} \end{cases}$

When j is even, $\phi(u^j_{1,i}) = \begin{cases} 3(2n-1)j - 6n + 3i - 2 & \text{if } i \text{ is odd} \\ 3(2n-1)j - 6n + 3i & \text{if } i \text{ is even,} \end{cases}$

$\phi(u^j_{2,i}) = \begin{cases} 3(2n-1)j - 3i - 2 & \text{if } i \text{ is odd} \\ 3(2n-1)j - 3i & \text{if } i \text{ is even.} \end{cases}$

For the vertex labeling ϕ, the induced edge labeling ϕ^* is as follows:

For $1 \le i \le n-1, 1 \le j \le m \phi^*(e^j_{1,i}) = 3(2n-1)(j-1) + 3i - 2, \phi^*(e^j_{2,i}) = 3(2n-1)j - 3i - 3n + 10$ and $\phi^*(e_j) = 3(2n-1)j - 2$ if $1 \le j \le m-1$.

Let $v_i v_j$ be the transformed edge in T for some indices $i, j, 1 \le i < j \le m$ and let P_1 be the ept that deletes the edge $v_i v_j$ and adds the edge $v_{i+t} v_{j-t}$ where t is the distance of v_i from v_{i+t} and the distance of v_j from v_{j-t}. Let P be a parallel transformation of T that contains P_1 as one of the constituent epts.

Since, $v_{i+t} v_{j-t}$ is an edge in the path $P(T)$, it follows that $i+t+1 = j-t$ which implies $j = i + 2t + 1$. Therefore, i and j are of opposite parity, that is, i is odd and j is even or vice-versa.

The induced label of the edge $v_i v_j$ is given by $\phi^*(v_i v_j) = \phi^*(v_i v_{i+2t+1}) = \left\lceil \frac{\phi(v_i) + \phi(v_{i+2t+1})}{2} \right\rceil$
$= 3(2n-1)(i+t) - 2, 1 \le i \le m$ and
$\phi^*(v_{i+t} v_{j-t}) = \phi^*(v_{i+t} v_{i+t+1}) = \left\lceil \frac{\phi(v_{i+t}) + \phi(v_{i+t+1})}{2} \right\rceil$
$= 3(2n-1)(i+t) - 2, 1 \le i \le m$. Therefore, $\phi^*(v_i v_j) = \phi^*(v_{i+t} v_{j-t})$. Let be the edges of $T\hat{o}2P_n$.

It can be verified that the induced edge labels of $T\hat{o}2P_n$ are $1, 4, 7, \ldots, 6mn - 3m - 5$. Hence, $T\hat{o}2P_n$ is a one modulo three mean graph. $\square$

An example for one modulo three mean labeling of $T\hat{o}2P_4$ where T is a T_p-tree with 10 vertices is given in Figure 5.

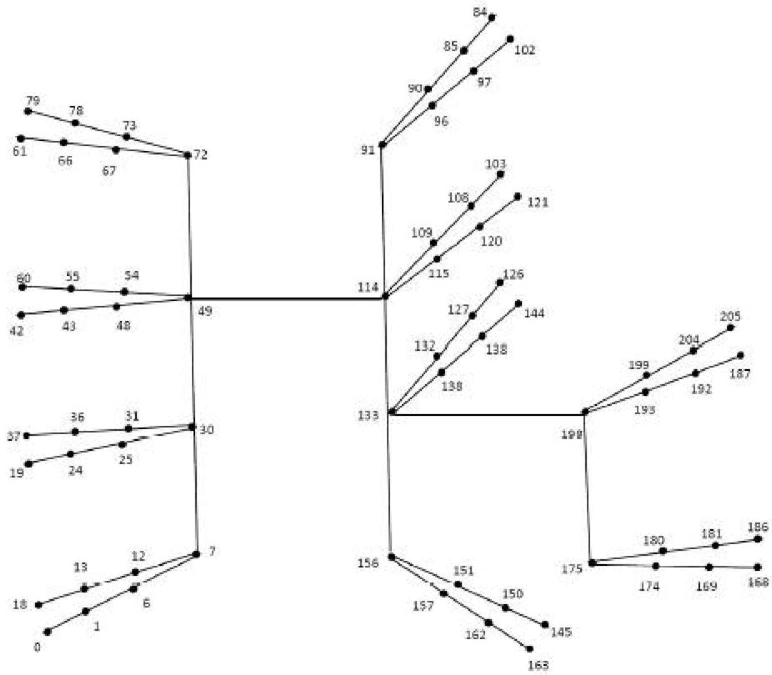

Figure 5

3. Conclusion

The concept of one modulo three mean labeling was introduced in [4]. In this paper we extend the study on one modulo three mean labeling and prove that graphs $T \odot \overline{K_n}, T\hat{o}K_{1,n}, T\hat{o}P_n$ and $T\hat{o}2P_n$ are one modulo three mean graphs.

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